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■■■■

CHAPTER 01

Introduction

1.1 Introduction

Concrete is one of the most common building material used for constructive civil engineering structures and is essential in the infrastructure development of any nation. Concrete is used in the construction of various types of structures which include buildings, bridges, piers, chimneys, pavements, dams, hydraulic structures, conveying pipes, liquid retaining tanks/structures, assembly halls, auditorium, swimming pools, bunkers etc. and the list goes on.

1.2 Types of Concrete

1.2.1 Plain Concrete

Concrete can be defined as a mass which is made from any cementing material and consists of sand, gravel and water. Mixing of such naturally occurring materials along with a cementing material result in a partial solid mass that can be molded in any shape and form, when wet, and which becomes hard on drying. Concrete is being used as a building material probably from the last 150 years.

Concrete is a highly successful building material and has gained wide popularity because of the following reasons:

1. Concrete is highly durable even under hostile environmental conditions.
2. It can be easily casted into any shape and size.
3. It is relatively cheaper and widely available.

The most important property of concrete is its compression resisting ability i.e. compressive strength, which supersedes any other building material. At present we have concrete grades ranging from 5 MPa to 100 MPa.

The major drawback of concrete is that it cannot resist significant tension. The tensile strength of concrete is about 10% of its compressive strength. Thus, the use of plain concrete as a building material is limited to places where tensile stresses/strains never develop. For example pedestals, mass concreting in dams etc.

1.2.2 Reinforced Concrete

Concrete has gained so much importance and popularity because of the development of reinforced concrete. Introducing the reinforcing bars in concrete makes the concrete an excellent composite building

material which can resist significant amount of tensile stresses/strains also. Construction of load bearing building elements like beams, slabs etc. is made possible due to the reinforced concrete only. Steel bars embedded in the tension zone of concrete make it able to take tension.

In reinforced concrete, strain compatibility is assumed to exist i.e. there exists a perfect bond between the concrete and steel bars so that strain in concrete is equal to the strain in steel at the interface of concrete and steel.

Moreover, since the failure of concrete is brittle in nature which takes place without giving any warning, introduction of steel in concrete makes it a ductile material which gives sufficient warning before collapse.

Now tensile stresses occur either directly (e.g. direct tension, flexural tension) or indirectly (e.g. shear which causes tension along the diagonal planes). Temperature and shrinkage effects may also induce tensile stresses. At all such locations, steel is invariably provided which is in fact inevitable, that passes across the tensile cracks. Insufficient steel causes propagation of cracks which can lead to complete failure.

Embedding reinforcing bars in compression zone of concrete increases the compressive strength of member (e.g. In columns, doubly reinforced beams etc.).

1.2.3 Prestressed Concrete

Development of prestressed concrete took place along with the reinforced concrete. It is a high strength concrete with high tensile wires embedded in concrete and tensioned before the application of actual working load. While doing so, the concrete can be compressed to such an extent that when the structure is actually loaded, there is almost no tension developed in the beam section. Prestressed concrete is frequently used where, even a hair line crack is not admissible like, high pressure vessels, pipes, water tanks etc. and at locations which are subjected to fatigue loading like long span bridges or rail sleepers etc.

1.3 Importance of Design Codes in the Design of Structures

Different countries have formulated their own codes for laying down the guidelines for the design and construction of structures. These codes came into picture after a collaborative effort of highly experienced structural engineers, construction engineers, academicians and other eminent fellows of respective areas. These codes are revised periodically based on current research and trends (e.g. IS 456: 1978 and IS 456: 2000). Codes serve the following objectives/purposes:

1. They ensure structural stability/safety by specifying certain minimum design requirements.
2. They make the task of a designer rather simple by making available results in the form of tables and charts.
3. They ensure a consistency in procedures adopted by the various designers in the country.
4. They protect the designer against structural failures that are caused by improper site construction practices i.e. codes have legal sanctity and one can have a stand on the basis of these design codes.

1.3.1 Basic Indian Standard Codes for Structural Design

Some of the basic Indian Standard codes for reinforced concrete published by the BIS (Bureau of Indian Standards) are:

1. IS 456: 2000 Plain and reinforced concrete-Code of practice.
2. IS 875: 1987 (Part-I to V) Code of practice for design loads.
3. IS 1893: 2002 Criteria for earthquake resistant design of structures.
4. IS 13920: 1993 Ductile detailing of reinforced concrete structures subjected to seismic forces.

1.4 Characteristic Strength of Concrete

Due to wide variation in the characteristics of concrete constituents (sand, coarse aggregates etc.), concrete is subjected to considerable variation in strength. Also, due to non-homogeneous nature of concrete, specimens taken from the same mix may give different compressive strengths in tests. This variation can be controlled by strict quality control and quality assurance.

Table 1.1: *Characteristic compressive strength compliance requirement (as per Cl. 16.1 and 16.3 of IS 456: 2000)*

Specified Grade	Mean of the Group of 4 Non-Overlapping Consecutive Test Results in N/mm²	Individual Test Results in N/mm²
M20 or above	≥ f_{ck} + 0.825 × Established standard deviation (rounded off to nearest 0.5 N/mm²) **or** f_{ck} +4 N/mm², whichever is greater	≥ f_{ck} − 4 N/mm²

Statistically, the variation in concrete strength is studied in terms of **standard deviation** and/or **coefficient of variation**.

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean strength}}$$

Experimentally, it is found that probability distribution of concrete strength (for a particular concrete mix as found out by compressive strength tests in laboratory on a large number of specimens) follows **normal/Gaussian distribution**. The **coefficient of variation varies generally in the range of 0.01 to 0.02**. With higher degree of quality control, this variation can be reduced.

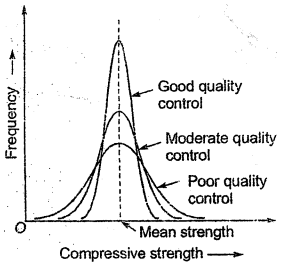


Fig.1.1 *Influence of quality control on the frequency distribution of concrete strength*

1. Strength of concrete in uniaxial compression is determined by testing a standard cube of 150 mm size and is loaded till its failure.
2. The cube specimen is tested after 28 days of casting and curing.
3. The strength of cube is always expressed nearest to 0.5 N/mm².
4. As per IS 456: 2000, there should be three specimens in a sample.
5. Strength of sample is expressed as an average of three specimens of the sample.
6. Individual variation in the strength of cubes should not vary by more than ±15% of average strength and if the variation is more then the test results are discarded.

Do You Know ?

Why it is needed to define characteristic strength of concrete?

Due to significant variation in the compressive strength of concrete (tested on concrete cube/cylinder specimens), it is quite essential to ensure that a certain minimum strength of concrete can always be obtained from a given mix. This is obtained by defining **Characteristic Strength** of concrete (which is applicable for other materials also).

Characteristic Strength is defined as that strength of the material below which not more than 5% of the tests results are expected to fall. This is shown in the Fig.1.2.

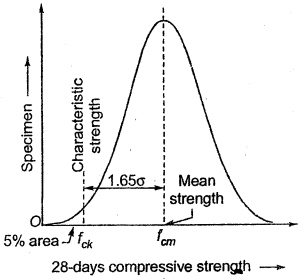


Fig.1.2 *Idealised normal distribution of concrete strength*

Thus, mean strength of concrete has to be significantly greater than the characteristic strength of concrete.

1.5 Grade of Concrete

The desired properties of concrete are its compressive strength, tensile strength, shear strength, bond strength, density, durability, impermeability etc. Among these properties, the most important property is the compressive strength of concrete. This is measured by standard tests on concrete cube/cylinder specimens. Many other properties of concrete can be inferred from its compressive strength.

The grade of concrete is expressed in terms of its characteristic compressive strength (of 150 mm cube at 28 days) expressed in N/mm^2 or MPa. e.g. Different grades of concrete are M20, M25, M30, M40 and so on.

Do You Know?

In the recent revised version of IS 456: 2000, minimum grade of concrete is based on considerations of durability and the type of environment that the structure is exposed to. Minimum concrete grade in RCC has been upgraded from M15 to M20 in IS 456: 2000. Minimum grade of concrete as per exposure conditions is shown in Table 1.2.

Table 1.2: *Minimum grade of concrete based on exposure conditions as per IS: 456*

Exposure Condition	Minimum Concrete Grade
Mild	M20
Moderate	M25
Severe	M30
Very Severe	M35
Extreme	M40

1.6 Concrete Mix Design

Design of concrete mix for a particular grade of concrete involves proper selection of relative proportions of cement, sand and coarse aggregates. While designing a concrete mix, it is always tried to obtain a minimum strength which is equal to characteristic strength of concrete but concrete must also have the desired workability (when fresh/green), impermeability and durability (in hardened state). Table 1.3 depicts the various grades of concrete based on concrete mix design. Concrete mix design is classified as nominal mix design and the design mix.

Table 1.3: *Various grades of concrete as per IS 456: 2000*

Concrete Grade	Type of Concrete
M10	Ordinary grade concrete
M15	
M20	
M25 - M55	Standard grade concrete
M60 - M100	High Strength concrete

Note: Provision of IS 456: 2000 do not apply to grades of concrete M60 and above.

1.6.1 Nominal Mix Design

Concrete mix design is a process that needs experience. Earlier, concrete mixes were specified in terms of fixed ratios like 1:2:4, 1:1.5:3 and so on for cement, sand and coarse aggregates respectively (by mass or by volume). This is a very rough method of concrete mix design which often gives wrong translations of concrete grades like M15, M20, M25, M30 etc.

IS 456: 2000 provides a more precise nominal mix proportions for M5, M7.5, M10, M15 and M20 grades of concrete in terms of total mass of aggregates, proportions of fine to coarse aggregates and volume of water to be used per 50 kg (i.e. 1 bag) of cement (which is in volume equal to 34.5 liters). Nominal mix concrete can only be used in ordinary concrete constructions involving concrete grade not higher than M20. For higher grades of concrete, design mix concrete is adopted.

Table 1.4: *Proportions of nominal mix concrete as per IS 456: 2000*

Concrete Grade	Weight of FA and CA in 50 kg of cement	FA:CA	Weight of water (in kg) per 50 kg (1 bag) of cement
M5	800	Generally 1:2 but subject to an upper limit of 1:1.5 and a lower limit of 1:25	60
M7.5	625		45
M10	480		34
M15	330		32
M20	250		30

FA : Fine Aggregate, CA : Coarse Aggregate

Traditional nominal mix of 1 : 2 : 4 (cement : sand : coarse aggregate, by weight) with 33 grade of OPC conforms approximately to M15 concrete grade. This nominal mix with higher grades of cement (43, 53 grades) yields higher grades of concrete (M 20 and above).

1.6.2 Design Mix Concrete

Design mix concrete is based on the principles of "mix design" and is always preferred over nominal mix of concrete. It yields concrete of desired quality and is more economical than the nominal mix. The IS recommendations of the mix design are given in IS 10262: 1982 and SP 23: 1982.

1.7 Steps Involved in Mix Design of Concrete as per IS Recommendation

Step 1. Determine the mean target strength (f_{cm}) from the desired characteristic strength (f_{ck}) as:

$$f_{cm} = f_{ck} + 1.65\sigma$$

where, σ is the standard deviation that depends on quality control as listed in Table 8 of IS 456: 2000. The same table has been reproduced here as Table 1.5.

Table 1.5: *Standard deviation for various concrete grades*

Concrete Grade	Assumed Standard Deviation (N/mm^2)
M10	3.5
M15	
M20	4
M25	
M30	5
M35	
M40	
M45	
M50	

Step 2. Determine the water-cement ratio based on 28 days strength of cement and the mean target strength of concrete. This ratio must not exceed the limits specified in Table 5 of IS 456: 2000 part of which is reproduced here as Table 1.6.

Step 3. Determine the water content based on requirements of workability. Select the type of proportion of fine and coarse aggregate (by mass) based on aggregate grading and type. Water requirement is usually in the range of 170-200 litres per cubic metre of concrete (without admixtures) and ratio of fine and coarse aggregates is generally taken as 1:1.5, 1:2, or 1:2.5.

Table 1.6: Minimum cement content and maximum water cement ratio based on exposure conditions

Exposure Condition	Minimum cement content (in kg/m^3)	Maximum free water cement ratio
Mild	300	0.55
Moderate	300	0.50
Severe	320	0.45
Very Severe	340	0.45
Extreme	360	0.40

Step 4. Determine the cement content (in kg/m^3) as:

$$\text{Cement Content} = \frac{\text{Water content}}{\text{Water-cement ratio}}$$

Cement content should not be less than that specified in Tables 4 and 5 of IS 456: 2000 for durability considerations.

Do you know? IS 456: 2000 restricts the use of cement beyond 450 kg/m^3 in order to control shrinkage and thermal cracks.

Step 5. Determine the masses of coarse and fine aggregates based on absolute volume principle as:

$$\frac{C}{\rho_c}+\frac{FA}{\rho_{FA}}+\frac{CA}{\rho_{CA}}+V_W+V_a=1$$

Here *C*, *FA* and *CA* denotes the masses of cement, fine aggregates (sand) and coarse aggregates respectively and ρ_c, ρ_{FA} and ρ_{CA} denotes the mass densities of cement, fine aggregates (sand) and coarse aggregates respectively.
V_w = Volume of water and V_a = Volume of air voids.

Step 6. Determine the weight of ingredients per batch based on capacity of the concrete mixer.

Example 1.1 Find the quantities of cement, fine aggregates (FA) and coarse aggregates (CA) for 1 m³ of concrete. The void ratio in cement is 60%, in FA is 40% and in CA is 44%.

Take Materials properties as:

Mix is 1 : 2 : 4 with water-cement ratio of 0.59. One bag of cement weighs 50 kg (neglecting the empty weight of bag) and density of cement is 1440 kg/m³. Density of FA is 1780 kg/m³ and coarse aggregate (CA) is 1650 kg/m³. Volume of one bag of cement is 34.7 litres. Assume volume of air in concrete as 3% per m³ of concrete.

Solution:

Since the proportion specified is 1 : 2 : 4 without saying anything that whether it is 'by weight' or 'by volume'. In such cases, it is always recommended to use minimum proportions 'by weight'.

Thus 1 : 2 : 4 ⇒ 1 kg cement, 2 kg FA, 4 kg CA.

Now Bulk density (γ) = $\frac{\text{Mass}}{\text{Total volume}}=\frac{M}{V}$

and Mass density (γ_m) = $\frac{\text{Mass}}{\text{Volume of solid content}}=\frac{M}{V_s}$

$\therefore$ $\gamma=\frac{M}{V}=\frac{M}{V_s+V_v}$ where, V_v = Volume of voids

$=\frac{M/V_s}{1+V_v/V_s}=\frac{\gamma_m}{1+e}$

Now for cement, $e = 0.6$

⇒ Mass density of cement, $\gamma_{mc}=\gamma_c(1+e)=1440\,(1+0.6)=2304$ kg/m³

For fine aggregates, $e = 0.4$

⇒ Mass density of fine aggregates, $\gamma_{mFA}=\gamma_{FA}(1+e)=1780(1+0.4)=2492$ kg/ m³

For coarse aggregates, $e = 0.44$

Mass density of coarse aggregates, $\gamma_{mCA}=\gamma_{CA}(1+e)=1650(1+0.44)=2376$ kg/ m³

Let Volume of concrete = 1 m³ and mass of cement in 1 m³ concrete = x kg

$\therefore$ Volume of air = 3% of volume of concrete = 0.03 m³

Now Volume of concrete = Vol. of cement + Vol. of FA + Vol. of CA + Vol. of air + Vol. of water

⇒ $1=\frac{x}{2304}+\frac{2x}{2492}+\frac{4x}{2376}+0.03+\frac{0.59x}{1000}$

⇒ $x = 276.35$

$\therefore$ Mass of cement in 1 m³ concrete = x = 276.35 kg

Mass of water in 1 m³ concrete = $0.59x$ = 163.05 kg

Mass of fine aggregates in 1 m³ concrete = $2x$ = 552.7 kg

Mass of coarse aggregates in 1 m³ concrete= $4x$ = 1105.4 kg

Example 1.2 Find the quantities of cement, sand and coarse aggregates in 1 m³ of concrete for a mix proportion of 1 : 1.15 : 2.5 (by volume). The water cement ratio required is 0.56 (by weight).

Bulk density of cement = 1500 kg/m³
Bulk density of sand = 1780 kg/m³
Bulk density of coarse aggregates = 1650 kg/m³
Assume volume of entrained air per cubic meter of concrete as 3%.
Specific gravity of cement = 3.15
Specific gravity of sand = 2.65
Specific gravity of coarse aggregates = 2.3

Solution:

Mix proportion is 1 : 1.15 : 2.5 (by volume)

Let volume of cement per m³ of concrete = x m³

$\therefore$ Volume of cement : FA : CA = $x : 1.15x : 2.5x$ (m³)

Also, Water-cement ratio = 0.56

⇒ $\frac{\text{Mass of water}}{\text{Mass of cement}}=0.56$

Now, bulk density of cement, γ_c = 1500 kg/m³

Bulk density of sand, γ_{FA} = 1780 kg/m³

Bulk density of coarse aggregates, γ_{CA} = 1650 kg/m³

$\therefore$ x m³ of cement ≡ $1500x$ kg of cement

$1.15x$ m³ of sand ≡ $1780(1.15x)$ kg of sand

$2.5x$ m³ of coarse aggregates ≡ $1650(2.5x)$ kg of coarse aggregates

and Mass of water = 0.56 times of mass of cement

= 0.56 $(1500x)$ kg of water per m³ of concrete

$\therefore$ Volume of water = $\frac{0.56(1500x)}{1000}$ m³ ($\because \gamma_w = 1000$ kg/m³)

Given, volume of air = 0.03 m³ per m³ of concrete

$\therefore$ Volume of concrete = Vol. of cement + Vol. of sand + Vol. of coarse aggregates + Vol. of air + Vol. of water

⇒ $1=\frac{1500x}{3.15\times1000}+\frac{1780(1.15x)}{2.65\times1000}+\frac{1650(2.5x)}{2.3\times1000}+0.03+\frac{0.56(1500x)}{1000}$

$$\Rightarrow \quad 0.97 = 0.4762x + 0.7725x + 1.793x + 0.84x$$

$$\Rightarrow \quad x = 0.25$$

$$\therefore \quad \text{Weight of cement} = 1500x = 375 \text{ kg}$$

$$\text{Weight of sand} = 1780\,(1.15x) = 511.75 \text{ kg}$$

$$\text{Weight of coarse aggregates} = 1650\,(2.5x) = 1031.25 \text{ kg}$$

$$\text{Weight of water} = 0.56\,(1500x) = 210 \text{ kg}$$

1.8 Behaviour of Concrete under Uniaxial Compression

Strength of concrete is determined by the compressive strength test on a standard 150 mm concrete cube in a compression testing machine as per IS 516: 1959. The test specimens are generally tested after 28 days of casting and continuous curing. The loading is **strain-controlled** and load is generally applied at a uniform **strain rate of 0.001 mm/mm/minute**. The maximum stress attained in this loading process is called as **cube strength** of concrete.

Remember

In USA, instead of 150 mm cubes, standard test cylinders of height to diameter ratio of 2, e.g. 150 mm diameter and 300 mm height cylinders are used. **Cylinder strength comes out to be lower than the cube strength of concrete** for the same quality of concrete.

1.8.1 Influence of Specimen Size on Strength of Concrete

Height to width ratio of the specimen and the cross sectional dimensions of the specimen affects the compressive strength (maximum stress attained) obtained from compression testing machine.

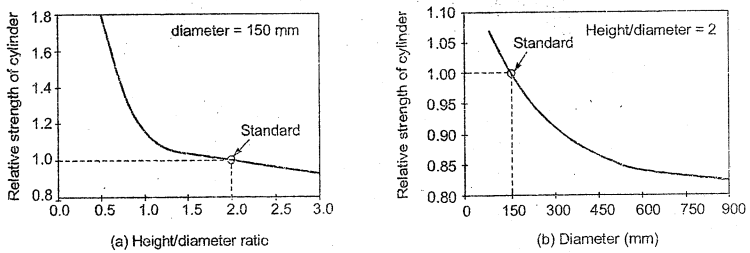


Fig.1.3 *Influence of (a) height/diameter ratio and (b) diameter on cylinder strength*

In case of cylindrical test specimen of 150 mm diameter and height to diameter ratio of 2, the compressive strength of the concrete specimen increases to about 80% as the height/diameter ratio is reduced from 2 to 0.5. With the same height/diameter ratio being equal to 2, the strength decreases by about 17% as the diameter of the cylindrical specimen is increased from 150 mm to 900 mm.

The loading plates and the top/bottom surface of the concrete specimen offers some frictional resistance called as platen restraint which introduces shear stress at the top and bottom surfaces of the specimen and this effect diminishes as the distance between the platen surfaces increases.

For this reason, standard concrete cube (height/diameter ratio equal to 1) specimen gives higher compressive strength than the cylindrical specimen (height/diameter ratio equal to 2).

Remember: Cube strength of concrete is nearly 1.25 times the cylinder strength.

1.8.2 Stress Strain Curve of Concrete in Compression

These stress strain curves of concrete for various grades are somewhat linear in the initial phase of loading. The non-linearity in the curve becomes quite significant when the stress in concrete reaches to about one third to one half of the maximum value.

For concrete, the maximum stress is reached at a strain which is approximately equal to 0.002, beyond which an increase in strain is accompanied by a decrease in stress. Usually at failure, the strain in concrete ranges from 0.003 to 0.005.

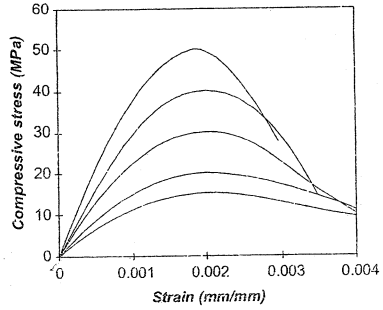


Fig.1.4 *Typical stress-strain curves of concrete in compression*

1.9 Behaviour of Concrete in Tension

Concrete is quite weak in tension and is not supposed to and also not designed to take any direct tension. But tensile stresses occur in concrete due to flexure, shear, shrinkage and temperature stresses etc. Pure shear causes tension on diagonal planes for which sufficient knowledge of direct tensile strength of concrete is important for assessing the shear strength of beams with unreinforced webs. For assessing the moment at first crack, the knowledge of flexural tensile strength of concrete is necessary. The direct tensile strength of concrete is about 7 to 15 % of the compressive strength of concrete.

It is difficult to have direct tension test on concrete as it requires pure axial tensile stress free from any secondary stresses and misalignment in the tension testing machine. Thus indirect tension tests are performed using **flexure test** or **cylinder splitting test**.

1.9.1 Modulus of Rupture of Concrete

While performing flexure test, a standard simply supported plain concrete beam of square or rectangular section is used, which is subjected to three points loading until failure. For a linear stress distribution across the section, the theoretical maximum tensile stress which is developed in the extreme fiber is called as **modulus of rupture** (f_{cr}) **of concrete**. It is given by,

$$f_{cr} = \frac{M}{Z}$$

Where, M is the bending moment that causes failure and Z is the section modulus.

But the actual stress distribution across the section is not linear. IS 456: 2000 recommends the use of following formula for modulus of rupture of concrete (f_{cr}):

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

where, stress units are in MPa or N/mm^2.

1.9.2 Splitting Tensile Strength of Concrete

Owing to limitations of direct tensile strength test of concrete, cylinder splitting test is performed which gives more uniform results. In this test, a standard plain concrete cylinder (as used in compression test) is

loaded on its sides along a diameter. Failure occurs by splitting of the cylinder along the plane of loading. This type of loading produces a uniform tensile stress across the plane of loading.

The splitting tensile strength (f_{ct}) is obtained as:

$$f_{ct} = \frac{2P}{\pi dL}$$

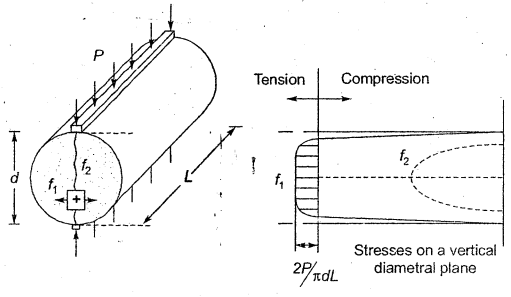


Fig.1.5 *Cylinder splitting test for tensile strength*

Where,

P is the maximum load applied at failure, d is the diameter of the cylinder specimen, L is the length of the cylinder specimen.

Remember IS 456: 2000 does not provide any empirical formula for splitting tensile strength (f_{ct}) as it does for modulus of rupture of concrete (f_{cr}). For normal density concrete, the splitting tensile strength is about 2/3rd of the modulus of rupture of concrete.

1.9.3 Stress Strain Curve of Concrete in Tension

Concrete is very weak in tension and thus has low failure strain in uniaxial tension, being in the range of 0.001 to 0.002. The stress - strain curve of concrete in tension is approximated as a straight line from origin to the point of failure. The modulus of elasticity of concrete in tension is taken to be the same as that of compression. Since the tensile strength of concrete is very low and is invariably ignored in design, the tensile stress-strain relationship of concrete is of very minimal practical value.

1.10 Modulus of Elasticity and Poisson's Ratio of Concrete

Concrete is not a truly elastic material i.e. it cannot regain its original shape from the deformed shape after removal of loading. It is not only inelastic, it is non-linear also i.e. stress – strain relationship is not linear. Thus, conventional elastic constants (like modulus of elasticity, Poisson's ratio etc.) cannot be defined for a non-linear, inelastic material like concrete. Even though we have to define these constants for concrete in order to make possible the analysis of structures which are in fact based on linear elastic analysis.

The Young's modulus of elasticity is the ratio of axial stress to axial strain under uniaxial loading within the linear elastic range and it is a constant. For concrete under uniaxial compression, this is valid only in the very initial portion of the stress - strain curve which is nearly linear i.e. loading is of low intensity and of short duration. Sustained loading introduces the effect of creep even at low loading intensities.

In situations, where long term effects of loading on the structure are negligible like wind or earthquake loading, we define initial tangent modulus which is in fact a measure of dynamic modulus of elasticity of concrete. In the usual linear static analysis, we define static modulus of elasticity. When loads on the structure are of long duration (dead and live loads), the long term effects of creep decreases the effective modulus of elasticity of concrete.

It is difficult to separate the long term strains due to creep and shrinkage from short term elastic strains. Thus, while estimating deflection of reinforced concrete beam, the total deflection is assumed to be the sum of instantaneous elastic deflection (due to loads) and long term deflections due to creep and shrinkage. Short term static modulus of elasticity of concrete (E_c) is used in computing the instantaneous elastic deflection.

Various definitions of *modulus of elasticity of concrete* (E_c) are there. Some of them are initial tangent modulus, tangent modulus (at specified stress level), secant modulus (at specified stress level) etc. Secant modulus at a stress level of about 1/3rd the cube strength of concrete is generally found acceptable in expressing the average value of E_c under the usual service load conditions.

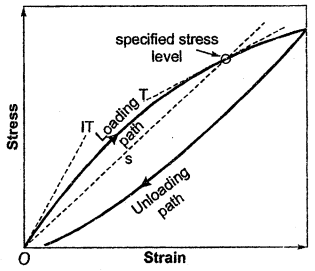


Fig.1.6 *Various descriptions of modulus of elasticity of concrete*

(IT = Initial tangent, T = tangent, s = secant)

As per IS 456: 2000, short term static modulus of elasticity of concrete (E_c, in MPa) is given in terms of characteristic strength of concrete as:

$$E_C = 5000\sqrt{f_{ck}}$$

Do you know? Earlier version of code i.e. IS 456:1978 gave this value as $5700\sqrt{f_{ck}}$, which was an over estimation of the elastic modulus.

1.10.1 Poisson's Ratio

It is the ratio of lateral strain to the longitudinal strain under uniform uniaxial stress. For concrete, a wide range of values of Poisson's ratio are obtained which range from 0.1 to 0.3. For design purposes, a value of 0.15 or 0.2 is considered.

1.11 Effect of Duration of Loading on Stress Strain Curve

The standard compression test gets over within 10 minutes with loading being applied at a uniform strain rate of 0.001 mm/mm/minute. When load is applied at a faster strain rate (as impact loading), the modulus of elasticity and strength of concrete increase but the failure strain decreases. When the load is applied at a slower rate (duration greater than 10 minutes to as long as 1 year or more) then there is a slight decrease in the modulus of elasticity and significant increase in the failure strain.

1.12 Creep of Concrete

Creep of concrete is covered in more detail in forth coming chapters. At present, it is worth to note that creep of concrete is having the following ill effects on concrete structures:

1. It increases the deflections of certain concrete elements like beams and slabs.
2. It increases the deflection of very long / slender columns.
3. It slowly transfers the load from concrete to reinforcing steel over a period of time.
4. It causes loss of prestress in prestressed concrete members.

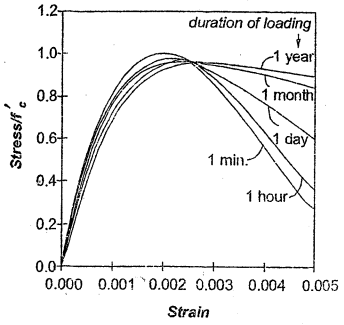


Fig.1.7 *Influence of duration of loading (strain-controlled) on the stress-strain curve of concrete*

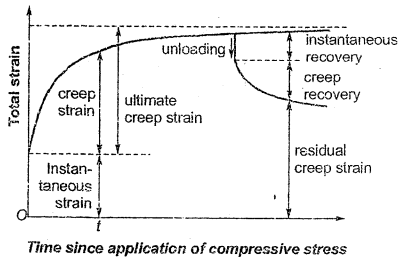


Fig.1.8 *Typical strain-time curve for concrete in uniaxial compression*

Following factors influence the creep of concrete:

1. High cement content increases the creep of concrete.
2. High water-cement ratio increases creep of concrete.
3. Creep increases when aggregate content is low.
4. It increases when air entrainment is high in concrete.
5. Low relative humidity increases creep.
6. Small size/thickness of members show large amount of creep.
7. Early loading of concrete members increases creep.
8. Long term sustained loading increases creep of concrete.

Long term sustained loading on concrete at a constant stress results in creep strains and a decrease in the compressive strength of concrete.

1.13 Compressive Strength of Concrete in the Design of Structures

The compressive strength of concrete in an actual structure cannot be exactly same as compressive strength obtained by uniaxial compression test in laboratory even for the samples obtained from the concrete mix which is used in actual construction. This is because of the effect of loading duration, member sizes and strain gradient.

The maximum strength of concrete is taken as 0.85 times the specified cylinder strength for the design of RCC members (for both in compression and flexure/bending). This comes out to be approximately equal to 0.67 times the characteristic cube strength. IS 456: 2000 also limits the failure strain to 0.002 in direct compression and 0.0035 in flexure.

Remember



When the predominant loading on the structure is of short term nature (like wind or earth quake loading on towers, chimneys etc.) and NOT the sustained loading then it is too conservative to limit the compressive strength of concrete to 0.85 times the cylinder strength or 0.67 times the characteristic cube strength. In such cases, a suitable higher value of compressive strength is taken.

CHAPTER 02

Basic Design Concepts

2.1 Introduction

Concrete is such a wonderful construction material that an *Italian architect* once said:

"Concrete has liberated us from the rectangle".

It is because of the flexibility and mouldability property of concrete (when wet) that we are now having structure of any shape and size. It has replaced the conventional building materials like stone, timber and steel. Any reinforced concrete structure (be it simple or complex), is an assembly of basic structural elements like beams, columns, footings, slabs and walls. Thus, design of reinforced concrete structures lies in the design of these basic structural elements i.e. the design of beams, columns, slabs, walls, footings etc. Numerous theories were proposed in the last century for the design of concrete structures. In this chapter, we will look into the various theories for design of concrete structure.

2.2 Necessity of Designing Reinforced Concrete Structures

The principal aim of structural design is that the structures should perform its intended function *safely* at ultimate loads within their life time and also *serviceable* at service or working loads.

The term 'safety' includes the following parameters:

1. Reserve strength of the material of the structure.
2. Limited or permissible deformation(s) in the structure.
3. Durability of the structure in the long run.

Thus, safety implies that the possibility of the failure of the structure (partial or complete failure) is very low even at ultimate loads considering appropriate factor of safety.

Serviceability implies that the structure should perform its intended function very well at working loads. It includes deflection, vibration, crack widths, durability, permeability, acoustics, thermal insulation etc.

Thus, the basic objectives to fulfil serviceability criteria can be summarized as:

1. Properly designed structures should perform its intended function at service loads quite satisfactorily.
2. Structure should bear all the loads and should deform within the permissible limits.
3. The structures should be durable enough against the adverse environmental conditions.

4. The designed structure must adequately resist the probable hazardous effects of structural misuse and fire.

Remember

Increasing the factor of safety in the design of structures increases the safety and serviceability of the structure but at the same time it increases the cost of structure also. Here comes the role of economy in the design of structures. While considering the overall economy of a structure, the increased cost associated with the increased factor of safety must be properly weighed against the possibility of structural failure.

2.2.1 Objectives of Structural Design

The rational design of a structure must satisfy the following requirements:

1. **Stability:** The structure must be stable enough to resist the failure of structure in terms of overturning, sliding, buckling of the structure or parts thereof under the severe action of loads viz. both permanent (dead load) and temporary (superimposed live load etc.).
2. **Strength:** The structure must be able to carry safely the stresses induced by the severe most possible combination of loads acting on the structure.
3. **Serviceability:** The structure must perform its intended function i.e. it must be serviceable. This implies that the deflections, vibrations, crack-widths, permeability to water etc. are within the permissible limits.
4. **Aesthetics:** The structure must be in harmony with the surroundings and should look pleasing. It is purely an architectural consideration.
5. **Economy:** At last, economy plays the most important role in structural design. The cost of the structure and its associated facilities must not be so gargantuan that it may dictate the overall functional requirement of the structure.

Do You Know?

It is a very common misconception that use of more cement in concrete is better provided the cost of cement is not a constraint. But this is NOT TRUE. Excessive use of cement in concrete leads to cracking of concrete because large amount of heat of hydration is generated from the hydration of cement in concrete and also due to plastic shrinkage of cement paste. It also leads to long term effects of creep and drying shrinkage resulting in large deflections and cracking in the structure.

2.3 Hydraulic and Non-Hydraulic Cements

Cements obtained from the calcination of gypsum and lime stones are **non-hydraulic** because the products of hydration of this type of cement are not resistant to water however the addition of pozzolanic materials in these cements make them **hydraulic**.

2.4 Tests on Cement

Production of quality concrete primarily depends on the quality of cement used for making the concrete. Tests on cement are conducted keeping in view the recommendations of IS 269: 1976 and IS 4031: 1988 in order to assess the following:

1. **Chemical composition:** It is done to determine the oxides of calcium, silica, aluminium, iron, sulphur and magnesium and to check whether these impurities are within the permissible limits.

2. **Fineness of cement:** It measures the size of cement particles expressed in terms of specific surface (defined as surface area per unit mass). Higher the fineness of cement, higher is the hydration rate of cement leading to early development of strength.
3. **Consistency:** It is done to determine the quantity of water to be mixed in order to produce a standard cement paste.
4. **Initial and final setting time:** It measures the rate at which cement paste solidifies. Initial setting time indicates the time after which cement paste becomes unworkable while the final setting time indicates the time to reach the state of complete solidification of cement paste. It is measured using Vicat's apparatus.
5. **Soundness :** It indicates the quality of cement paste by virtue of which it does not undergo large change in volumes.
6. **Strength :** It is measured by performing tests on hardened cement concrete/cement paste and is expressed in terms of stress at failure of the specimen subjected to compression and tension tests.

Do You Know?

Earlier the strength of cement was specified as 7 days strength but later on it was found that many other types of cement like the Rapid Hardening Portland Cement gains strength much earlier than the strength corresponding to 7 days strength of other types of cement however, the ultimate strength of both the types of cement (viz. rapid hardening cement and ordinary Portland cement) at the end of 28 days is more or less same. Most of the types of cement gain a significant amount of long term strength at about 28 days only.

2.5 Methods to Increase the Durability of Concrete against Chemical Attack

By reducing the permeability of concrete in following ways:

1. Use of high strength concrete
2. By proper curing of concrete
3. By having a low water cement ratio
4. Using improved quality of admixtures
5. By having the maximum possible compaction of concrete
6. By the use of well graded, dense aggregates
7. By minimizing the possibility of cracks at the design phase itself

By providing protection to reinforcing steel in following ways:

1. By providing adequate clear cover to reinforcing steel as per relevant stipulations of IS codes.
2. Using coated steel or suitable corrosion resistant steel.
3. By preventing the corrosion of steel by the methods like sacrificial protection etc.
4. By the use of suitable corrosion resistant cement.
5. Avoid use of alkali reactive aggregates.
6. By preventing ponding on roof slabs and on other concrete surfaces by proper drainage of water.
7. By having a proper control on chloride and sulphate content in the concrete mix.

2.6 Design Philosophies for the Design of Reinforced Concrete Structures

Numerous design philosophies have been proposed in the last century for the design of reinforced concrete structures. Every **design philosophy** is based on certain assumptions.

Cl. 18.2 of IS 456: 2000 accepts three methods of design of reinforced concrete structures viz. *Limit State Method, Working Stress Method* and *Method based on experimental approach.* **Working Stress Method** is the earliest codified design philosophy which is based on linear elastic theory. This method is now out dated and in the latest version of IS 456: 2000, the provisions of *working stress method* of design has been shifted from the main text of the code to **Annexure 'B'** so as to have larger emphasis on *Limit State Method.*

Ultimate Load Method of design is based on the strength of concrete at ultimate loads. This method was in fact a replacement of traditional WSM of design which was incorporated in the **IS: 456** in the year 1964.

Later on it was realized that there involves a lot of uncertainty in the precise estimation of loads and material properties. Every time designing a structure for the most possible severe combination of loads make the structure highly safe which in fact may not be subjected to that much loads in actual life and at the same time it increases the cost and works out to be uneconomical. However, these uncertainties can be handled by the **theory of probability.** The risk associated with the design of structure was quantified in terms of **probability of failure** of the structure. This type of probability based method is called as **reliability based method.** The major drawback of these methods is that they are highly complicated in terms of mathematical calculations.

In order to overcome the above stated problem of reliability based methods, instead of determining the actual **probability of failure** of the structure, we introduce **many (partial) safety factors.** The European Committee for Concrete **(CEB)** and the International Federation of Prestressing **(FIP)** were among the first to introduce the concept of **Limit State Method of Design,** which is a **reliability based method.** Later on, based on the recommendations of **CEB-FIP,** the limit state method of design got introduced in the British Code (BS-8110-1997) and in the **Indian Code (IS 456:1978).**

2.6.1 The Working Stress Method (WSM) of Design

It is the most traditional method for the design of **reinforced concrete, structural steel and timber.** The basic design philosophy of this method is:

1. The material behaves in a linear elastic manner.
2. Adequate safety can be ensured by restricting the stresses in the materials that are induced due to the application of working loads/service loads.

Now the question arises:

"How to justify the assumption that material behaves in a linear elastic manner?"

The answer to the above question lies in the second assumption, where the allowable stresses are kept well below the strength of material i.e. the allowable stresses lie in the linear phase of stress-strain curve so the assumption of linear elastic behavior of material is justified.

The second assumption as stated above also introduces the concept of **factor of safety** which is expressed as:

$$\text{Factor of safety} = \frac{\text{Strength of the material}}{\text{Allowable stress in the material}}$$

The stresses induced due to the application of service or working loads are determined using the principles of **Strength of Materials.** But reinforced concrete is a **composite material** having two altogether different materials viz. concrete and steel. To apply the principles of Strength of Materials to a composite structure like reinforced concrete, the concept of **strain compatibility** is employed wherein it is assumed that there exists a perfect bond between the two materials (steel and concrete) and that the strain in steel is equal to the strain in the surrounding concrete due to that perfect bond.

It is quite evident that the stress is related to the strain and thus it follows that stress in steel is related to the strain in steel. But strain in steel is equal to the strain in surrounding concrete. Therefore, the stress in steel is indirectly related to the strain in the surrounding concrete. This indirect relation (between the steel and concrete) is expressed in terms of **modular ratio (m)** as:

$$\text{Modular Ratio} = \frac{\text{Stress in steel } (f_s)}{\text{Stress in concrete } (f_c)}$$

$$= \frac{\text{Strain in steel } (\varepsilon_s) \times \text{Modulus of elasticity of steel } (E_s)}{\text{Strain in concrete } (\varepsilon_c) \times \text{Modulus of elasticity of concrete } (E_c)}$$

$$= \frac{E_s}{E_c} \qquad (\text{Since } \varepsilon_c = \varepsilon_s)$$

Limitations of WSM of design

1. It fails to give relative importance to the different types of loads that act on a structure i.e. dead loads, live loads, snow load, seismic loads, temperature load etc. All of these act on the structure with different uncertainties. This often leads to over conservative designs and sometimes under conservative designs (when two loads are acting oppositely i.e. dead and wind loads, live and seismic loads etc.).
2. WSM of design gives large sections (compared to LSM and ULM) of the designed reinforced concrete members.

Remember



Although WSM of design has been put at annexure in IS 456: 2000 and superseded by LSM, it still remains the accepted method of design for certain type of structures like reinforced concrete water tanks **(IS 3370),** chimneys **(IS 4998)** and bridges **(IRC-21).**

2.6.2 The Ultimate Load Method (ULM) of Design

This method is an improvement over the traditional WSM of design and takes into account the shortcomings of the earlier method. The ULM is also called as the **load factor method** or the **ultimate strength method.** In this method, the non-linear stress strain behavior of steel and concrete are accounted for and stresses induced in the structure at the verge of failure at ultimate loads are considered. The problems associated with the modular ratio (m) are entirely avoided in this method. The safety in the design of structure is taken care by the concept of **load factor** which is expressed as:

$$\text{Load Factor} = \frac{\text{Ultimate or design load}}{\text{Working or service load}}$$

This concept of load factor makes it possible to assign different factors of safety (in terms of load factors) to different types of loads (like dead loads, live loads, seismic loads, wind loads, snow loads etc.) and can be suitably combined; which was a major drawback in the WSM of design.

Limitations of ULM of design

1. The major drawback of this method is that, one cannot say with 100% assurance that if a structure performs well at ultimate loads (strength), the same structure will perform its function satisfactorily at service loads also (serviceability).
2. Another drawback of this method is that, the assumed nonlinear stress strain behavior of concrete and steel is relevant only if nonlinear analysis is performed on the structure. But nonlinear analysis of structures is too cumbersome to be done for routine type of structures.

For assessing the distribution of stresses at ultimate loads, we use the stress distribution at service loads magnified by the load factor, but this approach is NOT correct owing to the fact that as the loading is increased from the service load level to the ultimate load level, significant inelastic and nonlinear behavior of materials comes in with considerable stress redistribution.

2.6.3 The Limit State Method (LSM) of Design

The earlier design methods includes the working stress method (WSM) and the ultimate load method (ULM) of design. **WSM is based on service loads conditions** alone whereas the **ULM is based on ultimate load conditions** alone. However, LSM takes into account the safety at ultimate load and serviceability at service loads. LSM employs different safety factors at ultimate loads and service loads. These multiple safety factors are based on probabilistic approach with separate approaches for each type of failure, type of materials and types of loads.

Limit State: Limit state is the state of 'about to collapse' or 'impending failure', beyond which, the structure is not of any practical use i.e. either the structure collapses or becomes unserviceable. In LSM, two types of limit states are defined which are:

1. **Limit state of collapse**: This limit state deals with the strength of the structure in terms of collapse, overturning, sliding, buckling etc.

   Various limit states of collapse are:

   - Flexure
   - Compression
   - Shear
   - Torsion

2. **Limit state of serviceability**: This limit state deals with the deformation of the structure to such an extent that the structure becomes unserviceable due to excessive deflection, cracks, vibration, leakage etc.

   Various limit states of serviceability are:

   - Deflection
   - Excessive vibrations
   - Corrosion
   - Cracking (Do not consider the tensile strength of concrete)



If a structure has attained the limit state of serviceability and then the loads are removed, then the structure will return to its original state. However, if the structure has attained the limit state of collapse, then the structure will not return to its original shape.

2.7 Load and Resistance Factor Design

Load and resistance factor design (LRFD) is the simplest of all the design methods that are currently in use. According to this method, the following condition must be satisfied:

$$\text{Design resistance} \geq \text{Design load}$$

$$\phi R_n \geq \gamma S_n \quad \ldots(i)$$

Where, R_n and S_n are nominal/characteristic values of resistance and load effects respectively and ϕ, γ represent the **resistance factor** and **load factor** respectively.

Resistance factor (ϕ) takes into account the possible uncertainties in the determination of material characteristics like Poisson's ratio, modulus of elasticity etc. and is less than unity whereas the load factor (γ) takes into account the possible overloading of the structure and is greater than unity.

From eq.(i), it can be inferred that,

$$S_n \leq \frac{R_n}{(\gamma/\phi)}$$

The above safety concept is being adopted by **Working Stress Method (WSM)** of design. Here (γ/ϕ) represents the **factor of safety** which is applied to material strength to arrive at the permissible stresses.

From eq.(i), it can also be inferred that,

$$R_n \geq \frac{S_n}{(\phi/\gamma)}$$

The above safety concept is adopted in **Ultimate Load Method (ULM)** of design. Here, (ϕ/γ) represents the **load factor** which is applied to loads in order to arrive at ultimate load.

2.7.1 'Multiple Partial Safety Factor' Method of Design

This method is adopted by the IS 456: 2000 and is expressed as:

$$R_d \geq S_d$$

where, R_d is the design resistance of the material which is computed at little bit lower material strength ($0.67 f_{ck}/\gamma_c$ for concrete and f_y/γ_s for steel). Thus, it involves two partial safety factors viz. for concrete and for steel. S_d is the design load which is calculated at the enhanced load effect involving separate safety factors for dead load, live load, earthquake load, wind load etc.

2.8 IS 456: 2000 Recommendations for LSM of Design

The salient features of the limit state method (LSM) of design as described in the code (**IS 456: 2000**) are as follows:

1. **Characteristic Strength and Characteristic Load:**

   Characteristic strength of a material is that strength below which not more than 5% of the test results are expected to fall i.e. not more than 1 in 20 test results should fall below this characteristic value. For concrete, this value is called as *characteristic strength of concrete* and for steel, this value is called as *yield strength of steel.*

   For Example: For M30 concrete, characteristic strength (f_{ck}) is 30 N/mm^2 and for Fe415, characteristic strength or yield strength (f_y) is 415 N/mm^2.

NOTE: M5 and M7.5 grades of concrete are lean concrete mixes used for simple bases and foundation of masonry walls.

IS 456: 2000 does not allow concrete grade lower than M20 in reinforced concrete and this grade of concrete should not be lower than M30 in coastal areas.

Characteristic load (Cl. 36.2 of IS 456: 2000) is that load which has the 95% probability of not being exceeded during the life time of the structure.

2. **Partial safety factor for materials:** The design strength of concrete and steel are obtained by dividing the respective characteristic strengths by appropriate partial safety factors for concrete

and steel. For concrete, f_{ck} is the characteristic cube strength and in actual structures, the characteristic strength of concrete is taken as $0.67f_{ck}$. Thus, **design strength of concrete is $0.67f_{ck}/\gamma_c$**. Similarly, **design strength of steel is f_y/γ_s**.

At ultimate loads i.e. **at ultimate limit state**, partial safety factor for concrete $\gamma_c = 1.5$ and partial safety factor for steel $\gamma_s = 1.15$. A higher partial safety factor is assigned for concrete because there is greater possibility of variation in quality of concrete than steel, particularly in fields.

$$\text{Thus, permissible stress in concrete} = \frac{0.67\ f_{ck}}{1.5} = 0.446\ f_{ck}$$

$$\text{and permissible stress in steel} = \frac{f_y}{1.15} = 0.87f_y$$

At serviceability limit states, $\gamma_c = \gamma_s = 1$. It is taken as unity because we are interested in actual deflections and crack widths at service loads rather than ultimate deflections and crack widths at ultimate loads.

3. **Partial safety factor for loads: IS 456: 2000** recommends the following load combinations for Dead Loads (DL), Live Loads (LL) and Earthquake Load (EQ) for assessing the ultimate load:
   (a) 1.5(DL + LL)
   (b) 1.5(DL + EQ)
   (c) 0.9 DL + 1.5 EQ
   (d) 1.2(DL + LL + EQ)

   In the last load combination i.e. 1.2 (DL + LL + EQ), a reduced load factor of 1.2 instead of 1.5 is used since it is quite rational to assume that all the three loads will not act at their peak values simultaneously during the event of an earthquake. For the purpose of structural design, the design load resistance with partial safety factors for materials should be greater than the maximum load effect that arises from the above load combinations.

   For serviceability conditions, IS:456 recommends the following load combinations:
   (a) 1.0 (DL + LL)
   (b) 1.0 (DL + EQ)
   (c) DL + 0.8 LL + 0.8 EQ

Table 2.1 : *Values of partial safety factors for loads (Table 18 of IS 456: 2000)*

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL/IL	WL or EQ	DL	LL/IL	WL or EQ
1. DL + LL	1.5	1.5	–	1.0	1.0	–
2. DL + WL or EQ	1.5 or 0.9*	–	1.5	1.0	–	1.0
3. DL + LL + WL or EQ	1.2	1.2	1.2	1.0	0.8	0.8
(*) This value is considered for stability against overturning or where there is a possibility of stress reversal.						

Design Load = Partial safety factor for load × Characteristic Load

Remember: EQ and WL are not considered simultaneously in design.

4. **Design stress-strain curve for concrete:** **IS 456: 2000** recommends the following characteristic and design stress strain curve in flexural compression for concrete as shown in Fig.2.1.

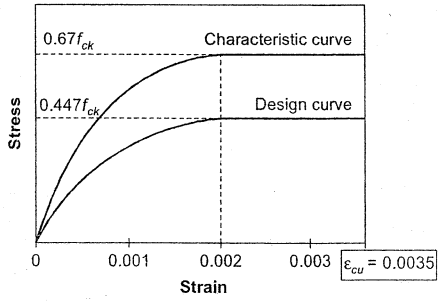


Fig.2.1 *Characteristic and design stress strain curve of concrete in flexural/bending compression*

Maximum strength in characteristic curve is $0.67f_{ck}$ for concrete structures. (Characteristic strength for concrete cube is f_{ck}). For limit state design, a partial safety factor for concrete $\gamma_c = 1.5$ is applied to characteristic strength of concrete ($= 0.67f_{ck}$). Thus **design stress strain curve of concrete is obtained by dividing the ordinates of characteristic stress strain curve of concrete by partial safety factor of concrete γ_c (=1.5)**. Thus maximum stress in design stress strain curve of concrete is $0.67f_{ck}/1.5 = 0.446f_{ck}$.

For uniformly compressed concrete (as in axially loaded columns), the maximum strain in concrete is taken as 0.002 or 0.2% and the corresponding maximum design stress of concrete is $0.446f_{ck}$.

NOTE

The stress strain curve of concrete under pure compression is NOT given by the IS 456: 2000, because limit state of collapse under pure compression of concrete is not relevant in design considerations of structures.

When concrete is subjected to axial compression and flexure, then the maximum strain in concrete is limited in between 0.002 and 0.0035 depending upon the neutral axis location with respect to section of concrete under consideration. However, the maximum design stress level for this case remains equal to $0.446f_{ck}$.

5. **Design stress strain curve for reinforcing steel:** The design yield strength of steel f_{yd} is obtained by dividing the yield strength or characteristic strength of steel (f_y) by partial safety factor of steel (γ_s) which is equal to 1.15.

Thus, $$f_{yd} = \frac{f_y}{\gamma_s} = \frac{f_y}{1.15} = 0.87f_y$$

The corresponding design yield strain is $\frac{0.87f_y}{E_s}$

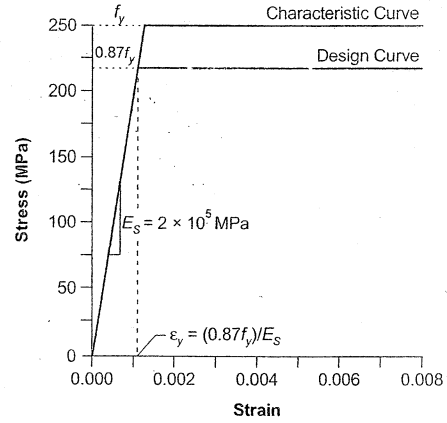


Fig.2.2 *Characteristic and design stress strain curve for mild steel (Fe 250)*

Cold worked bars (Fe 415 and Fe 500) does not have a definite yield point and thus the transition from linear elastic behavior to nonlinear behavior is assumed

to occur at a stress level of $0.8f_y$ in characteristic curve and $0.8f_{yd}$ in design stress strain curve of steel. Full design yield strength occurs at a proof strain of 0.2% or 0.002. Thus, design stress of $0.8f_{yd}$ corresponds to zero inelastic strain and the design stress of $1.0f_{yd}$ corresponds to inelastic strain of 0.002. Design yield strain of HYSD steel is,

$$\varepsilon_y = 0.002 + \frac{0.87f_y}{E_s}$$

2.9 Various types of Young's Modulus of Elasticity of Concrete (E_c)

1. **Initial tangent modulus of elasticity:** It is the slope of stress – strain curve of concrete at origin. It is the maximum value of modulus of elasticity of concrete (E_c). IS 456: 2000 defines this value as $5000\sqrt{f_{ck}}$.
2. **Tangent modulus of elasticity:** The slope of tangent at any point on the stress – strain curve is referred to as tangent modulus of elasticity.
3. **Secant Modulus of elasticity:** It is the slope of the line joining any point on stress – strain curve to the origin.
4. **Long term modulus of elasticity:** It considers the effect of creep and is defined as:

$$E_{long} = \frac{E_c}{(1+\theta)} = \frac{5000\sqrt{f_{ck}}}{(1+\theta)}$$

where, θ = Creep coefficient = $\dfrac{\text{Ultimate creep strain}}{\text{Elastic strain}}$

Table 2.2: *Values of creep coefficient as per IS 456: 2000*

Age of Loading	Creep Coefficient (θ)
7 Days	2.2
28 Days	1.6
1 Year	1.1

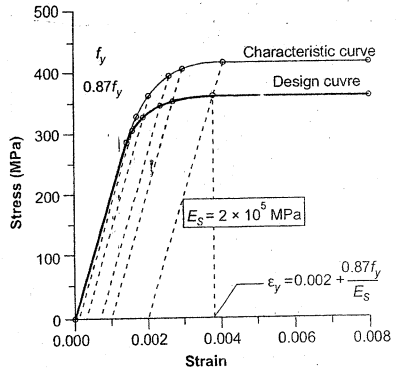


Fig. 2.3 *Characteristic and design stress strain curve for cold worked Fe 415 grade steel*

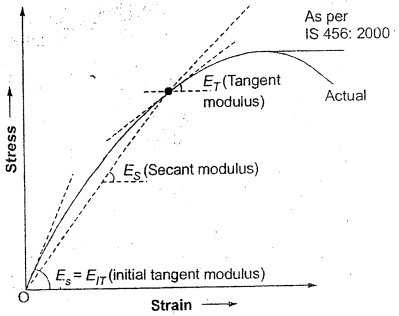


Fig. 2.4 *Various Types of Modulus of Elasticity of Steel*

Remember: For design purposes, we should take secant modulus but actually we are using initial tangent modulus i.e. $5000\sqrt{f_{ck}}$.

2.9.1 Modulus of Rupture of Concrete

As per IS 456: 2000, it is defined as:

$$f_r (\text{N/mm}^2) = 0.7\sqrt{f_{ck}}$$

2.10 Properties of Reinforcement and its use in Reinforced Concrete Structures

The reinforcing steel is classified according to the yield strength of the steel that is to be used in construction for structural purposes. This specified yield strength is called as **grade of steel**. The grade of steel (e.g. Fe 250, Fe 415 etc.) represents yield strength of steel (expressed as N/mm² or MPa).

As stated earlier, for certain types of steel, yield point cannot be located distinctly on the stress strain curve. For such types of steel, the stress corresponding to 0.2% (or 0.002) offset strain is defined as yield point.

The slope of the elastic portion of stress strain curve is called as **modulus of elasticity** of steel (E_s) and for all types of steel it is taken as **2 x 10⁵ N/mm²**.

2.10.1 Types of Reinforcing Steel

1. **Mild Steel (Fe 250):** It can either be ORDINARY or HOT ROLLED with characteristic strength of 250 N/mm².
2. **Medium Tensile Steel:** It can also be either ORDINARY or HOT ROLLED.
3. **Cold Twisted Bar:** These are HYSD (HIGH YIELD STRENGTH DEFORMED) bars. e.g. Fe 415 and Fe 500 HYSD bars.
4. **TMT Bars:** These are THERMO MECHANICALLY TREATED (TMT) bars. e.g. Fe 415, Fe 500, Fe 600 TMT bars. Their outer shell has very high tensile strength but inner core is soft and ductile. These are usually coated with anticorrosive coating.

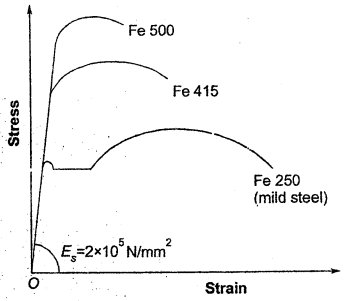


Fig. 2.5 *Stress strain baheviour of mild and HYSD steel*

About Hot Rolled Mild Steel Reinforcement: In case of Hot Rolled mild steel reinforcement, an yield plateau is observed at upper yield point resulting in large deformation in steel and further resulting in large cracking of structure.

Yield Plateau: It can be eliminated by cold working.

Cold Working: Steel reinforcement is stressed beyond yield plateau either by stretching or by twisting and then it is unloaded. This process is called as cold working. Cold twisted bars are made by this process.

2.10.2 Yield Strength for HYSD Bars

Table 2.3: *Permissible tensile stresses of steel bars*

Types of bar	Permissible Tensile Stress (N/mm²)			
	Mild Steel (Fe 250)	Medium Tensile Steel	CTP/HYSD Steel	Fe 500
Tensile Strength				
Dia upto 20 mm	140	190	230	275
Dia > 20 mm	130	190	230	275
Compression bar in column	130	130	190	--

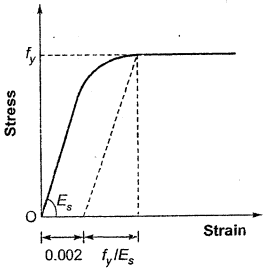


Fig. 2.6 *Stress strain behaviour of HYSD/ CTP steel bars*

For HYSD bars, yield stress is read at 0.2% proof strain i.e. at strain of 0.002 by drawing a line parallel to stress-strain curve.

Total strain at f_y, $\varepsilon_y = 0.002 + \frac{f_y}{E_s}$

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2.10.3 Reinforcement Bar Sizes available in India

The following bar sizes i.e. nominal bar diameters (in mm) are available in the Indian market viz.:

5, 6, 8, 10, 12, 16, 18, 20, 25, 28, 32, 36, 40, 45, 50. Among these, most commonly available bar sizes are 6, 8, 10, 12, 16, 20, 25, 32, 40.

2.10.4 Hot Rolled Versus Cold Worked Steel Bars

Hot rolled mild steel: The stress-strain curve of hot rolled mild steel is featured by an initially straight linear elastic part of the curve which is followed by the occurrence of an **yield plateau** (strain increases at constant stress) and followed by **strain hardening zone** wherein the stress increases with increasing strain till peak tensile strength is reached. Then this is followed by a decreasing limb of the stress strain curve till fracture occurs. For **Fe 250, ultimate tensile strength is 412 MPa and minimum percentage elongation is 20 to 22%.**

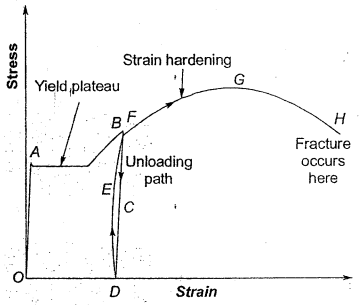


Fig. 2.7 *Effect of cold-working of mild steel bars*

Cold worked bars involve the process of stretching and twisting of mild steel beyond the yield plateau which is followed by the release of load. This specimen on reloading will follow the path *DEF* as shown in the figure below. The process of unloading after the yield stress and then again reloading gives rise to a **hysteresis loop**. The unloading and reloading curves are nearly straight and parallel to *OA* as shown below. Thus the reloading process follows a linear elastic path (with the same modulus of elasticity E_s as that of mild steel) till the point where the unloading process started. Then the material (steel) enters into the strain hardening stage. The process of cold working increases the yield strength of steel but at the same time, it reduces the ductility of steel. **Cold working also decreases the margin between the yield strength and the ultimate strength of steel.**

Do You Know ?

Some Interesting Facts about Mild Steel (Fe250):

1. In LSM, mild steel (Fe 250) which has a clearly defined yield point, the steel yields i.e. reaches the stress 'f_y' even at strains less than $\left(\frac{0.87f_y}{E_s} + 0.002\right)$ and thus at values of x_u which are slightly greater than $x_{u\,lim}$, the stress in mild steel remains $0.87f_y$.
2. Ductility requirement may be partially satisfied with the use of Fe 250 steel even if x_u is slightly greater than $x_{u\,lim}$.
3. Due to the ability of mild steel to yield even at strains less than $\left(\frac{0.87f_y}{E_s} + 0.002\right)$, the actual under-reinforced behavior gives values of percentage of tensile reinforcement (p_t) greater than $p_{t\,lim}$. But **IS 456: 2000** limit the use of p_t values greater than $p_{t\,lim}$ for design purposes.

2.11 Cover Requirements as per IS 456: 2000

In order to prevent the reinforcement from corrosion, a layer of concrete should be there in reinforced concrete structures. In actual construction practice, the **specified minimum cover is very difficult to maintain.** Thus, **IS 456: 2000** specifies tolerance levels ranging from 0 mm to +10 mm i.e. no reduction in clear cover is permitted but the nominal cover can be increased up to 10 mm.

IS 456: 2000 has eliminated the restriction on clear cover up to 75 mm but this does not convey the message to use large covers as the use of very large covers (like 100 mm or more) lead to increased crack widths (size greater than 0.3 mm) and allows the ingress of moisture and chemical attack to concrete.

Table 2.4: *Min. conctete grade and nominal cover requirements*

Exposure Conditions	Minimum Concrete Grade	Nominal Cover (mm)
Mild	M20	20
Moderate	M25	30
Severe	M30	45
Very Severe	M35	50
Extreme	M40	75

Note :
- Nominal cover can be decreased by 5 mm for main reinforcing bars of less than 12 mm diameter.
- Nominal cover can be decreased by 5 mm if concrete grade is M35 or higher.

Table 2.5: *Nominal Cover to Meet the Fire Resistance Requirement as per Table 16 A of IS 456: 2000*

Fire Resistance (hours)	Nominal Cover (mm)						
	Beams		Slabs		Ribs		Columns
	Simply Supported	Continuous	Simply Supported	Continuous	Simply Supported	Continuous	
≤0.5	20	20	20	20	20	20	40
1	20	20	20	20	20	20	40
1.5	20	20	25	20	35*	20	40
2	40*	30	35*	25	45	35*	40
3	60	40*	45	35*	55	45	40
4	70	50	55	40	65	55	40

(*) Requires attention to additional measures necessary to reduce the risks of spalling.

2.12 Spacing of Reinforcement

Proper placement of reinforcement is an important component in the reinforced concrete design. The required area of steel reinforcement is provided in the form of bars adequately placed in the concrete. Closely placed bars i.e. densely reinforced concrete members create problem in pouring of concrete and its compaction. On the other hand, too widely spaced bars cannot control the cracks in concrete due to shrinkage.

2.13 Other Important Considerations in Reinforced Concrete

Given below are some other important considerations for the design and preparation of concrete mixes for reinforced concrete..

1. **Mixing:** Cl. 10.3 of **IS 456: 2000** states that concrete should be mixed in a mechanical mixer for at least **two minutes** in order to have uniform distribution of materials and to have a uniform consistency.
2. **Transporting, placing, compaction and curing:** Cl. 13 of **IS 456: 2000** states that concrete must be transferred to the formwork immediately after mixing to avoid segregation of concrete constituents, loss of workability and possibility of mixing of foreign impurity in the concrete. Moreover, adequate precautions must be taken to prevent loss of water from concrete due to evaporation.

The **compaction of concrete** should start immediately after placing and before the initial setting time so that it is not disturbed after the start of initial setting time. While placing concrete, it must be ensured that reinforcing bars should not get displaced from their positions and formwork should not get displaced.

Compaction of concrete around the reinforcement should be done with mechanical vibrators but excessive vibration should be avoided as it leads to segregation of concrete.

Curing prevents the loss of moisture from concrete thereby making available sufficient water for proper hydration of cement in concrete mix.

3. **Formwork:** Cl. 11 of IS 456: 2000 states that properly designed formwork should be used in order to maintain its rigidity during placing and compaction of concrete. The stripping time of formwork should be such that the concrete attains strength of at least twice the stress that the concrete may be subjected to when formwork is removed from the concrete.
4. **Assembly of reinforcement:** Cl. 12 of IS 456: 2000 states that reinforcement bars designed for flexural moments, shear forces and axial forces must be properly accommodated and a proper bar bending schedule should be prepared. Reinforcement bars should be placed over blocks, spacers, supporting bars etc. to maintain their position and the required concrete cover. **High strength bars must NOT be re-bent.** Reinforcement bars must be properly assembled to have a good flow of concrete through it without segregation.
5. **Sampling and strength of concrete mix:** Cl. 15 of IS 456: 2000 recommends that random samples of concrete cubes shall be made from the fresh concrete which is then properly cured and tested at 28 days as per IS 516: 1959. The number of samples will depend on the total quantity of concrete as per Cl. 15.2.2 of IS 456: 2000.

   Table 2.6: Min. frequency of sampling of concrete

Quantity of concrete in work (m^3)	Number of samples
0 - 5	1
6 - 15	2
16 - 30	3
31 - 50	4
51 and above	4 plus one additional sample for each 50 m^3 or part thereof.

   At least one sample shall be taken from each shift. Where concrete is produced continuously (e.g. in RMC Plants), there, frequency of sampling may be agreed upon mutually by the supplier and the buyer.
6. **Acceptance Criteria:** Cl. 16 of IS 456: 2000 states the limit on acceptance criteria i.e. concrete should be considered satisfactory when both the mean strength of any group of four consecutive test results and any individual test result on compressive strength and flexural strength comply with the limits specified in Cl. 16.1 and Cl. 16.2 of IS 456: 2000.
7. **Inspection and testing of structures:** Cl. 17 of IS 456: 2000 lays down the systematic procedure for inspection which covers materials, records, workmanship and construction.
8. **It is always desirable to limit the number of different types of beams to a few standard sized beams** which makes the reinforcement detailing process simpler and also greatly helps in the construction of formwork at site.
9. **Unit weight of reinforced concrete:** Cl. 19.2.1 of IS 456: 2000 specifies the unit weight of reinforced concrete to be taken as 25 kN/m^3.

2.14 Major Reasons of Structure Failure

There are so many reasons for a building failure. By the term **'failure'**, it implies failure in terms of either collapse or serviceability or both. Some of the major reasons of structure failure are as follows:

Failure during construction or soon after	Failure after a long time of construction
Design fault/significant shift from actual design	Collapse/failure of primary load carrying member by accident or otherwise
Poor detailing	Change in use of the structure leading to over loading
Inferior quality of materials	Factors which are beyond human control like fire, earthquake, blast etc.
inferior construction quality	Lack of proper repair and maintenance
Substandard formwork and/or scaffolding	Exposure to adverse environment which was not envisaged in design

2.15 List of Major Indian Standard (IS) Codes Relating to Reinforced Concrete

2.15.1 Codes for Cement

IS 269: 1989	:	Specification for 33 Grade Ordinary Portland cement
IS 8112: 1989	:	Specification for 43 Grade Ordinary Portland cement
IS12269: 1987	:	Specification for 53 Grade Ordinary Portland cement
IS 8041: 1990	:	Specification for rapid hardening Portland cement
IS 4031: 1988	:	Methods of physical tests for hydraulic cement
IS 6909: 1990	:	Specification for supersulphated cement
IS 8042: 1978	:	Specification for Portland white cement
IS 12600: 1989	:	Specification for low heat Portland cement
IS 1489: 1991	:	Specification for Portland pozzolana cement Part – I Flyash based Part - II Calcined clay based
IS 8043: 1991	:	Specification for hydrophobic Portland cement
IS 12330: 1988	:	Specification for sulphate resisting cement
IS 6452: 1989	:	Specification for high alumina cement for structural use

2.15.2 Codes for steel used as reinforcement in RCC structures

IS 432 (Part-I): 1982	:	Specification for mild steel and medium tensile steel bars for concrete reinforcement
IS 1786: 1985	:	Specification for high strength deformed steel bars for concrete reinforcement
IS 1608: 1995	:	Mechanical testing of metals-Tensile testing
IS 2062: 1999	:	Steel for general structural purposes-Specification
IS 1566: 1982	:	Specification for hard drawn steel wire fabric for concrete reinforcement

2.15.3 Codes for concrete

IS 10262: 1982	:	Recommended guidelines for concrete mix design
IS 7861 (Part-I): 1975	:	Code of Practice for extreme weather concreting (Part-I recommended for hot weather concreting)
IS 3370 (Part –I): 1965	:	Code of Practice for storage of liquids (Part-I : General)

IS 516: 1959	:	Methods of tests for strength of concrete
IS 1199: 1959	:	Methods of sampling and analysis of concrete
IS 1343: 1980	:	Code of Practice for Prestressed Concrete

2.15.4 Codes for aggregates, admixtures and water to be used in RCC

IS 383: 1970	:	Specification for coarse and fine aggregates from natural sources for concrete
IS 2386 (Parts 1 to 8)	:	Methods of tests for aggregate for concrete
IS 1344: 1981	:	Specification for calcined clay pozzolana
IS 3812: 1981	:	Specification for flyash for use as pozzolana and admixture
IS 9103: 1999	:	Specification for admixtures for concrete
IS 3025 (Parts 17 to 32)	:	Methods of sampling and test (physical and chemical) for water and waste water.

2.16 Major Challenges for a Structural Designer

In order to design a structure economically , a structural designer faces the following challenges:

1. Analysing a structure on the basis of highly simplified structural analysis theories which are far from actual material (steel, concrete) behavior.
2. Construction of structure by the unorganized sector of construction workers and there always exists a possibility of human error.
3. Structure subjected to a completely unpredictable natural environment.

Objective Brain Teasers

Q.1 By the term '*safety*' in reinforced concrete, we actually mean:
(a) Material reserve strength
(b) Limited deformation in structure
(c) Structural durability
(d) All of the above

Q.2 If ϕ is the resistance factor which takes into account the material uncertainties and γ is the load factor, then as per ULM of design, the ratio ϕ/γ should be:
(a) Less than unity (b) Greater than unity
(c) Equal to unity (d) Can't say

Q.3 A structure will return to its original state when it has reached:
(a) The limit state of collapse
(b) The limit state of serviceability
(c) Both (a) and (b)
(d) Structure will never return to its original shape.

Q.4 Which of the following load combination is NOT recommended by IS 456: 2000 for limit state of serviceability? .
(a) 1.0(DL + LL)
(b) 1.5(DL + LL)
(c) DL + 0.8LL + 0.8EQ
(d) 1.0(DL + EQ)

Q.5 The design stress-strain curve of mild steel (Fe 250) in limit state method can be obtained from the corresponding characteristic stress-strain curve by:
(a) Multiplying the ordinates of characteristic curve by 1.15
(b) Multiplying the ordinates of characteristic curve by 1.5
(c) Dividing the ordinates of characteristic curve by 1.15
(d) Dividing the ordinates of characteristic curve by 1.5

Q.6 IS 456: 2000 defines *Modulus of Elasticity* of concrete as 5000 $\sqrt{f_{ck}}$. This in the stress strain curve of concrete refers to:
(a) Secant modulus of elasticity
(b) Initial tangent modulus of elasticity
(c) Tangent modulus of elasticity
(d) Long term modulus of elasticity

Q.7 Which of the following curve is NOT covered by IS 456: 2000?
(a) Stress strain curve of concrete in tension.
(b) Stress strain curve for mild steel.
(c) Stress strain curve of concrete in compression.
(d) Stress strain curve for cold worked steel.

Q.8 The number of concrete cube samples required to be taken for 35 m^3 of concreting work as per IS 456: 2000 is:
(a) 1 (b) 2
(c) 3 (d) 4

Q.9 As per IS 456: 2000 recommendations, which of the following is true?
(a) High strength bars can be re-bent.
(b) High strength bars must not be re-bent.
(c) There is no need to prepare bar bending schedule.
(d) Reinforcement bars should not be placed over supporting bars.

Q.10 Which of the following reinforced concrete design philosophy do not distinguish between the different load cases?
(a) Limit State Method
(b) Working Stress Method
(c) Ultimate Load Method
(d) All of the above

Q.11 If 'f_{ck}' is the characteristic strength of concrete cube then design strength of concrete is
(a) 0.67 f_{ck} (b) 0.447 f_{ck}
(c) f_{ck} (d) 0.5 f_{ck}

Q.12 Minimum grade of concrete that can be used in structures as per IS 456: 2000 is
(a) M15 (b) M20
(c) M25 (d) M30

Q.13 At what flexural stress, the first crack in RCC member made of M30 concrete will occur?
(a) 4.5 MPa (b) 5 MPa
(c) 3.83 MPa (d) 30 MPa

Q.14 The cover to reinforcement in a RC beam shall not be less than
(i) 25 mm
(ii) diameter of the bar (ϕ)
(iii) spacing between the bars
(iv) 5 mm
Which of the above statement(s) is/are true?
(a) (i) and (ii) (b) (i) only
(c) (ii) only (d) (i) and (iii)

Q.15 Shear span is defined as the region where
(a) shear force is constant
(b) shear force is zero
(c) flexural moment is constant
(d) flexural moment is zero

Answers

1. (d)	2. (a)	3. (b)	4. (b)	5. (c)
6. (b)	7. (a)	8. (d)	9. (b)	10. (b)
11. (b)	12. (b)	13. (c)	14. (a)	15. (a)

Hints:

4. (b)
1.5 (DL+LL) is recommended for limit state of collapse and not for limit state of serviceability.

11. (b)
Strength of concrete taken for actual structure $= 0.67\ f_{ck}$
Design strength of concrete

$$= \frac{0.67f_{ck}}{FOS} = \frac{0.67f_{ck}}{1.5} = 0.4467f_{ck}$$

13. (c)

$$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{30} = 3.83 \text{ MPa}$$

CHAPTER 03

Working Stress Method (WSM) of Design

3.1 Introduction

As discussed in the previous chapters, there are many design philosophies for the design of reinforced concrete members. In this chapter, we will discuss the **Working Stress Method** of design which is the most traditional method of design. With the recent advances in the understanding of the behavior of materials (concrete and steel), now we have more rational methods of design. Due to this reason only, most design codes in the world, have dispensed **Working Stress Method.** In the Indian code also i.e. IS 456: 2000, Working Stress Method has been put at annexure and major focus is on the recent **Limit State Method** of design.

3.2 Proceeding from Bending Moments to Flexural Stresses

From the principles of Structural Analysis, we arrive at bending moments, shear forces, reactions, axial forces etc. In the theory of bending, we are mainly concerned with the bending moments. The bending moment at any section of the beam produce normal stress (compressive and tensile). The theory of flexure for homogeneous materials is the part of Basic Structural Analysis or Solid Mechanics.

3.3 Analysis of Composite/Non-homogeneous Sections

The principles of the theory of flexure as studied in Solid Mechanics is NOT applicable for non-homogeneous materials. Reinforced concrete is non-homogeneous and thus the results developed in the theory of flexure cannot be applied directly to reinforced concrete.

3.4 Stress-strain Distribution

One of the fundamental assumptions of theory of flexure is **"plane sections remain plane before and after the bending".** This assumption is true only if there exists a perfect bond between the two materials in order to act as an integral unit without any possibility of slip in between.

Thus, there will be a linear strain variation. The two materials bonded together and located at the same distance from the neutral axis will have same amount of strain (ε) in both of them. The corresponding stress in the respective materials will be $f_1 = E_1\varepsilon$ and $f_2 = E_2\varepsilon$. Where, E_1 and E_2 are modulus of elasticity of the two materials.

From strain compatibility,

$$\varepsilon_1 = \varepsilon_2$$

$$\frac{f_1}{E_1} = \frac{f_2}{E_2}$$

$$f_2 = \frac{E_2}{E_1} f_1 = mf_1 \qquad \text{where,} \quad m = \frac{E_2}{E_1} = \text{modular ratio}$$

3.5 Transformed Section

In order to analyse of composite materials by the use of the linear elastic principles of Structural Analysis, it is necessary to transform the composite section into a single homogeneous section. This is made possible by the concept of **modular ratio (*m*).**

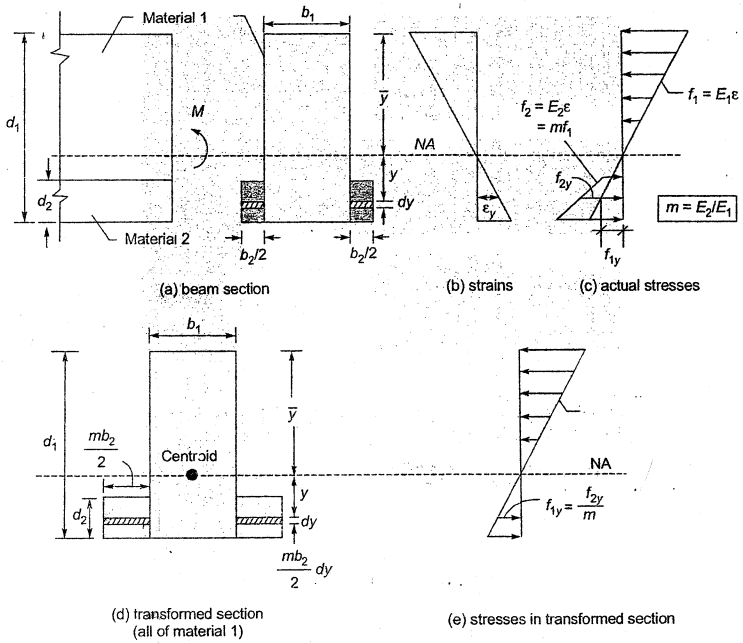


Fig. 3.1 *Transformed Section*

Let there be an infinitesimal element in material 2 located at a distance '*y*' from the neutral axis.

Thus

$$dF_2 = f_2(b_2 \cdot dy)$$

But

$$f_2 = \left(\frac{E_2}{E_1}\right) f_1 = mf_1 \text{ as proved earlier}$$

Therefore $dF_2 = mf_1(b_2 \cdot dy) = f_1(mb_2 \cdot dy)$

Thus material 2 can be transformed to materials 1 by replacing original width b_2 by 'mb_2', where m is the modular ratio $\left(=\frac{E_2}{E_1}\right)$. The term **width** implies dimension parallel to the neutral axis.

NOTE: In the transformed section, the magnitude of resultant forces, their direction and line of action does not change.

In the transformed section, as the material is homogeneous (of one material only), the principles of linear elastic analysis is applicable.

3.6 Modular Ratio

In case of working stress method of analysis, the composite section of reinforced concrete is transformed into an equivalent section of single material of concrete. For this, we need to define modular ratio which is the ratio of elastic modulus of steel to concrete.

However, the short term modulus of elasticity of concrete does not take into account the long term effects of creep and shrinkage and thus it is not considered in defining the modular ratio (m). However, partly taking into account the long term effects of creep and shrinkage, **Cl. B-1.3 of IS 456: 2000** defines modular ratio (m) as:

$$\boxed{m = \frac{280}{3\sigma_{cbc}}}$$

or $$m\sigma_{cbc} = \frac{280}{3} = 93.33 = \text{constant}$$

where, σ_{cbc} = Permissible stress in concrete in bending compression

Table 3.1: *Values of σ_{cbc} and m for different grades of concrete*

Grade of Concrete	σ_{cbc} (MPa)	Modular Ratio (m)
M15	5	18.67
M20	7	13.33
M25	8.5	10.98
M30	10	9.33
M35	11.5	8.11
M40	13	7.18
M45	14.5	6.44
M50	16	5.83

3.6.1 A Note on Modular Ratio (m)

The two different materials of reinforced concrete i.e., the concrete and the steel are transformed into a single material using the modular ratio (m). The modular ratio (m) is expressed as:

$$m = \text{Modular ratio} = \frac{\text{Modulus of elasticity of steel}}{\text{Modulus of elasticity of concrete}} = \frac{E_s}{E_c}$$

The modulus of elasticity of steel = $E_s = 2 \times 10^5$ N/mm^2

The short-term modulus of elasticity of concrete $E_c = 5000\sqrt{f_{ck}}$

Above expression does not take into account the effect of creep and shrinkage and other long term effects.

Thus, $$m = \frac{2\times10^5}{5000\sqrt{f_{ck}}}$$

But it is given as $$m = \frac{280}{3\sigma_{cbc}}$$

(this partially takes into account the long-term effects of creep and shrinkage)

3.7 Transformed Area of Reinforcement-Tension Steel

The area of tension reinforcement steel (A_{st}) is converted to an equivalent concrete area as mA_{st}. Cl. B-2.1.1 of IS 456: 2000 states that this transformation is applicable not only for flexural members but also for direct tension members. The corresponding stress in steel 'f_{st}' is given from the equivalent transformed concrete stress f_c (at the level of tension steel) as $f_{st} = mf_c$.

3.8 Transformed Area of Reinforcement-Compression Steel

The modular ratio for compression steel (eg. Steel in columns, compression steel in doubly reinforced beams) is greater than that for tension steel. This is due to the long term effects of creep and shrinkage of concrete along with non-linearity in material behavior at higher stress levels results in much higher compressive strains in compression steel rather than those indicated by linear elastic theory. Thus **IS 456: 2000** recommends transformed area of compression steel to be equal to 1.5 mA_{sc} and **NOT** mA_{sc}. The corresponding stress in compression steel f_{sc} is given as $f_{sc} = 1.5\, mf_c$. Where, f_c is the corresponding stress in equivalent transformed concrete.

3.9 Cracking Moment

The very first crack in the extreme tension fibre of a beam appears when the stress reaches the value of **modulus of rupture of concrete (f_{cr})**. Assuming a linear stress-strain relationship for concrete in compression and tension with the same modulus of elasticity of concrete, the corresponding **cracking moment** is given by:

$$\text{Cracking moment } (M_{cr}) = f_{cr}\frac{I_T}{y_T}$$

where, I_T = Second moment of area or moment of inertia of transformed concrete about neutral axis

y_T = Distance of extreme tension fibre from neutral axis

When the concrete beam is very lightly loaded such that the applied moment (M) is less than the cracking moment (M_{cr}) then section is **uncracked section**, and both concrete and steel takes part in resisting tension.

3.10 Behaviour of Reinforced Concrete in Flexure

When a reinforced concrete beam is loaded, it behaves differently according to the stage of loading that is being superimposed on the beam. In general, this behavior can be studied right from *uncracked phase to the* final ultimate stage that leads to collapse because of flexural resistance capacity of the section is being exceeded.

3.10.1 The Uncracked Phase in Reinforced Concrete

Initially, there is no externally applied load on the beam. Now gradually, as the beam is being loaded, the corresponding moment at a particular section in the beam increases. When this applied moment at any section of the beam (M) is less than the cracking moment (M_{cr}), then the maximum tensile stress f_T (at the extreme fibre of beam) is less than the flexural tensile strength of concrete f_{cr}. This phase is called as **uncracked phase** and the whole section takes part (i.e. effective) in resisting the applied moment M.

The limiting case of **uncracked phase** occurs when the applied moment (M) becomes equal to the **cracking moment (M_{cr})**.

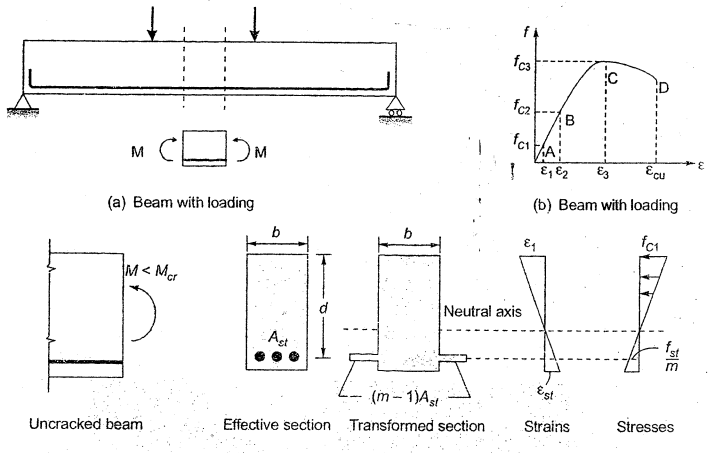


Fig. 3.2 *Uncracked phase of concrete*

3.10.2 The Cracked Phase in Reinforced Concrete

As the externally applied moment M (due to external loading) exceeds the cracking moment M_{cr}, the tensile stress in the concrete exceeds the flexural tensile strength of concrete. At this stage, cracks get initiated from the bottom most fibre (in tension) of concrete. As the loading is increased further, these cracks widen and they start propagating towards the neutral axis of the beam. As the concrete has cracked, it is not effective in resisting the tension anymore. The effective area of concrete gets reduced. The tension which was resisted by concrete just before the commencement of crack is now gets transferred to the reinforcing steel at the location of cracked section of the beam. Any further increase in the loading is now entirely resisted by the reinforcing steel in tension side of the section.

Now tension in the steel increases suddenly from the value of zero. This leads to increase in the tensile strain in the steel bars at the cracked section. This large increase of strain at the level of reinforcing steel results in shifting the neutral axis upwards.

It is to be noted that, because cracks appear in concrete at very low tensile stresses, it is quite conservative to ignore the tensile strength of concrete. Cl. B-1.3(b) of IS 456: 2000 states that all tensile stresses are taken up by the reinforcement and none by the concrete, except otherwise specifically permitted.

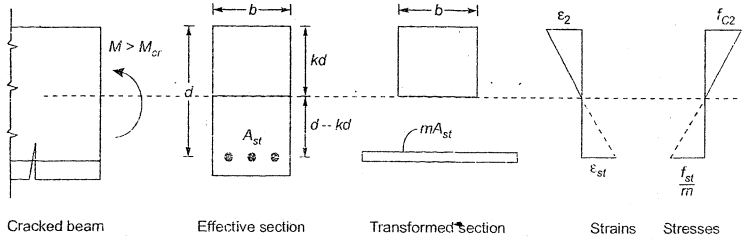


Fig. 3.3 *Cracked phase of concrete (with linear stress distribution)*

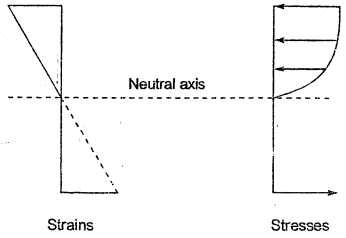


Fig. 3.4 *Cracked concrete with nonlinear stress distribution*

3.11 Location of Reinforcing Bars in Beam Section

In order to effectively resist the flexural tension due to the applied moment, the reinforcing bars must be placed as far from the neutral axis as possible provided the requirements of minimum cover to reinforcing bars and spacing of bars are satisfied.

3.12 Usefulness of Concrete in Tension Side

The concrete in the tension side is not at all useless but it serves many other important functions which are given below:

1. It holds the reinforcing bars in place.
2. It takes part in resisting the shear and torsion.
3. It increases the flexural stiffness of the beam i.e. it increases second moment of area (I) and thereby reduces deflections.
4. It protects the steel from fire and corrosion.

3.13 Permissible Stresses in Concrete and Steel

3.13.1 Permissible Stresses in Concrete in Tension

Cl. B-2.1.1 of IS 456: 2000 specifies permissible stress in direct tension for different grades of concrete. Although full tension is to be taken by reinforcement only, the actual tensile stress in concrete shall not exceed the respective permissible stresses in order to prevent cracks in concrete. The factor of safety of concrete in direct tension ranges from 8.5 to 9.5.

Concrete grade	M10	M15	M20	M25	M30	M35	M40	M45	M50
Tensile stress (N/mm^2)	1.2	2.0	2.8	3.2	3.6	4.0	4.4	4.8	5.2

3.13.2 Permissible Stresses in Concrete in Compression

Table 21 of IS 456: 2000 gives the values of permissible stresses in concrete in direct compression, bending compression.

Table 3.2: Permissible Stresses in Concrete (N/mm²)

Concrete grade	Permissible stress in compression	
	Bending (σ_{cbc})	Direct (σ_{cc})
M10	3.0	2.5
M15	5.0	4.0
M20	7.0	5.0
M25	8.5	6.0
M30	10.0	8.0
M35	11.5	9.0
M40	13.0	10.0
M45	14.5	11.0
M50	16.0	12.0

The factor of safety of concrete in bending compression, direct compression are taken as 3, 4 respectively.

3.13.3 Permissible Stress in Steel

Table 22 of IS 456: 2000 specifies permissible stresses in steel reinforcement for various steel grades, bar diameters and types of stress.

In the above table, FOS for steel = 1.8. (This is much lower than concrete due to better quality control during the production of steel)

Table 3.3: Permissible Stresses in Steel (N/mm²)

Type of stress	Mild steel (Fe 250)	HYSD (Fe 415)
Tension		
(i) Bar dia upto 20 mm	140	230
(ii) Bar dia greater than 20 mm	130	230
Compression	130	190

3.13.4 Increase in Permissible Stresses

Cl. B-2.3 of IS 456: 2000 recommends an increase in the permissible stresses in concrete and steel given in Table 21 and Table 22 up to a limit of 33.33%. This increase in permissible stresses is made where stresses due to wind loading, seismic forces, temperature loads, shrinkage effects etc. are combined with those due to dead loads, live loads and impact loads. Wind and seismic forces needn't be considered simultaneously.

3.13.5 Significance of Permissible Stress of Concrete in Direct Tension (σ_{td})

Concrete in not assumed to take any tension, the actual tensile stress in concrete is always tried to keep below the permissible stress of concrete in direct tension to avoid cracks in concrete. As stated earlier the FOS of concrete in direct tension ranges from 8.5 to 9.5.

3.14 Assumptions in the analysis of beams by working stress method (at service loads)

The working stress method is based on elastic theory of analysis and following assumptions are made as per Cl. B-1.3 of IS 456: 2000:

1. Plane sections remain plane before and after the bending.
2. All tensile stresses are taken up by steel reinforcement only.

3. Under working loads, the stress strain relationship of steel and concrete is linear.
4. The modular ratio (m) has the value $280/3\sigma_{cbc}$. Where, σ_{cbc} is the permissible compressive stress in concrete in flexure in N/mm² as per Table 21 of IS 456: 2000.

3.15 Design of Reinforced Concrete Structures

Design of reinforced concrete structures started purely on the basis of an empirical approach. After that linear elastic theory was proposed in which concrete and steel were assumed to be elastic. It was further assumed that the load deflection relationship is linear and obeys Hooke's law. This method is called as **working stress method** since the loads for design of structures are service loads or the working loads. However the failure of structure occurs at a much higher load. The ratio of failure load to working load is called as **factor of safety**.

i.e., $$\boxed{\text{FOS} = \frac{\text{Failure load}}{\text{Working load}}}$$

Thus stresses in concrete and steel are allowed not to exceed a particular value called as **permissible stresses**. As per Cl. B-2 of IS 456: 2000, permissible stress are determined by dividing the characteristics values by factor of safety i.e.,

$$\text{Permissible stress} = \frac{\text{Characteristic strength}}{\text{Factor of safety (FOS)}}$$

The value of factor of safety depends on:

(i) the grade/type of material (ii) the type of stress

For example: Permissible stress of concrete (M 20) in bending compression

$$\sigma_{cbc} = \frac{\text{Characteristic strength of M20 concrete}}{\text{FOS of concrete in bending compression}}$$

$$= \frac{20}{3} = 6.67 \simeq 7 \text{ N/mm}^2$$

Where, the subscript in σ_{cbc} implies "concrete in bending compression (*cbc*)"

3.16 Singly Reinforced Sections

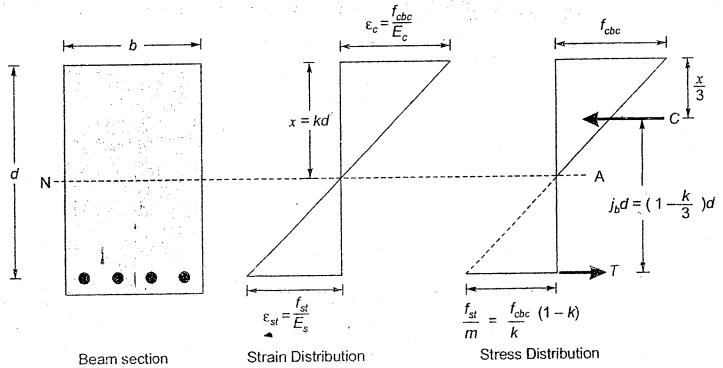


Fig. 3.5 *Singly reinforced beam section*

The above figure represents strain and stress distribution in a beam section subjected to pure flexure.

where, b = width of beam section
d = Effective depth of beam section
f_{cbc} = Actual stress in concrete in bending compression at the top fibre $\not>\sigma_{cbc}$
f_{st} = Actual stress in steel at the level of centroid of steel reinforcement $\not>\sigma_{st}$
$x = kd$ = depth of neutral axis from the top fibre.
k = Neutral axis factor

$jd = \left(1-\frac{k}{3}\right)d$ = Lever arm i.e., distance between the lines of action of compression (C) and tension (T)

Here, stress at the level of centroid of steel reinforcement is $\frac{f_{st}}{m}$ due to transformation of steel into an equivalent area of concrete ($= mA_{st}$)

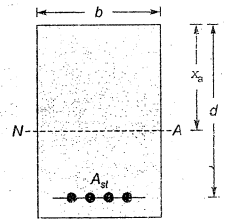
Natural Axis

Actual depths of NA is calculated by equating the moment of area on both sides of NA.

Equating moment of area

$$bx_a = \frac{x_a}{2} = mA_{st}(d-x_a)$$

$$\frac{bx_a^2}{2} = mA_{st}(d-x_a)$$



Solving above equation will give x_a.

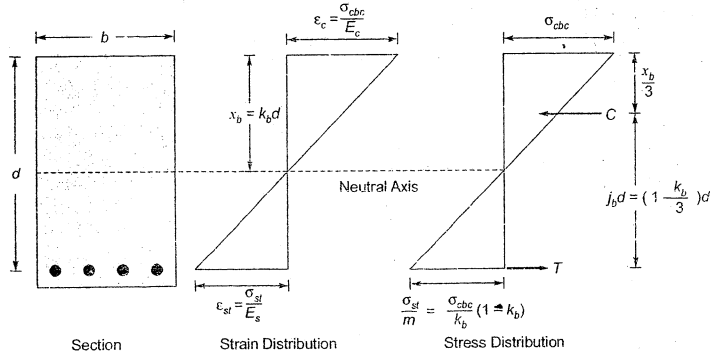
Actual depth of NA is used for

(i) Calculating stress in the section due to any given moment.

(ii) Calculating MOR of the section

3.16.1 Singly Reinforced Balanced Section

In balanced section, both f_{cbc} and f_{st} reach their permissible values of σ_{cbc} and σ_{st} respectively at the same time.



From stress distribution diagram,

$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1-k_b), \quad \text{but} \quad m = \frac{280}{3\sigma_{cbc}}$$

$$\therefore \quad \frac{\sigma_{st}.3\sigma_{cbc}}{280} = \frac{\sigma_{cbc}}{k_b}(1-k_b)$$

$$\Rightarrow \quad k_b = \frac{280}{3\sigma_{st}+280} = \text{Neutral axis factor for balanced section.}$$

Thus the neutral axis for balanced section i.e., $x_b = k_b d$ is known as critical depth of NA.

Also Lever arm $= j_b d = \left(1-\frac{k_b}{3}\right)d$

Total compressive force $= C = \frac{1}{2}\sigma_{cbc} b x_b = \frac{1}{2}\sigma_{cbc} b k_b d$ (It acts at a distance of $x_b/3$ from top)

Total tensile force $= T = \sigma_{st} A_{st}$

Moment of resistance of balanced section

$$\therefore M_b \text{ (with respect to compression)} = C(j_b d) = \frac{1}{2}\sigma_{cbc} b k_b d\left(1-\frac{k_b}{3}\right)d = \frac{1}{2}\sigma_{cbc} k_b j_b (d^2 b)$$

$$\text{and } M_b \text{ (with respect to tension)} = T(j_b d) = \sigma_{st} A_{st}(j_b d) = \left(\frac{p_{t\,bal}}{100}\right) b\sigma_{st} j_b d^2$$

where, $p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100$ = Percentage of tensile steel for balanced section.

Moment of resistance is also expressed as,

$$M_b = R_b b d^2 \qquad \text{where, } R_b = \frac{1}{2}\sigma_{cbc} k_b j_b = \left(\frac{p_{t\,bal}}{100}\right)\sigma_{st} j_b$$

and lever arm factor $j_b = \left(1-\frac{k_b}{3}\right)$

Also For balanced section, $C = T$ gives,

$$A_{st}\sigma_{st} = \left(\frac{\sigma_{cbc}}{2}\right) b k_b d$$

$$\Rightarrow \quad \boxed{\left(\frac{A_{st}}{bd}\right) = \frac{\sigma_{cbc} k_b}{2\sigma_{st}}}$$

$$\Rightarrow \quad p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100 = \left(\frac{50\sigma_{cbc} k_b}{\sigma_{st}}\right)$$

3.16.2 Under Reinforced Section-Singly Reinforced

Owing to the fact that bar diameters available cannot meet the requirement of steel area required and thus it is not possible to design a balanced reinforced section. Then percentage of steel provided in beams is always kept below the value of '$p_{t\,bal}$' i.e., percentage of tension steel for balanced section.

Salient Features of Under-reinforced Sections

(i) $x_a < x_b$

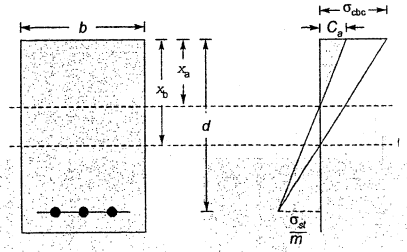
(ii) Failure of beam is due to failure of reinforcement therefore failure will be ductile.

(iii) Stresses in concrete are always less than the maximum permissible value.

(iv) This type of failure is preferred.

Calculation of MOR

$x_a < x_b$, $C_a < \sigma_{cbc}$, $t_a = \sigma_{st}$



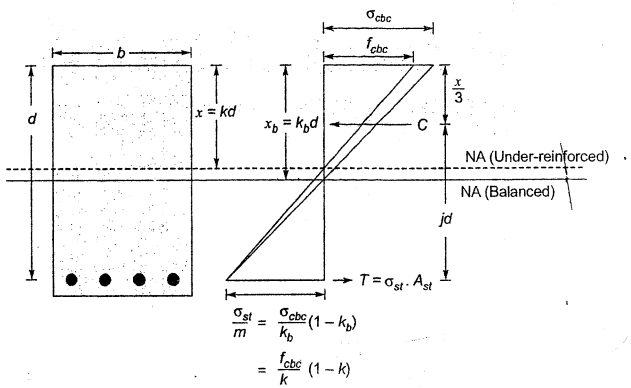
From compression side

$$MR = \frac{1}{2} B x_a C_a \left(d - \frac{x_a}{3}\right)$$

where $C_a = \dfrac{x_a \sigma_{st}}{(d - x_a)^m}$

For tension side

$$MR = \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)$$



$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1 - k_b) = \frac{f_{cbc}}{k}(1 - k)$$

Equating the moment of compression area and tension area about NA

$$bkd\left(\frac{kd}{2}\right) = m\left(\frac{p_{t\,bal}}{100} \times bd\right)(d - kd)$$

$$\Rightarrow \quad k^2 + \left(\frac{p_t m}{50}\right)k - \left(\frac{p_t m}{50}\right) = 0$$

$$\therefore \quad k = -\left(\frac{p_t m}{100}\right) + \left[\left(\frac{p_t m}{100}\right)^2 + \left(\frac{p_t m}{50}\right)\right]^{1/2}$$

(Ignoring the negative value as 'k' cannot be negative)

Moment of resistance of under-reinforced section is,

$$M = C(\text{lever arm}) = T(\text{lever arm})$$

$$= \sigma_{st} A_{st} d\left(1 - \frac{k}{3}\right) = \left(\frac{p_t bd^2}{100}\right)\sigma_{st}\left(1 - \frac{k}{3}\right) = Rbd^2$$

where $R = \left(\dfrac{p_t}{100}\right)\sigma_{st}\left(1 - \dfrac{k}{3}\right)$ = Moment of resistance factor

The actual stress in concrete in the top fibre (f_{cbc}) will not reach σ_{cbc} in an under-reinforced section and f_{cbc} is determined by equating compressive force of concrete (C) to tensile force of steel (T) i.e.,

$$C = T$$

where, $C = \left(\dfrac{1}{2}\right) f_{cbc} bkd$

and $T = \sigma_{st} A_{st}$

Here the actual stress in steel 'f_{st}' will reach σ_{st}.

Thus $\left(\dfrac{1}{2}\right) f_{cbc} bkd = \sigma_{st} A_{st} \Rightarrow f_{cbc} = \dfrac{2\sigma_{st} A_{st}}{bkd}$

Expressing A_{st} as, $A_{st} = \dfrac{p_t}{100} bd$

We have, $f_{cbc} = \dfrac{2\sigma_{st}}{bkd}\left(\dfrac{p_t}{100} bd\right) = \dfrac{\sigma_{st} p_t}{50k}$

3.16.3 Over-reinforced Sections

Salient feature of over-reinforcement section:

(i) $x_a < x_b$

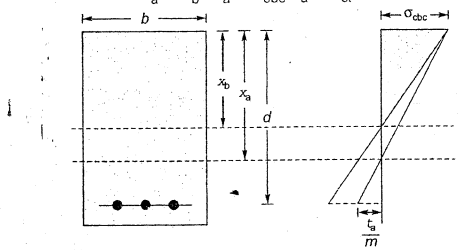
(ii) Failure of beam is due to failure of concrete therefore failure will be prittle failure.

(iii) Stress in steel is always than the maximum permissible value.

(iv) This type of sections are not preferred due to brittle failure.

Calculation of MOR

$x_a > x_b$, $C_a = \sigma_{cbc}$, $t_a < \sigma_{st}$



From compression side

$$MR = \frac{1}{2} b x_a \sigma_{cbc} \left(d - \frac{x_a}{3} \right)$$

From tension side

$$MR = t_a A_{st} \left(d - \frac{x_a}{3} \right)$$

where $$t_a = \frac{(d - x_a) m \sigma_{cbc}}{x_a}$$

3.17 Doubly Reinforced Beam Section

A doubly reinforced beam is required whenever size of beam is fixed and beam has to resist higher moment (i.e., more than that of MOR of singly reinforced balanced section).

- Modular ratio for compression steel = 1.5 m
- Critical depth of NA will be same as that of singly reinforced section.

3.17.1 Calculation of NA

$$\frac{b x_a^2}{2} + 1.5 m A_{sc} (x_a - d') - A_{sc} (x_a - d') = m A_{st} (d - x_a)$$

$$\frac{b x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d') = m A_{st} (d - x_a)$$

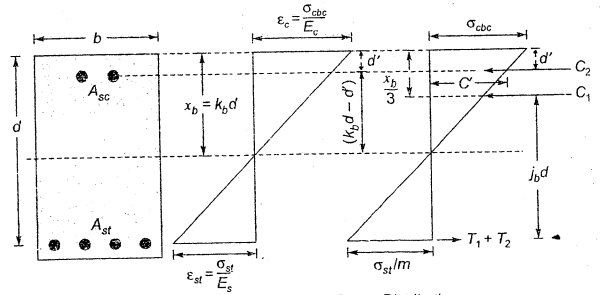
3.17.2 Calculation of MOR

MR = Moment resisted by concrete above NA + Moment resisted by compression reinforcement

$$MR = C_1 (LA)_1 + C_2 (LA)_2$$

$$MR = b x_a \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) + (1.5m - 1) A_{sc} C' (d - d_c)$$

NOTE: Hanger bars must not be confused with compression reinforcement bars. Hanger bars are provided to hold stirrups.



3.17.3 Detailed Analysis of Doubly Reinforced Balanced Beam

$$M = M_b + M'$$

where, M' = Additional moment to be resisted by compression steel

$$A_{st} = A_{st1} + A_{st2}$$

where, A_{st2} = Additional tensile steel for compression steel (A_{sc})

Modular ratio for compression steel is taken as 1.5 m.

C_1 and T_1 are compressive and tensile forces corresponding to balanced section and

Now $$C = T$$

$$\Rightarrow \quad C_1 + C_2 = T_1 + T_2$$

But $$C_1 = T_1;\ C_2 = T_2$$

But $$T_2 = (p_t - p_{t\,bal}) \frac{bd}{100} \sigma_{st}$$

and $$C_2 (k_b d - d') = C' \times (1.5m - 1) A_{sc} (k_b d - d') \qquad \left[C' = \frac{\sigma_{cbc}}{k_b d} (k_b d - d') \right]$$

$$\therefore \quad C_2 (k_b d - d') = \frac{\sigma_{cbc}}{k_b d} (k_b d - d') (1.5m - 1) A_{sc} (k_b d - d')$$

$$\Rightarrow \quad C_2 = \frac{A_{sc} (1.5m - 1) \sigma_{cbc} (k_b d - d')}{k_b d}$$

Additional moment, $$(M') = C_2 (d - d') = \left(\frac{p_c bd}{100} \right) (1.5m - 1) \sigma_{cbc} \frac{(k_b d - d')}{k_b d} (d - d')$$

$$= \left(\frac{p_c}{100} \right) (1.5m - 1) \sigma_{cbc} \left(1 - \frac{d'}{k_b d} \right) \left(1 - \frac{d'}{d} \right) bd^2$$

Also, $$M' = T_2 (d - d')$$

$$= (p_t - p_{t\,bal}) \frac{bd}{100} \sigma_{st} (d - d') = \frac{(p_t - p_{t\,bal})}{100} \sigma_{st} \left(1 - \frac{d'}{d} \right) bd^2$$

From $C_2 = T_2$ we have,

$$p_c (1.5m - 1) \sigma_{cbc} \left(1 - \frac{d'}{k_b d} \right) = (p_t - p_{t\,bal}) \sigma_{st}$$

$$\therefore \quad \text{Total moment (M)} = M_b + M' = M_b + \frac{(p_t - p_{t\,lim})}{100} \sigma_{st} \left(1 - \frac{d'}{d} \right) bd^2$$

$$A_{st} = A_{st1} + A_{st2} = \left(\frac{p_{t\,bal}}{100} \right) bd + \frac{M'}{\sigma_{st} (d - d')}$$

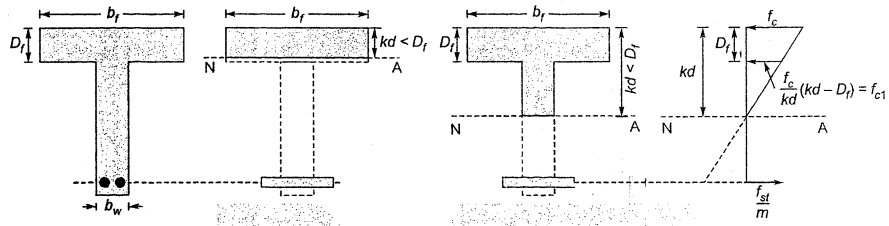
The compression steel (A_{sc}) is expressed in terms of additional tensile reinforcement (A_{st2}) as:

$$\frac{A_{sc}}{A_{st2}} = \frac{p_c}{p_t - p_{t\,bal}} = \frac{\sigma_{st}}{\sigma_{cbc} (1.5m - 1) \left(1 - \frac{d'}{kd} \right)}$$

Table 'M' of SP-16 gives the values of $\frac{A_{sc}}{A_{st2}}$ for different values of $\left(\frac{d'}{d} \right)$ and σ_{cbc} with true values of σ_{st} i.e., σ_{st} = 140 N/mm² and σ_{st} = 230 N/mm².

SP-16 gives values of p_t and p_c for various $\left(\frac{d'}{d}\right)$ values and different values of $\frac{M}{bd^2}$ for four concrete grades and two steel grades.

3.18 Singly Reinforced Flanged Section



(a) Section (b) NA lies in flange (c) NA lies in web (d) Stress distribution

The neutral axis may lie either in the flange ($kd \le D_f$) or in the web ($kd > D_f$). When neutral axis (NA) lies in flange ($kd \le D_f$), the flanged beam behaves like a rectangular beam only (since concrete on tension side of NA assumed to be ineffective) of width b_f i.e., width of flange and effective depth 'd'. Thus, this case can be analysed just like singly reinforced rectangular beams as described in earlier sections by just replacing 'b' by 'b_f'.

In other case, when NA lies in web ($kd > D_f$), the area of concrete in compression consists of flange area and a part of web area above the NA. The exact location of NA is determined by cracked section analysis of the transformed section. Equating moment of compression and tension area of cross-section about NA.

$$(b_f - b_w)D_f\left(kd - \frac{D_f}{2}\right) + b_w(kd)\left(\frac{kd}{2}\right) = mA_{st}(d - kd)$$

The above equation is true only if $kd > D_f$.

Now the concrete in compression consists of two opposite rectangular areas viz. $b_f(kd)$ and $[-(b_f - b_w)(kd - D_f)]$. Thus from $C = T$, we have

$$C = T$$

$$\Rightarrow \quad \frac{1}{2}(f_c)b_f(kd) - \frac{1}{2}f_{c1}(b_f - b_w)(kd - D_f) = f_{st}\,A_{st}$$

Here, f_{c1} is obtained from similar triangles as

$$f_{c1} = \frac{f_c}{kd}(kd - D_f)$$

Also moment about the tension steel is given by,

$$M = \frac{1}{2}(f_c)b_f(kd)\left(d - \frac{kd}{3}\right) - \frac{1}{2}f_{c1}(b_f - b_w)(kd - D_f)\left[(d - D_f) - \frac{(kd - D_f)}{3}\right]$$

As derived in earlier sections,

$$k_b = \frac{280}{280 + 3\sigma_{st}}$$

By comparing lever arm factor (k) with (k_b), the type of section can determined.

If $k < k_b$ then section is under-reinforced and $f_{st} = \sigma_{st}$ and $f_c < \sigma_{cbc}$

If $k > k_b$ then section is over-reinforced and $f_c = \sigma_{cbc}$ and $f_{st} < \sigma_{st}$

3.19 Doubly Reinforced Flanged Section

If neutral axis (NA) lies in flange ($kd \le D_f$), then the section can be analysed just like doubly reinforced rectangular beam section of size ($b_f \times d$).

If NA lies in web ($kd > D_f$) then the section is analysed as per the following procedure. Equating moment of compression and tension area of cross-section about NA.

$$(b_f - b_w)D_f\left(kd - \frac{D_f}{2}\right) + b_w(kd)\left(\frac{kd}{2}\right) + (1.5m - 1)A_{sc}(kd - d') = mA_{st}(d - kd)$$

The net compressive force in concrete is given just as described in earlier section as:

$$C_c = \frac{f_c}{2}(b_f)(kd) - \frac{1}{2}(f_c)(b_f - b_w)\frac{(kd - D_f)^2}{kd} + (1.5m - 1)A_{sc}\,f_c\left(\frac{kd - d'}{kd}\right)$$

Moment of resistance of beam section is given by,

$$M = \left[\frac{1}{2}(f_c)b_f(kd)\left(d - \frac{kd}{3}\right) - \frac{1}{2}(f_c)(b_f - b_w)\frac{(kd - D_f)^2}{2}\left(d - D_f - \frac{kd - D_f}{3}\right)\right] + (1.5m - 1)A_{sc}f_c\frac{(kd - d')}{kd}(d - d')$$

3.20 Limitations of WSM of Design

Use of WSM of design is not only limited to concrete structures but earlier it was used for the design of timber and steel structures also. However, it has the following drawbacks:

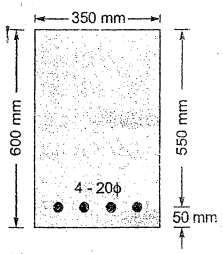
1. Assumption of linear elastic behaviour of concrete does not hold good. Also, the assumption of stresses in concrete and steel within the permissible limits is not true due to long term effects of creep and shrinkage which are not taken into account.
2. Actual factor of safety is not known precisely.
3. Uncertainties of various types of loads acting simultaneously is not being considered in this method.
4. It is NOT advantageous to use high strength deformed bars as compression reinforcement since the permissible stresses are relatively low and is independent of the grade of steel use as compression reinforcement.
5. In case where large moments are encountered, the area of compression steel (A_{sc}) may even exceed the area of tension steel (A_{st}).

Thus, WSM has been gradually replaced by LSM of design. In IS 456: 2000, the WSM has been placed in Annexure - B and gives greater emphasis on LSM of design. Cl. 18.2.2 of IS 456: 2000 recommends to use WSM, where LSM cannot be easily applied. (e.g. in water retaining structures)

NOTE: Design of water retaining structures and tension structures are not covered by IS 456: 2000.

Example 3.1 A reinforced concrete beam of size 350 mm × 600 mm (effective cover of 50 mm) is made up of M 20 concrete and reinforced with 4-20 bars of Fe 415 steel. Calculate the cracking moment of the beam and stresses due to an applied moment of 55 kNm. (Take 'm' for M 20 concrete as 13.33)

Solution:



Modular ratio (m) = 13.33

Modulus of rupture of M 20 concrete $(f_{cr}) = 0.7\sqrt{f_{ck}}$

$= 0.7\sqrt{20} = 3.13$ N/mm²

Section modulus $(Z) = \dfrac{bD^2}{6}$

$= \dfrac{350\times600^2}{6} = 21\times10^6$ mm³

∴ Cracking moment (M_{cr}) (using gross area) $= f_{cr}.Z = 3.13\,(21\times10^6)$ Nmm $= 65.73$ kNm

Transformed Section

Area of tension steel $(A_{st}) = 4\dfrac{\pi}{4}(20)^2 = 1256.64$ mm²

Now transformed area (A_T) = concrete area + transformed steel area

$= (A_g - A_{st}) + mA_{st}$

$= A_g + (m-1)A_{st}$

$= bD + (m-1)A_{st}$

$= 350\times600 + (13.33-1)\,1256.64$

$= 225.5\times10^3$ mm²

Depth of NA (y)

Taking moment of transformed area about the top edge

$$A_T y = (bD)\frac{D}{2} + (m-1)A_{st}d$$

$$\Rightarrow \quad y = \frac{\dfrac{(350)(600)^2}{2} + (13.33-1)1256.64(550)}{225.5\times10^3}$$

$= 317.17$ mm

Thus distance of NA from topmost compression fibre = 317.17 mm = y_c

and distance of NA from tension steel = 550 − 317.17 = 232.83 mm = y_s

distance of NA from bottom most tension fibre = 600 − 317.17 = 282.83 mm = y_t

∴ Second moment of area/MOI of transformed section

$$= I_T = \frac{by_c^3}{3} + \frac{by_t^3}{3} + (m-1)A_{st}y_s^2$$

$$= \frac{350\times317.17^3}{3} + \frac{350\times282.83^3}{3} + (13.33-1)1256.64(232.83)^2$$

$= 3722.4\times10^6 + 2639.5\times10^6 + 840\times10^6$

$= 7201.9\times10^6$ mm⁴

∴ Cracking moment $(M_{cr}) = f_{cr}\dfrac{I_T}{y_T} = 3.13\times\dfrac{7201.9\times10^6}{282.83}$ Nmm $= 79.7$ kNm

Thus cracking moment from gross area (= 65.73 kNm) is under estimated as compared to cracking moment using transformed area (= 79.7 kNm)

Stresses due to applied moment of 55 kNm

Applied moment (M) = 55 kNm < M_{cr}

Thus uncracked section analysis can be done.

∴ Maximum compressive stress in concrete

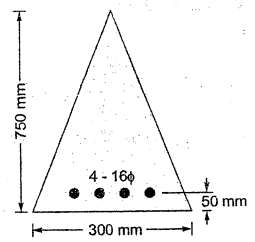
$$= f_c = \frac{M}{I_T}y_c = \frac{55\times10^6}{7201.9\times10^6}\times317.17 = 2.42\ \text{N/mm}^2$$

Maximum tensile stress in concrete $= \dfrac{M}{I_T}y_T = f_c\left(\dfrac{y_T}{y_c}\right) = 2.42\dfrac{(282.83)}{317.17}$

$= 2.16$ N/mm² < f_{cr} (= 3.13 N/mm²)

Maximum tensile stress in steel $= f_{st} = mf_c\left(\dfrac{y_s}{y_c}\right) = 13.33(2.42)\dfrac{232.83}{317.17} = 23.68$ N/mm²

Example 3.2 Calculate moment of resistance (MR) of the section as shown in figure below. Concrete grade is M20 and steel is Fe415.



Solution:

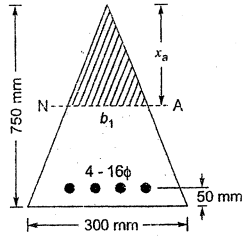
$\sigma_{cbc} = 7$ N/mm², $m = 13$, $\sigma_{st} = 230$ N/mm² and $d = 700$ mm

Limiting depth of neutral axis, $x_c = \dfrac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}\times d = 198.4$ mm

Actual depth of neutral axis, x_a will be calculated from similar triangle as,

$$\frac{b_1}{x_a} = \frac{300}{750}$$

$$b_1 = 0.4\,x_a$$



Equating moment of areas of both the sides about NA

$$\frac{1}{2}b_1 x_a \frac{x_a}{3} = mA_{st}(d-x_a)$$

$$\frac{1}{2}0.4x_a\frac{x_a^2}{3} = 13\times 804.24(700-x_a) \qquad \left[\because A_{st}+\frac{\pi}{4}\times(16)^2\times 4 = 804.24\ \text{mm}^2\right]$$

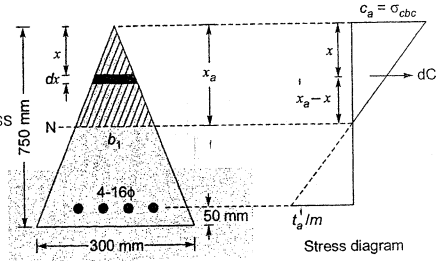
$$x_a = 371.48\ \text{mm} \approx 372\ \text{mm}$$

$$b_1 = 0.4\times 372\ \text{mm} = 148.8\ \text{mm}$$

$$x_a > x_c$$

It is an over reinforcement section.

$$\therefore \quad t_a < \sigma_{st}$$



Stress diagram

Let us consider an elementary strip of thickness 'dx' at a distance x from top.

Width of strip $= b_x = 0.40x$

Compressive force in strip $= dC = b_x dx C_x$

Now, from similar triangle in stress diagram

$$\frac{C_x}{(x_a-x)} = \frac{7}{x_a}$$

$$C_x = \frac{(x_a-x)}{x_a}\times 7 = \frac{372-x}{372}\times 7 = \frac{372-x}{53.14}$$

Moment of resistance of this elementary strip

$$dM_R = dC\times \text{lever arm} = dC(d-x) = b_x dx\left(\frac{372-x}{53.14}\right)\times(700-x)$$

∴Moment of resistance of the section

$$M_R = \int_0^{x_a} dM_R = \frac{1}{53.14}\int_0^{x_a} 0.4x(700-x)(372-x)dx = 33.19\ \text{kNm}$$

Example 3.3 Design a simply supported reinforced concrete beam using WSM of an effective span of 7.0 m. The beam is subjected to a live load of 42 kN/m. Width of the beam is 350 mm. Use M30 concrete and Fe500 steel.

Solution:

Given: Effective span (l) = 7.0 m

Width of the beam (b) = 350 mm

Assuming an initial depth of the beam as $\frac{\text{span}}{10} = \frac{7000}{10} = 700$ mm

Self weight of the beam = 0.7 × 0.35 × 25 kN/m = 6.12 kN/m

Live load = 42 kN/m (given)

Total load = 42 + 6.12 kN/m = 48.12 kN/m

Maximum bending moment for simply supported beam (M)

$$= \frac{wl^2}{8} = \frac{48.12\times 7^2}{8}\text{kNm} = 294.77\ \text{kNm}$$

Depth of the beam required from bending moment consideration

$$d = \sqrt{\frac{M}{Qb}}$$

Where, $Q = \frac{1}{2}cjk$

$$k = \frac{mc}{mc+t} = \frac{9\times 10}{9\times 10+275} = 0.246$$

$$j = 1-\frac{k}{3} = 0.918$$

$$Q = \frac{1}{2}\times 10\times 0.918\times 0.246 = 1.129 \simeq 1.13$$

$$\therefore \quad d = \sqrt{\frac{294.76\times 10^6}{1.13\times 350}}\ \text{mm} = 863.3\ \text{mm}$$

$D = 863.3 + 50$ mm = 913.3 mm (Assuming effective cover 50 mm)

Adopt $D = 950$ mm

∴ Self weight of the beam = 0.95 × 0.35 × 25 kN/m = 8.31 kN/m

Total load = 42 + 8.31 kN/m = 50.31 kN/m

$$M = \frac{wl^2}{8} = 50.31\times\frac{7^2}{8} = 308.16\ \text{kNm}$$

$$d = \sqrt{\frac{308.16\times 10^6}{1.13\times 350}}\ \text{mm} = 882.7\ \text{mm}$$

Overall depth of the beam (D) = 882.7 + 50 mm = 932.7 mm < 950 mm (OK)

∴ Provide $D = 950$ mm, $d = 900$ mm

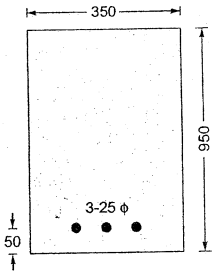
and $b = 350$ mm

Area of steel required

$$A_{st} = \frac{M}{\sigma_{st}\left(d-\frac{X_c}{3}\right)} = \frac{M}{\sigma_{st}(jd)}$$

$$= \frac{308.16\times 10^6}{275(0.918\times 900)} = 1356.31\ \text{mm}^2$$

Using 25 mm dia. bars, no. of bars required $= \frac{1356.31}{\frac{\pi}{4}\times 25^2} = 2.76 \simeq 3$ (say)



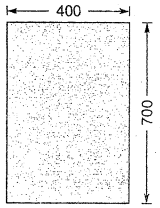
Provide 3-25 ϕ bars as tension reinforcement.

Example 3.4 An RCC rectangular beam of size 400 × 700 mm overall depth is to be designed for BM of (i) 120 kNm (ii) 260 kNm. Calculate area of steel required for above moments if M 25 concrete and Fe 415 steel are used. Calculate steel required just for balanced section. Assume effective cover as 50 mm.

Solution:

Given: M 25 and Fe 415

$B = 400$ mm, $D = 700$ mm



Assume effective cover = 50 mm

$\therefore$ Effective depth, $d = 700 - 50 = 650$ mm

$\sigma_{st} = 230$ N/mm^2, $\sigma_{cbc} = 8.5$ N/mm^2, $m = 11$

Step-1: Moment of resistance of singly reinforced balanced section

$$Q = \frac{1}{2}cjk$$

$$k = \frac{mc}{mc+t} = \frac{11\times8.5}{11\times8.5\times230} = 0.289$$

$$J = 1-\frac{k}{3} = 0.903$$

$$Q = \frac{1}{2}\times8.5\times0.903\times0.289 = 1.109$$

$$MR = QBd^2 = 1.109\times400\times650^2 = 187.49 \text{ kNm}$$

Step-2: Area of steel for balanced section

$$A_{st} = \frac{MR_{bal}}{\sigma_{st}\times jd} = \frac{187.4\times10^6}{230\times0.903\times656} = 1389 \text{ mm}^2$$

Step-3: Design for BM = 120 kNm

$$BM < MR_{bal}$$

So we need a under reinforced section

Condition $x_a < x_c$, $C_a < \sigma_{cbc}$, $t_a = \sigma_{st} = 230$ N/mm^2

Equating BM = MR (from tension side)

$$BM = \sigma_{st}A_{st}\left(d-\frac{x_a}{3}\right)$$

$$120\times10^6 = 230\times A_{st}\left(650-\frac{x_a}{3}\right) \quad ...(i)$$

From compression side

$$BM = B\times x_a\times\frac{C_a}{2}\left(d-\frac{x_a}{3}\right)$$

$$120\times10^6 = 400\times x_a\times\frac{C_a}{2}\left(650-\frac{x_a}{3}\right) \quad ...(ii)$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{(d-x_a)}$$

$$C_a = \frac{t_a\times x_a}{m(d-x_a)} = \frac{230\times x_a}{11(650-x_a)} \quad ...(iii)$$

From eq. (ii) and (iii),

$$120\times10^6 = 200\times x_a\times\frac{230\times x_a}{11(650-x_a)}\times\left(650-\frac{x_a}{3}\right)$$

$$(120\times10^6)\times11(650-x_a) = 200\times230\times x_a^2\left(650-\frac{x_a}{3}\right)$$

$$-1.32\times10^9 x_1 + 8.58\times10^{11} = 29.9\times10^6\times x_a^2 - 15333.34\, x_a^3$$

Solving by trial and error method

$$x_a = 154.2 \text{ mm}$$

Area of steel required

$$A_{st} = \frac{BM}{\sigma_{st}\left(650-\frac{x_a}{3}\right)} = \frac{120\times10^6}{230\times\left(650-\frac{154.2}{3}\right)} = 871.58 \text{ mm}^2$$

If balanced section formula is used

$$A_{st} = \frac{BM}{\sigma_{st}\cdot(jd)} = \frac{120\times10^6}{230\times(0.903\times650)} = 888.8 \text{ mm}^2$$

In under reinforced section we can use the formula of balanced section because this difference is negligible.

NOTE: For beams, balanced section formula can be used for under reinforced section with some minor error.

Step-4: Design for BM = 260 kNm

BM > MR_{bal}

So we need either an over reinforced section or a doubly reinforced section

Condition for over reinforced section

$$C_a = \sigma_{cbc}$$

$$x_a > x_c$$

$$t_a < \sigma_{st}$$

BM = MR

$$260\times10^6 = bx_a\frac{C_a}{2}\left(d-\frac{x_a}{3}\right) = 400\times x_a\times\frac{8.5}{2}\left(650-\frac{x_a}{3}\right)$$

$$152.94\times10^3 = 650x_a - \frac{x_a^2}{3}(0.33x_a^2)$$

$$x_a = 273.71 \text{ mm}$$

$\Rightarrow$

$$260\times10^6 = t_a\times A_{st}\left(d-\frac{x_a}{3}\right)$$

$$A_{st} = \frac{260\times10^6}{t_a\times\left(650-\frac{273.71}{3}\right)}$$

Now,

$$\frac{C_a}{x_a} = \frac{t_a/m}{d-x_a}$$

$$t_a = \frac{C_a m(d-x_a)}{x_a} = \frac{8.5\times11\times(650-273.71)}{273.71} = 128.54 \text{ N/mm}^2$$

$$A_{st} = \frac{260\times10^6}{128.54\times\left(650-\frac{273.71}{3}\right)} = 3619 \text{ mm}^2$$

If balanced section formula is used

$$A_{st} = \frac{260\times10^6}{\sigma_{st}\times jd} = \frac{260\times10^6}{230\times6.903\times650} = 1925 \text{ mm}^2$$

This formula should never be used for over reinforced section.

Example 3.5 The size of a rectangular beam is 400 × 650 mm. Area of steel used

(i) 3 Nos – 16 mm ϕ bar

(ii) 6 Nos. – 25 mm ϕ bar

M 30 concrete and Fe 415 steel are used

Calculate MR of the section in two cases and area of steel required for the balanced section also. Effective cover is 50 mm.

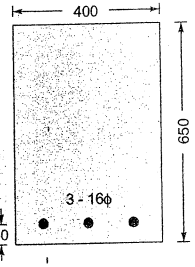
Solution:

For Case (i) 3 Nos. 16 mm ϕ bar

b = 400 mm, D = 650 mm and d = 650 – 50 = 600 mm

$$A_{st} = 3\times\frac{\pi}{4}\times16^2 = 603.2 \text{ mm}^2$$

For M 30, m = 9, C = 10, t = 230



1. Critical depth of NA

$$x_c = kd = \frac{mc}{t+mc}\times d = \frac{9\times10}{230+9\times10}\times600$$
$$= 168.75 \text{ mm}$$

2. Actual depth of NA

Equating moment of area

$$\frac{bx_a^2}{2} = mA_{st}(d-x_a)$$

$$\frac{400\times x_a^2}{2} = 9\times603.2\,(600-x_a)$$

$$200\,x_a^2 = 3257280 - 5428.8\,x_a$$

$$x_a = 114.76 \text{ mm}$$

3. $x_a < x_c$, It is under reinforced

$$C_a < \sigma_{cbc}$$
$$t_a = \sigma_{st} = 230 \text{ N/mm}^2$$

4. Compressive stress in concrete

$$\frac{C_a}{x_a} = \frac{t_a/m}{d-x_a} = \frac{\sigma_{st}/m}{d-x_a}$$

$$C_a = \left(\frac{\sigma_{st}/m}{d-x_a}\right)x_a = \frac{114.76}{(600-114.76)}\times\frac{230}{9} = 6.04 \text{ N/mm}^2$$

From compression side

$$MR = bx_a\frac{C_a}{2}\left(d-\frac{x_a}{3}\right) = 400\times114.76\times\frac{6.04}{2}\left(600-\frac{114.76}{3}\right)\times\frac{1}{10^6}$$

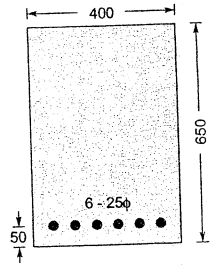
$$MR = 77.87\times10^6 = 77.87 \text{ kNm}$$

From tension side

$$MR = \sigma_{st}.A_{st}\left(d-\frac{x_a}{3}\right) = 230\times603.2\times\left(600-\frac{114.76}{3}\right)\times\frac{1}{10^6} = 77.9 \text{ kNm}$$

For Case (ii) 6 Nos. 25 mmϕ bar

$$A_{st} = \frac{\pi}{4}\times25^2\times6 = 2945.24 \text{ mm}^2$$



1. The value of critical depth of NA (x_c) = 168.75 mm

2. Actual depth of NA

$$b\frac{x_a^2}{2} = mA_{st}(x_a)$$

$$400\frac{x_a^2}{2} = 9\times2945.24\,(600-x_a)$$

$$x_a = 223.41 \text{ mm}$$

3. $x_a > x_c$, it is an over reinforced section

$$C_a = \sigma_{cbc} = 10 \text{ N/mm}^2$$
$$t_a < \sigma_{st}$$

4. From compression side

$$MR = \frac{1}{2}bx_aC_a\times\left(d-\frac{x_a}{3}\right) = \frac{1}{2}\times400\times223.41\times10\left(600-\frac{223.41}{31}\right)\times\frac{1}{10^6}$$

$$MR = 234.8 \text{ kNm}$$

5. Tensile stress in steel

$$\frac{C_a}{x_a} = \frac{t_a/m}{d-x_a}$$

$$t_a = \frac{(d-x_a)\sigma_{cbc}m}{x_a} = \frac{(600-223.41)\times10\times9}{223.41} = 151.7 \text{ N/mm}^2$$

From tension side

$$MR = t_aA_{st}\left(d-\frac{x_a}{3}\right) = 151.7\times2945.24\times\left(600-\frac{223.4}{3}\right)\times\frac{1}{10^6}$$

$$MR = 234.84 \text{ kNm}$$

For the case of balanced section

$$x_a = x_c$$
$$\sigma_{cbc} = C$$
$$t = \sigma_{st}$$

$$\Rightarrow \quad MR = bx_a\frac{C_a}{2}\left(d-\frac{x_a}{3}\right) = 400\times168.75\times\frac{10}{2}\left(600-\frac{168.75}{3}\right)\frac{1}{10^6}$$

$$MR = 183.52 \text{ kNm}$$

Area of steel for balanced section

$$MR = t_aA_{st}\left(d-\frac{x_a}{3}\right)$$

$$183.52\times10^6 = 230A_{st}\left(600-\frac{168.75}{3}\right)$$

$$797936.4128 = 600A_{st} - \frac{168.75}{3}A_{st}$$

$A_{st} = 1467.47\ mm^2$

Second method for area of steel

$C = T$ (valid in both WSM and LSM)

$$bx_a\frac{C_a}{2} = t_a.A_{st}$$

$$A_{st} = \frac{bx_aC_a}{2t_a} = \frac{400\times168.75\times10}{2\times230} = 1467.47\ mm^2$$

Third method for area of steel : Equating moment of area on both sides of neutral axis

$$b\frac{x_a^2}{2} = mA_{st}(d-x_a)$$

$$b\frac{x_c^2}{2} = mA_{st}(d-x_c)$$

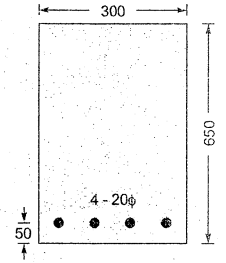
(for balanced section)

$$A_{st} = \frac{bx_c^2}{2m(d-x_c)} = 1467.4\ mm^2$$

Value	Under Reinforced	Balanced	Over Reinforced
Size	400 × 650 mm	400 × 650 mm	400 × 650 mm
A_{st}	603.2 mm^2	1467.69 mm^2	2945.24 mm^2
X_v	x_a = 114.76 mm	x_c = 168.75 mm	x_a = 223.4 mm
M_u	77.9 kNm	183.50 kNm	234.8 kNm
C_a	6.04 N/mm^2	10 N/mm^2	10 N/mm^2
t_a	$t = \sigma_{st}$ = 230 N/mm^2	$t = \sigma_{st}$ = 230 N/mm^2	t_a = 151.7 $N/mm^2 < \sigma_{st}$
Lever Arm	561.74 mm	543.75 mm	525.53 mm

Example 3.6 A rectangular beam of the size 300 x 650 mm is reinforced with 4 no. 20 mmϕ bars. Calculate stresses developed in steel and concrete due to BM of (i) 90 kNm (ii) 210 kNm (modular ratio method WSM). Modular ratio (m) = 15. Assume effective cover 50 mm.

Solution:



$$\text{Area} = \frac{\pi}{4}\times20^2\times4 = 1256.63\ mm^2$$

(i) For BM of 90 kNm

1. Actual depth of NA

$$b\frac{x_a^2}{2} = mA_{st}(d-x_a)$$

$$300\frac{x_a^2}{2} = 15\times4\times\frac{\pi}{4}(20)^2(600-x_a)$$

$$15x_a^2 = 11309670 - 18849.45x_a$$

$$x_a = 218.85\ mm$$

2. From compression side

$$BM = MR$$

$$90\times10^6 = bx_a\frac{C_a}{2}\left(d-\frac{x_a}{3}\right)$$

$$90\times10^6 = 300\times218.85\times\frac{C_a}{2}\left(600-\frac{218.85}{3}\right)$$

$$C_a = 5.2\ N/mm^2$$

From tension side

$$BM = t_a\times A_{st}\left(d-\frac{x_a}{3}\right)$$

$$90\times10^6 = t_a\times1256.64\times\left(600-\frac{218.85}{3}\right)$$

$$t_a = 135.9\ N/mm^2$$

2nd method for t_a

$$\frac{C_a}{x_a} = \frac{t_a/m}{d-x_a}$$

$$t_a = \frac{mC_a(d-x_a)}{x_a} = 135.84\ N/mm^2$$

For BM of 210 kNm linear interpolation can be done for stress in concrete and steel.

For 90 kNm moment, concrete stress = 5.2 N/mm^2

$\therefore$ For 210 kNm moment, concrete stress = $\frac{5.2}{90}\times210 = 12.13\ N/mm^2$

Similarly stress in steel = $\frac{135.9}{90}\times210 = 317.1\ N/mm^2$

Example 3.7 A simply supported rectangular beam of size 400 x 650 mm is to be provided for a simply supported span of 6.0 m. The beam has to support a live load of 55 kN/m (excluding self weight). Calculate area of steel required using WSM. Use M 30 concrete and Fe 500 steel. Effective cover = 50 mm.

Solution:

σ_{st} = 275 N/mm^2, m = 9, σ_{cbc} = 10 N/mm^2, b = 400 mm, D = 650 mm and d = 600 mm

Span = 6 m

Live load = 55 kN/m

Self weight = 0.4 x 0.65 x 1 x 25 = 6.5 kN/m

Total load = 55 + 6.5 = 61.5 kN/m

1. Maximum Bending moment = $\frac{wl^2}{8} = \frac{61.5\times6^2}{8} = 276.75$ kNm

$$k = \frac{mc}{mc+t} = \frac{9\times10}{9\times10+275} = 0.246$$

$$j = 1-\frac{k}{3} = 0.917$$

$$C = 10$$

$$Q = \frac{1}{2}\times cjk = \frac{1}{2}\times10\times0.917\times0.246 = 1.128$$

2. Moment of resistance of singly reinforced balanced section

$MR_1 = Qbd^2 = 1.128 \times 400 \times 600^2 = 162.432$ kNm

Total BM > MR_1 (so we need a doubly reinforced section)

3. $A_{st1} = \frac{MR_1}{\sigma_{st}(jd)} = \frac{162.432\times10^6}{275\times(0.917\times200)} = 1073.5\ mm^2$

4. Remaining BM, MR_2 = BM – MR_1 = 276.75 – 162.6 = 114.15 kNm

5. $A_{st2} = \frac{MR_2}{\sigma_{st}(d-d_c)} = \frac{114.15\times10^6}{275\times(600-50)} = 754.7\ mm^2$

6. Total $A_{st} = A_{st_1} + A_{st2}$ = 1073.5 + 754.7 = 1828.2 mm^2

7. Value of compressive stress in concrete at the level of compression steel

$$C' = \frac{(x_a - d_c)}{x_a} \times C_a \qquad (\because C_a = 10)$$

$$= \frac{(x_c - d_c)}{x_c} \times C_a \qquad (\because x_c = kd)$$

$$= \frac{(0.246 \times 600 - 50)}{0.246 \times 600} \times 10 = 6.612 \text{ N/mm}^2$$

8. $$A_{sc} = \frac{MR_2}{(1.5m-1)C'(d-d_c)} = \frac{114.15 \times 10^6}{(1.5 \times 9 - 1)6.612 \times (600 - 50)}$$

$$= 2511 \text{ mm}^2$$

Alternate formula for A_{sc}

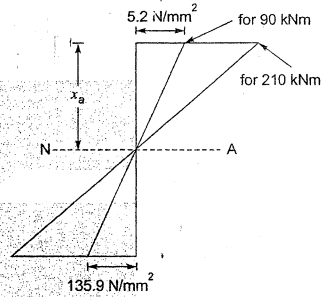
$$A_{sc} = (1.5m - 1)A_{sc}(x_a - d_c)$$

$$= mA_{st2}(d - x_a)$$

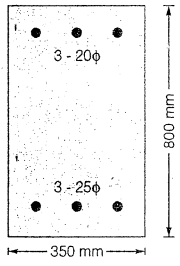
$$A_{sc} = \frac{mA_{st_2}(d - x_a)}{(1.5 \times 9 - 1)(x_a - d_c)}$$

$$= \frac{9 \times 754.7 \times (600 - 147.6)}{(1.5 \times 9 - 1) \times (147.6 - 50)}$$

$$= 2518 \text{ mm}^2$$



Example 3.8 Calculate the moment of resistance of a doubly reinforced beam as shown in figure using WSM. Use M 25 concrete and Fe 500 steel. Assume effective cover 50 mm.



Solution:

Given:

Width of the beam (b) = 350 mm

Overall depth of the beam (D) = 800 mm

Effective cover = 50 mm

Effective depth of the beam (d) = 800 – 50 mm = 750 mm

Area of tension steel (A_{st}) = $3 \times \frac{\pi}{4} \times 25^2 = 1472.62 \text{ mm}^2$

Area of compression steel (A_{sc}) = $3 \times \frac{\pi}{4} \times 20^2 = 942.5 \text{ mm}^2$

Design coefficient, $k = \frac{mC}{mC + t} = \frac{11 \times 8.5}{11 \times 8.5 + 275} = 0.254$

Limiting depth of neutral axis, $x_c = kd = 0.254 \times 750 \text{ mm} = 190.5 \text{ mm}$

Actual depth of neutral axis (x_a)

$$b\frac{x_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d_c) = mA_{st}(d - x_a)$$

$$\frac{350}{2}x_a^2 + (1.5 \times 11 - 1) \times 942.5(x_a - 50) = 11 \times 1472.62(750 - x_a)$$

$$x_a = 197.19 \text{ mm} > x_c (= 190.5 \text{ mm})$$

∴ Section is over reinforced

$$C_a = \sigma_{cbc} = 8.5 \text{ N/mm}^2$$

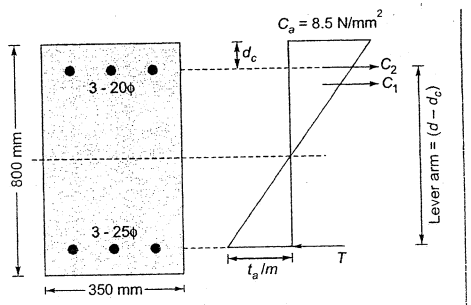
and $f_a < \sigma_{st}$

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a}$$

$$C' = \frac{x_a - d_c}{x_a} \times C_a$$

$$= \frac{197.19 - 50}{197.19} \times 8.5$$

$$= 6.345 \text{ N/mm}^2$$



Moment of resistance

$$M_R = bx_a\frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1)A_{sc}C'(d - d_c)$$

$$= 350 \times 197.19 \times \frac{8.5}{2}\left(750 - \frac{197.19}{3}\right) + (1.5 \times 11 - 1) \times 942.47 \times 6.344(750 - 50)$$

$$= 266.5 \text{ kNm}$$

Objective Brain Teasers

Q.1 Match the following

A. FOS of concrete in bending compression	I. 4
B. FOS of concrete in direct compression	Ii. 8.5 to 9.5
C. FOS of concrete in direct tension	III.3

(a) A-I, B-II, C-III (b) A-I, B-III, C-II

(c) A-II, B-I, C-III (d) A-III, B-I, C-II

Q.2 The ratio of working load to ultimate load is equal to

(a) FOS (b) $(FOS)^{-1}$

(c) Load factor (d) None of the above

Q.3 As per IS 456: 2000 provisions, stresses in concrete and steel can be increased upto _____ when the structure is subjected to wind loading and seismic forces.

(a) 25% (b) 50%

(c) $33\frac{1}{3}\%$ (d) 10%

Q.4 The factor of safety for stress in steel is taken as

(a) 1.15 (b) 1.3

(c) 1.8 (d) 2.0

Q.5 Analysis of flanged beams can be done just like rectangular beam when

(a) $kd \le D_f$ (b) $kd > D_f$

(c) $kd \le b_f$ (d) $kd > b_f$

Q.6 Which of the following is true?

The modular ratio (m) expressed as $m = \frac{280}{3\sigma_{cbc}}$

(a) does not take into account the long term effects of creep and shrinkage

(b) partially takes into account the long term effects of creep and shrinkage

(c) is the ratio of modulus of elasticity of concrete to steel

(d) (b) and (c)

Q.7 The product '$m\sigma_{cbc}$' is

(a) a real variable (b) a complex variable

(c) a complex constant (d) a real constant

Q.8 Cracking moment is useful while analysing a beam section as

(a) cracked section

(b) partially cracked section

(c) uncracked section

(d) plastic section

Q.9 Which of the following statement is false?
The concrete on tension side of neutral axis is useful because
(a) it resists the applied moment in cracked phase of concrete
(b) it holds the reinforcing bars in place
(c) it prevents the bars from corrosion
(d) option (b) and (c)

Q.10 For concrete members subjected to axial compression, the permissible shear stress in concrete shall be increased by a factor of

(a) $1-\frac{5P}{A_g f_{ck}}$ (b) $1+\frac{5P}{A_g f_{ck}}$

(c) $1+\frac{3P}{A_g f_{ck}}$ (d) $\frac{5P}{A_g f_{ck}}$

Q.11 In WSM of design, modular ratio (M) is defined as $280/3\sigma_{cbc}$. The extent of creep allowance taken in this definition of modular ratio (M) is
(a) Partial compensation
(b) No compensation
(c) Full compensation
(d) Insufficient data

Answers

1. (d)	2. (b)	3. (c)	4. (c)	5. (a)
6. (b)	7. (d)	8. (c)	9. (d)	10. (b)
11. (a)				

Hints:

2. (b)

$$FOS = \frac{\text{ultimate load}}{\text{working load}}$$

6. (b)

$$m = \frac{E_s}{E_c}$$

7. (d)

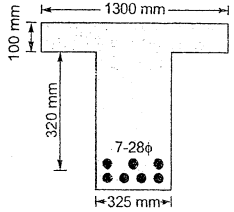
$$m\sigma_{cbc} = \frac{280}{3} = \text{real constant}$$

Conventional Practice Questions

Q.1 A beam caries a uniformly distributed service load of 38 kN/m including self weight on a simply supported span of 7.0 m. The beam width is 300 mm and effective depth is 655 mm and is reinforced with 4-25ϕ Fe 415 bars. Determine the stresses developed in concrete and steel at service load. Use M 20 concrete.

Ans. [10.4 N/mm^2, 209 N/mm^2]

Q.2 A T-beam section is as shown below. Using M20 concrete and Fe415 steel, determine the stresses developed in concrete and steel under a service moment of 150 kN-m.

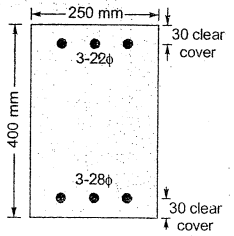


Ans. [4.3 N/mm^2, 93 N/mm^2]

Q.3 In Q.2 above, the allowable moment capacity of the beam at service load is _____

Ans. [244 kNm]

Q.4 A doubly reinforced beam section is as shown. Using M20 concrete and Fe415 steel, calculate the stresses in concrete and steel at a service moment of 125 kNm.



Ans. [11.7 N/mm^2, 170 N/mm^2, 218 N/mm^2]

Q.5 In Q.4 above, what is the allowable moment capacity of the section?

Ans. [74.5 kNm]

CHAPTER 04

Analysis and Design by Limit State Method (LSM)

4.1 Introduction

In any type of structure, the basic structural elements viz. beams, columns, walls, slabs, footings etc. are subjected to bending, transverse shear, torsion and axial tension or compression. In the subject of Strength of Materials, we talk of pure flexure i.e. shear stresses are zero. But in practical situations, the concept of pure flexure rarely exists; rather flexure is often associated with transverse shear and/or axial forces (tension or compression). Torsion is also sometimes encountered along with flexure.

In this chapter, we will look into the analysis of reinforced concrete beams subjected to pure flexure only. Furthermore, we will discuss the design aspects of reinforced concrete beams by **limit state method.** The additional effects associated with flexure viz. transverse shear, torsion etc. are dealt in separate chapters.

4.2 Analysis of Beams by LSM

In the previous chapters, we studied the WSM of analysis and design wherein the structure was subjected to working loads. This chapter deals with the **limit state method** of analysis and design at **ultimate loads.** Two types of Limit States are defined in Limit State Method of design viz. Limit State of Collapse and Limit State of Serviceability which are defined as below:

Limit State of Collapse: In this state, loads are corresponding to impending failure of structure and lead to complete collapse of the structure. It depicts the **imaginary behavior of structure** at the time of failure. It includes limit state of flexure, shear, compression and torsion.

Limit State of Serviceability: In this state, the loads and stresses which are applicable in the day-to-day service of the structure and structure is expected to perform its intended function. It depicts the **actual behavior** of structure at the time of service of the structure. It includes limit state of deflection, cracking, vibration, corrosion etc.

4.3 Assumptions in the Analysis Design by LSM

As per Cl. 38.1 of IS 456: 2000, the following assumptions are made while analyzing the reinforced concrete beam by LSM:

1. Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section. Thus, strain diagram is linear as shown in the Fig. 4.1.
2. The maximum compressive strain in concrete at the outermost fiber (ϵ_{cu}) is taken as 0.0035, regardless of whether the beam is under-reinforced or over-reinforced, because collapse invariably occurs by the crushing of concrete.
3. The design stress-strain curve of concrete in flexural compression as recommended by IS 456: 2000 is as shown in Fig. 4.2. The IS 456: 2000 also allows the use of any other possible shape of the stress-strain curve which results in substantial agreement with the results of the tests on reinforced concrete.
4. For design purposes, compressive strength of concrete may be assumed as 0.67 times the characteristic strength of concrete. The partial safety factor of $\gamma_c = 1.5$ shall be applied in addition to this.
5. The tensile strength of concrete is ignored i.e. not taken into account. Cl. B-1.3(b) of IS 456: 2000 states that all tensile stresses are to be taken up by reinforcement and none by concrete, except as otherwise specifically permitted.
6. The stress in reinforcement is derived from representative stress - strain curve for the type of steel used. The design stress-strain curves for mild steel and cold-worked bars are as shown in Fig. 4.4 and Fig. 4.5.

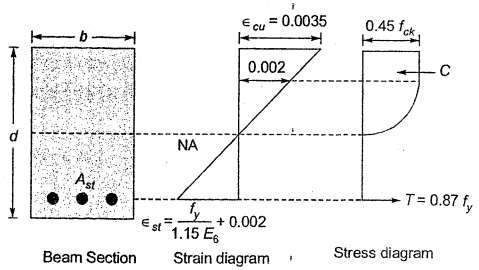


Fig. 4.1 *Singly reinforced beam section*

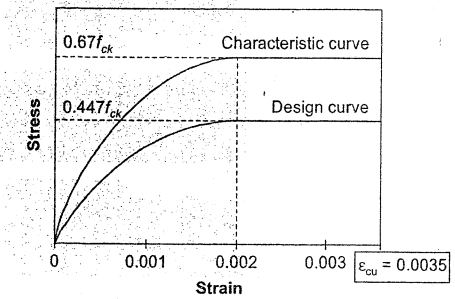


Fig. 4.2 *Stress-strain curve of concrete in flexural compression*

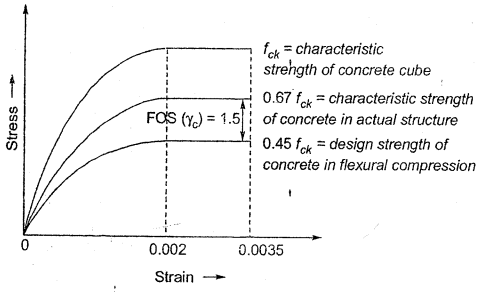


Fig. 4.3 *Characteristic, actual and design stress-strain curve of concrete in flexural compression*

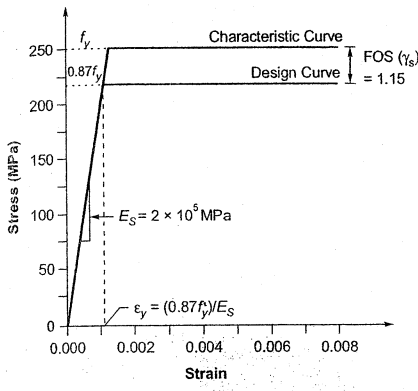


Fig. 4.4 *Characteristic and design stress strain curve for mild steel (Fe 250)*

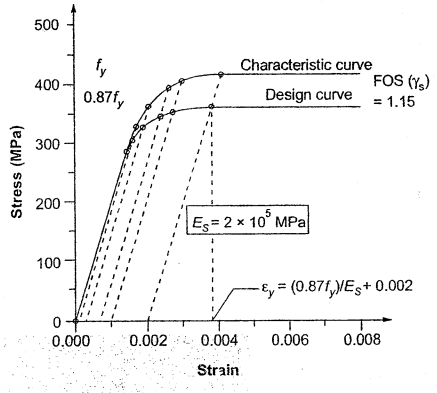


Fig. 4.5 *Characteristic and design stress strain curve for cold worked Fe 415 grade steel*

7. For design purpose, the partial safety factor for steel is taken as $\gamma_s = 1.15$ i.e.

$$\text{design stress of steel} = \frac{f_y}{1.15} = 0.87 f_y$$

8. The maximum strain (ϵ_{st}) in the tension reinforcement at the level of centroid of reinforcement steel at the ultimate limit state shall not be less than ϵ_{st} which is defined as:

$$\epsilon_{st} = \frac{0.87 f_y}{E_s} + 0.002$$

Mild steel has a well defined yield point $\left(\epsilon_y = \frac{0.87 f_y}{E_s}\right)$ but this is not so for HYSD bars. IS 456: 2000 specify a uniform criterion of yield for all grades of steel. This is done to ensure that the yielding of the tension steel takes place at the ultimate limit state, so that the consequent failure is ductile in nature and thus providing ample warning of the impending collapse by limiting the minimum permissible steel strain to $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$.

A very rare type of RCC failure is "FRACTURE OF REINFORCING STEEL" which happens due to extremely low amounts of reinforcing steel and under the conditions of dynamic loading.

Remember



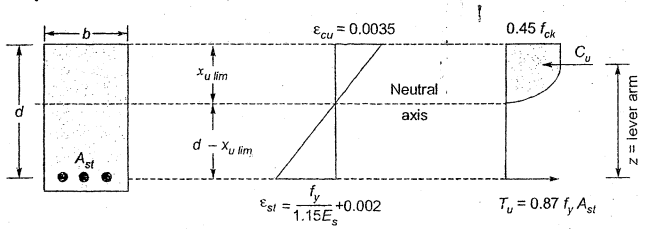
Limitation of the Assumption "Plane Sections Remain Plane before and after the Bending"

This assumption is NOT applicable in case of **deep beams** where warping of cross-section occurs due to shear deformations.

4.4 Analysis of Singly Reinforced Sections

4.4.1 Limiting Depth of Neutral Axis

It corresponds to the loading when concrete compression and steel tension, both reach their ultimate limit state simultaneously.



(a) Beam section (b) Strains (c) Stresses

Fig. 4.6 *Reinforced concrete rectangular section at ultimate limit state of flexure*

Strain diagram is used for calculating the limiting depth of neutral axis. From strain diagram, by similar triangles,

$$\frac{0.0035}{x_{u\,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{d - x_{u\,lim}}$$

$$\frac{d - x_{u\,lim}}{x_{u\,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{0.0035}$$

Thus, $$\frac{x_{u\,lim}}{d} = k = \frac{700}{0.87f_y + 1100}$$

$$\Rightarrow \quad x_{u\,lim} = kd = \left(\frac{700}{0.87f_y + 1100}\right)d$$

4.4.2 $\frac{x_{u\,lim}}{d}$ For Different Grades of Steel

The limiting depth of neutral axis corresponds to balanced failure at ultimate limit state i.e. concrete and steel will reach to their limiting strain in simultaneously. If $x_u < x_{ulim}$, then the section is under-reinforced leading to yielding in steel first and if $x_u > x_{ulim}$ then the section is over-reinforced leading to crushing of concrete first.

Table 4.1 : *Limiting depth of neutral axis*

Steel grade	Fe 250	Fe 415	Fe 500
$k = \frac{x_{u\,lim}}{d}$	0.53	0.48	0.46

4.4.3 Analysis of Singly Reinforced Rectangular Section

Analyzing a reinforced concrete member at ultimate limit state is equivalent to determining the ultimate moment of resistance (M_{uR}) of the reinforced concrete section. This is computed by,

$$M_{uR} = C_u \cdot z = T_u \cdot z$$

Where, C_u and T_u are the ultimate compression and tension forces in the concrete and steel respectively.

Analysis of Stress Diagram

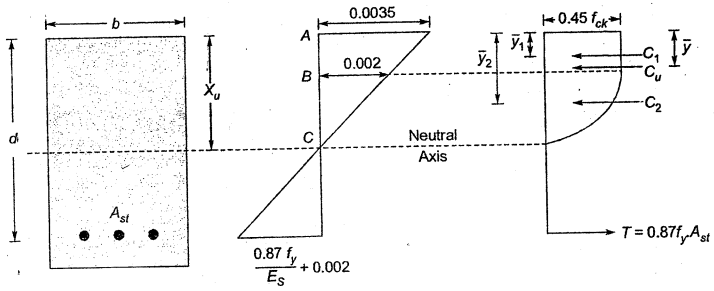


Fig. 4.7 *Analysis of stress diagram*

By similar triangles of strain diagram,

$$\frac{BC}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$BC = \frac{4}{7}AC = \frac{4}{7}x_u$$

$$AB = x_u - \frac{4}{7}x_u = \frac{3}{7}x_u$$

Compressive Force

For determining the value of C_u and its line of action, it is essential to analyze the concrete stress block under compression.

C_1 = width x area of stress diagram

$$= b \times 0.45\,f_{ck} \times \left(\frac{3}{7}\right)x_u = 0.193\,f_{ck}\,bx_u$$

$$y_1 = \left(\frac{1}{2}\right) \times \left(\frac{3}{7}\right)x_u = \left(\frac{3}{14}\right)x_u$$

$$C_2 = b \times \left(\frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7}x_u\right) = 0.1714\,f_{ck}\,bx_u$$

$$y_2 = \left(\frac{3}{7}\right)x_u + \left(\frac{3}{8}\right) \times \left(\frac{4}{7}\right)x_u = 0.643\,x_u$$

Total Compressive Force

$$C_u = C_1 + C_2$$

$$= 0.193 f_{ck}\,b\,x_u + 0.1714\,f_{ck}\,bx_u = 0.36\,f_{ck}\,bx_u$$

This total compressive force (C) acts at a distance $\bar{y}$ from top of compression fibre which is given by,

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2} = \frac{(0.193 \times 0.214 + 0.1714 \times 0.643)}{0.3644}x_u$$

$= 0.416 x_u$ (As per previous code IS 456: 1978)

$= 0.42 x_u$ (As per recent code IS 456: 2000)

Alternatively it can also be derived as,

The line of action of C_u is determined by the centroid of the concrete stress block in compression. Thus distance of C_u from the fibers subjected to the maximum compressive strain is:

$$0.362 f_{ck} b x_u \cdot \bar{x} = (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\cdot\left(\frac{1.5x_u}{7}\right)+\left(\frac{2}{3}\cdot\frac{4}{7}\right)\left(x_u - \frac{5}{8}\cdot\frac{4x_u}{7}\right)\right]$$

$$\Rightarrow \quad \bar{x} = 0.42 x_u$$

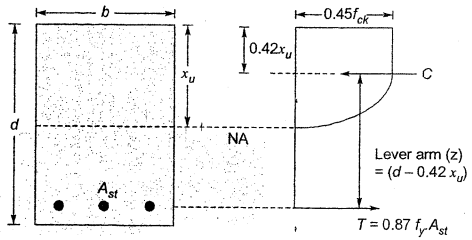
Tensile Force

$$T_u = f_{st} \cdot A_{st}$$

Where $f_{st} = 0.87 f_y$ for all $x_u \le x_{u\,lim}$

$\therefore \quad T_u = 0.87 f_y A_{st}$

The line of action of T_u is the centroid of tension steel.



Actual Depth of Neutral Axis

For any given section, the location of neutral axis will be such that it satisfies the condition of static equilibrium i.e., $C_u = T_u$. Thus,

Total force of compression = Total force of tension

$$C = T$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st} \qquad (f_{st} = 0.87 f_y \text{ if } x_u \le x_{u\,lim})$$

$$\Rightarrow \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \text{ for all } x_u \le x_{u\,lim}$$

NOTE: When $x_u > x_{u\,lim}$, then section is over-reinforced and $f_{st} \neq 0.87 f_y$ and in this case, a trial and error procedure is followed for determining the value of x_u.

Table 4.2: *Difference between Tension and Compression failure of a beam section*

TENSION FAILURE	COMPRESSION FAILURE
Tension reinforcement yields before limit state of collapse of beam.	Tension reinforcement does not yield prior to limit state of collapse of beam.
In this, the beam shows large amounts of deflection and excessive cracks before its actual failure. This type of failure gives ample warning and is said to be ductile failure.	In this, the beam fails by crushing of concrete. This type of failure is sudden without giving any warning and thus this type of failure is always to be avoided.

Moment of Resistance

For determining the moment of resistance M_R, the lever arm z i.e., the distance between compressive force (C) and tensile force (T) is given by:

$$z = d - 0.42 x_u$$

Thus moment of resistance w.r.t. compressive force of beam section

$$M_R = 0.362 f_{ck} b x_u (d - 0.42 x_u)$$

and the moment of resistance w.r.t. tensile force of beam section

$$M_R = f_{st} \cdot A_{st} (d - 0.42 x_u)$$

$$= 0.87 f_y A_{st} (d - 0.42 x_u) \quad \text{if } x_u \le x_{u\,lim}$$

Limiting Moment of Resistance

The limiting moment of resistance for a singly reinforced rectangular section is obtained when $x_u = x_{u\,lim}$.

Thus, $$M_{u\,lim} = 0.362 f_{ck} b x_{u\,lim} (d - 0.42 x_{u\,lim})$$

$$\Rightarrow \quad \frac{M_{u\,lim}}{f_{ck} b d^2} = 0.362 \left(\frac{x_{u\,lim}}{d}\right)\left(1 - 0.42 \frac{x_{u\,lim}}{d}\right)$$

The values of the above quantity i.e. $\frac{M_{u\,lim}}{f_{ck} b d^2}$ is obtained as 0.148, 0.138 and 0.133 for Fe 250, Fe 415 and Fe 500 respectively by substituting the values of $\frac{x_{u\,lim}}{d}$ as discussed earlier.

Comparison of x_u and x_{ulim}

(a) When $x_u < x_{ulim}$, then the section is under - reinforced

and, $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$ i.e., steel has been yielded ($f_{st} = 0.87 f_y$)

(b) When $x_u = x_{u\,lim}$, then section is **balanced** and use $x_{u\,lim}$ for calculation the moment of resistance (M_R)

$$M_{u\,lim} = 0.36 f_{ck} b x_{ulim} (d - 0.42 x_{ulim}) \qquad \text{(from compression side)}$$

$$M_{u\,lim} = 0.87 f_y A_{st} (d - 0.42 x_{ulim}) \qquad \text{(from tension side)}$$

(c) When $x_u > x_{u\,lim}$, then section is over-reinforced and M_u is limited to $M_{u\,lim}$ by limiting x_u to $x_{u\,lim}$. $M_{u\,lim}$ can be found using compression side formula as:

$$M_{ulim} = 0.36 f_{ck} b x_{ulim} (d - 0.42 x_{ulim})$$

Remember: Do not use tension side formula because $f_{st} \neq 0.87 f_y$ in this case. Due to more reinforcement, steel has not reached to its yield stress value.

Design Formula

While designing a beam section, x_u is limited to $x_{u\,lim}$ i.e.

$$x_u = x_{ulim}$$

and $$M_u = M_{ulim}$$

From compression side,

$$M_u = 0.362 f_{ck} b x_{ulim} (d - 0.42 x_{ulim})$$

Put $x_{ulim} = kd$

$\therefore \quad M_u = 0.362 f_{ck} b kd (d - 0.42 kd)$

$$= [0.362 f_{ck} k (1 - 0.42 k)] b d^2$$

Thus, $M_u = Qbd^2$

$\Rightarrow$ $d = \sqrt{\frac{M_u}{Qb}}$

where, $Q = 0.362\, f_{ck}\, k(1 - 0.42k)$

For example: For Fe 415, $k = 0.48$ $(\because x_{u\,lim} = 0.48d$ for Fe 415)

$\therefore$ $Q = 0.36\, f_{ck}\, 0.48\,(1 - 0.42 \times 0.48) = 0.138\, f_{ck}$

Lever Arm: For limiting section, lever arm is given as:

Lever arm $= d - 0.42\, x_{ulim}$

$= d - 0.42\, kd = (1 - 0.42\, k)\, d = jd$

where j = Lever arm factor $= 1 - 0.42k$

$= 1 - 0.42 \times 0.53 = 0.777$ For Fe 250

$= 1 - 0.42 \times 0.48 = 0.79$ For Fe 415

$= 1 - 0.42 \times 0.46 = 0.806$ For Fe 500

Thus if can be inferred that $j \simeq 0.8$ for all the thre grades of steel.

Reinforcement Steel Requirement

$$M_{u\,lim} = 0.87 f_y\, A_{st}\,(d - 0.42 x_{u\,lim})$$

$$A_{st} = \frac{M_{u\,lim}}{0.87 f_y\,(d - 0.42 x_{u\,lim})} = \frac{M_{u\,lim}}{0.87 f_y\, jd}$$

Alternate procedure for determining the area of steel required

Let $M_u = M_{u\,lim}$ and $x_u \le x_{u\,lim}$ so that $f_{st} = 0.87 f_y$

Now, $C = T$

$\Rightarrow$ $0.36\, f_{ck}\, b x_u = 0.87 f_y\, A_{st}$

$\Rightarrow$ $\left(\frac{x_u}{d}\right) = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$

$M_u = 0.36\, f_{ck} b x_u\,(d - 0.42\, x_u)$

$\Rightarrow$ $M_u = 0.36 f_{ck} b \left(\frac{x_u}{d}\right)\left(1 - 0.42\frac{x_u}{d}\right) d^2$

$\Rightarrow$ $0.42\left(\frac{x_u}{d}\right)^2 - \left(\frac{x_u}{d}\right) + \frac{M_u}{0.36 f_{ck} bd^2} = 0$

This is a quadratic equation in $\left(\frac{x_u}{d}\right)$ which on solving gives,

$$\frac{x_u}{d} = 1.202\left[1 - \sqrt{1 - \frac{4.598R}{f_{ck}}}\right]$$

where, the quantity $\frac{M_u}{bd^2}$ is sometimes denoted as 'R'

Now, $M_u = 0.87\, f_y A_{st}\,(d - 0.42 x_u)$

$\Rightarrow$ $A_{st} = \frac{M_u}{0.87 f_y\,(d - 0.42 x_u)}$

$$= \frac{M_u}{0.87 f_y d\left(1 - 0.42\frac{x_u}{d}\right)}$$

$\Rightarrow$ $$\frac{A_{st}}{bd} = \frac{M_u}{0.87 f_y bd^2\left(1 - 0.42\frac{x_u}{d}\right)}$$

By substituting the value of $\left(\frac{x_u}{d}\right)$ as computed above, area of steel required (A_{st}) can be determined.

Alternatively: Area of steel can also be determined without actually calculating $\left(\frac{x_u}{d}\right)$.

$M_u = 0.87\, f_y A_{st}\,(d - 0.416 x_u)$ for all $x_u \le x_{u\,lim}$

Now, $C = T$

$\Rightarrow$ $0.362\, f_{ck}\, b x_u = 0.87 f_y\, A_{st}$

$\Rightarrow$ $\frac{A_{st}}{bd} = \frac{p_t}{100} = \frac{0.362 f_{ck} x_u}{0.87 f_y d}$ where, $p_t = \frac{A_{st}}{bd} \times 100$ and $p_t \le p_{t\,lim}$

$\Rightarrow$ $\left(\frac{x_u}{d}\right) = \frac{0.87 f_y p_t}{100(0.362) f_{ck}}$

$\therefore$ $M_u = 0.87 f_y A_{st}\,[d - 0.416 x_u]$

$\Rightarrow$ $\frac{M_u}{bd^2} = \frac{0.87 f_y A_{st}}{bd}\left(1 - 0.416\frac{x_u}{d}\right)$

$$= 0.87 f_y \left(\frac{p_t}{100}\right)\left[1 - \frac{0.416(0.87 f_y p_t)}{100(0.362 f_{ck})}\right]$$

$$= 0.87 f_y \left(\frac{p_t}{100}\right)\left[1 - \frac{f_y p_t}{100 f_{ck}}\right]$$

$$= 0.87 f_y \left(\frac{p_t}{100}\right)\left[1 - \left(\frac{p_t}{100}\right)\frac{f_y}{f_{ck}}\right]$$

$\Rightarrow$ $\left(\frac{f_y}{f_{cu}}\right)\left(\frac{p_t}{100}\right)^2 - \left(\frac{p_t}{100}\right) + \frac{R}{0.87 f_y} = 0$ where, $R = \frac{M_u}{bd^2}$

This is a quadratic equation in $\left(\frac{p_t}{100}\right)$ which on solving gives,

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

The above expression can be directly used for calculating the area of tension reinforcement required.

Limiting Percentage of Tension Steel

Corresponding to limiting moment of resistance for singly reinforced sections, there is a limiting percentage of tension steel given by:

$$p_{t\,lim} = \frac{100\,A_{st\,lim}}{bd} \qquad \text{when, } x_u = x_{u\,lim}$$

Now, $C = T$

$$\Rightarrow \quad 0.362 f_{ck}\, b\, x_{u\,lim} = 0.87 f_y A_{st\,lim}$$

$$\frac{x_{u\,lim}}{d} = \frac{0.87 f_y}{0.362 f_{ck}} \cdot \frac{p_{t\,lim}}{100} \qquad \text{where, } p_{t\,lim} = \frac{A_{st\,lim}}{bd} \times 100$$

$$\Rightarrow \quad \frac{A_{st\,lim}}{bd}\left(\frac{0.87 f_y}{0.362 f_{ck}}\right) = \left(\frac{x_{u\,lim}}{d}\right)$$

$$\Rightarrow \quad p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right)$$

Under reinforced sections undergo ductile failure giving ample warning to the users prior to its complete collapse. For this, $p_t \le p_{t\,lim}$ and corresponding $x_u \le x_{u\,lim}$.

Thus,

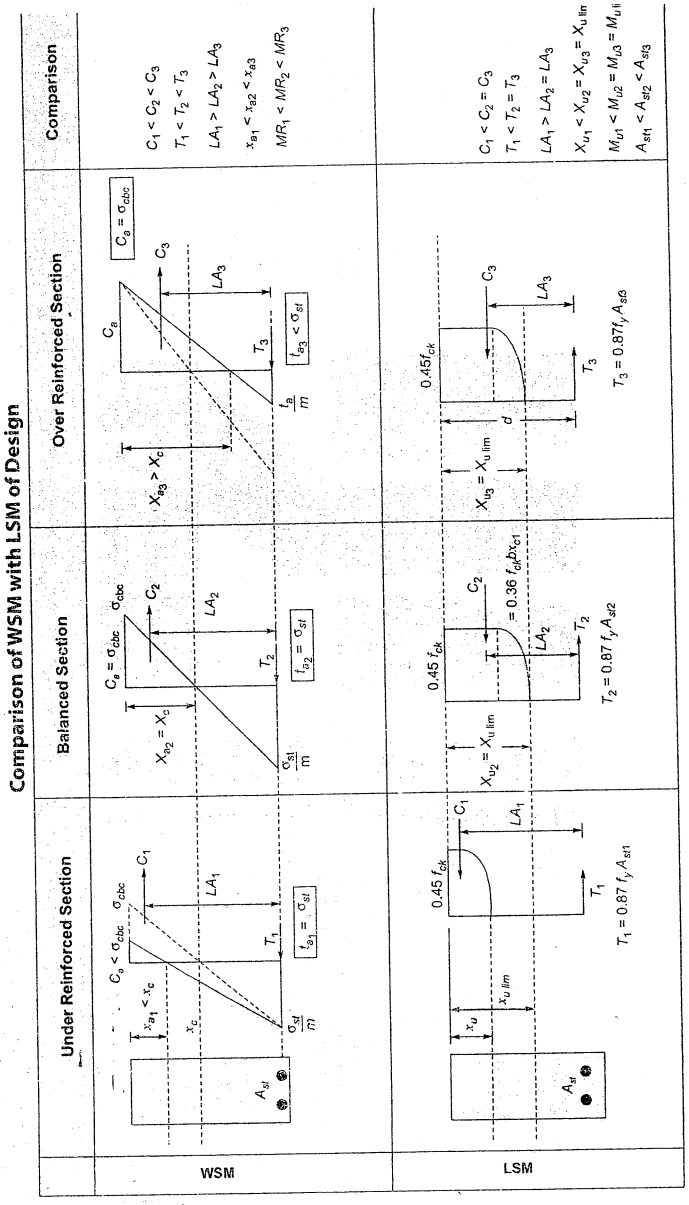
When $p_t = p_{t\,lim}$, then section is **balanced**.

When $p_t < p_{t\,lim}$, then section is **under-reinforced**.

When $p_t > p_{t\,lim}$, then section is **over-reinforced**.

Table 4.3: $p_{t\,lim}$ *(in %) for different grades of concrete and steel*

f_{ck} (N/mm^2)	Fe 250 (f_y = 250 N/mm^2)	Fe 415 (f_y = 415 N/mm^2)	Fe 500 (f_y = 500 N/mm^2)
20	1.77	0.96	0.77
25	2.21	1.2	0.96
30	2.65	1.44	1.15
35	3.09	1.68	1.34
40	3.54	1.92	1.53
45	3.98	2.16	1.72
50	4.42	2.4	1.91



Example 4.1 Determine the depth of neutral axis of a beam section of 250 mm x 400 mm size reinforced with 3-20 mm diameter bars of Fe 250 grade steel. The beam is made up of M 20 concrete and effective cover to reinforcement is 50 mm.

Solution:

Effective depth of the beam (d) = 400 mm – 50 mm = 350 mm

Area of tensile steel (A_{st}) = $3\times\left(\frac{\pi}{4}\right)\times 20^2 = 942.5\ \text{mm}^2$

Limiting depth of neutral axis ($x_{u\,lim}$) for Fe250 steel

= 0.53 d = 0.53 x 350 mm = 185.5 mm

Let depth of neutral axis (x_u) < $x_{u\,lim}$ so that $f_{st} = 0.87 f_y$

From statical equilibrium, $C = T$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times 250\times 942.5}{0.36\times 20\times 250} = 113.26\,\text{mm}$$

$< x_{u\,lim} (= 185.5\ \text{mm})$

So, the assumption of $x_u < x_{u\,lim}$ is true and thus stress in steel reaches to yield stress i.e. 0.87 f_y. Therefore, depth of neutral axis (x_u) = 113.26 mm.

Example 4.2 Determine the lever arm for a rectangular beam of size 300 mm x 450 mm with 50 mm effective cover which is made up of M 25 concrete and is reinforced with 4-20 mm diameter bars of Fe 415 grade of steel.

Solution:

Area of steel (A_{st}):

$A_{st} = 4\times\frac{\pi}{4}\times 20^2 = 1256.64\ \text{mm}^2$

Compressive force of concrete (C) = $0.36 f_{ck} b x_u$

Tensile force of steel (T) = $0.87 f_y A_{st}$ if $x_u \le x_{u\,lim}$

From statical equilibrium, $C = T$

$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$0.36 \times 25 \times 300\, x_u = 0.87 \times 415 \times 1256.64$

$x_u = 167.11$ mm

Also limiting depth of neutral axis ($x_{u\,lim}$) = 0.48 d (for Fe 415)

= 0.48 x (450 – 50) = 192 mm > x_u (= 167.11 mm)

Thus,

steel will reach its yield point first i.e. $f_{st} = 0.87 f_y$

Thus, lever arm (z) = $d - 0.42 x_u$

= (400 – 0.42 x 167.11) mm = 330.48 mm

Example 4.3 A rectangular beam of size 300 mm x 500 mm is required to resist a moment of 30 kNm. Find the reinforcement required for the beam if grade of concrete is M25 and steel is Fe500. Assume effective cover is 50 mm.

Solution:

Working moment to be resisted = 30 kNm

Factored moment to be resisted (M_u) = 1.5 x 30 kNm = 45 kNm

Let effective cover of the beam section = 50 mm

Thus, effective depth of beam section (d) = 500 mm – 50 mm = 450 mm

$$R = \frac{M_u}{bd^2} = \frac{45\times 10^6}{(300\times 450^2)} = 0.74\ \text{N/mm}^2$$

Area of steel required:

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right]$$

$$\Rightarrow \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2\times 500}\left[1-\sqrt{1-4.598\frac{0.74}{25}}\right] = 0.0017653$$

$$\Rightarrow \quad p_t = 0.176\%$$

$$\therefore \quad A_{st} = \frac{0.176}{100}\times 300\times 450 = 238.3\ \text{mm}^2$$

Depth of Neutral Axis:

From statical equilibrium, $C = T$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times 500\times 238.3}{0.36\times 25\times 300} = 38.18\,\text{mm}$$

Check:

Limiting depth of neutral axis ($x_{u\,lim}$) for Fe500 steel = 0.46 d = 0.46 x 450 mm = 207 mm > x_u (OK)

Thus, the steel reaches its yield point first i.e. $f_{st} = 0.87 f_y$

Also limiting percentage of tension steel (p_{tlim}),

$$p_{tlim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right)$$

$$p_{tlim} = 41.61\left(\frac{25}{500}\right)(0.46) = 0.957\% > 0.176\% \quad \text{(OK)}$$

Example 4.4 Determine the ultimate moment of resistance of a rectangular beam of width 350 mm with 550 mm effective depth which is casted with M30 grade of concrete and reinforced with 4 numbers of 25 mm diameter bars of Fe250 steel grade.

Solution:

Area of steel (A_{st}):

$$A_{st} = 4\times\frac{\pi}{4}\times d^2 = 4\times\frac{\pi}{4}\times 25^2 = 1963.5\,\text{mm}^2$$

Limiting depth of neutral axis (x_{ulim}) for Fe250 steel = 0.53 d = 0.53 x 550 = 291.5 mm

Actual depth of neutral axis (x_u):

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 250 \times 1963.5}{0.36 \times 30 \times 350} = 112.36\,\text{mm} < x_{u\,lim}\ (= 291.5\ \text{mm})$$

Thus, steel stress $f_{st} = 0.87\ f_y$ is correct.

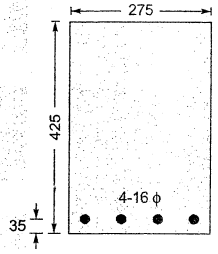
Ultimate moment of resistance of beam:

Since, the section is under-reinforced, the ultimate moment of resistance of the beam is given by

$$M_{uR} = 0.87 f_y A_{st} (d - 0.42 x_u)$$
$$= 0.87\,(250)\,(1963.5)\,(550 - 0.42 \times 112.36)\ \text{Nmm}$$
$$= 214.92\ \text{kNm} \simeq 215\ \text{kNm}$$

Example 4.5 Determine the lever arm and moment of resistance of the beam section as shown. Use M 20 concrete and Fe 250 steel.

Solution:



Given $b = 275\,\text{mm}$

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25\ \text{mm}^2$$

Effective cover = 35 mm

$\therefore$ Effective depth (d) = 425 – 35 = 390 mm

Limiting depth of neutral axis for Fe 250 steel ($x_{u\,lim}$) = 0.53 d

= 0.53 (390) = 207.09 mm

Let depth of neutral axis $(x_u) < x_{u\,lim}$

$\therefore$ $f_{st} = 0.87\ f_y$

From statical equilibrium $C = T$

$\Rightarrow$ $0.36\ f_{ck}\ b x_u = 0.87\ f_y\ A_{st}$

$\Rightarrow$ $$x_u = \frac{0.87(250)804.25}{0.36(20)275}$$

$= 87.86\ \text{mm} < x_{u\,lim}$

Thus assumption of $x_u < x_{u\,lim}$ is correct.

$\Rightarrow$ Steel will yield first i.e. $f_{st} = 0.87\ f_y$ is correct

$\therefore$ Lever arm (z) = $d - 0.42\ x_u = 390 - 0.42\ (87.86)$

= 353.45 mm

$\therefore$ Moment of resistance = 0.87(250) (804.25) (353.45) Nmm = 61.83 kNm

Ultimate moment of resistance corresponding to balanced section

$M_{u\,lim} = 0.148\ f_{ck}\ bd^2$ (for Fe 250)

$= 0.148(20)\ 275\ (390)^2$

= 123.81 kNm > 61.83 kNm

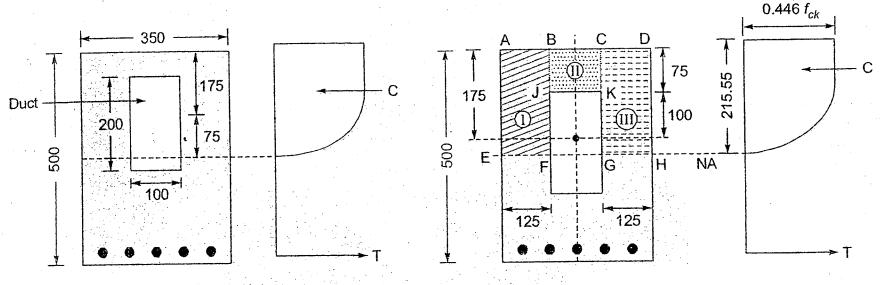
Thus the given beam section is under-reinforced ($x_u < x_{u\,lim}$)

with lever arm (z) = 353.45 mm

and moment of resistance of 61.83 kNm

Example 4.6 A beam of size 350 x 500 mm is having a duct of rectangular section of size 100 x 200 mm which is symmetric about the vertical axis of beam section and the centre of duct is at a distance of 175 mm from the top of the beam section. Compute the moment of resistance of the balanced section and thus the area of steel required. Assume M 20 concrete and Fe 415 steel. Assume effective cover is 50 mm.

Solution:



Given, M 20 concrete $\Rightarrow$ $f_{ck} = 20\ \text{N/mm}^2$

Fe 415 steel $\Rightarrow$ $f_y = 415\ \text{N/mm}^2$

$b = 350\ \text{mm}$

$D = 500\ \text{mm}$

Effective cover = 50 mm

$\therefore$ Effective depth (d) = 500 mm – 50 mm = 450 mm

Distance of top edge of duct from top of beam = CK = 75 mm

Limiting depth of neutral axis ($x_{u\,lim}$) = 0.48 d = 0.48 (450) = 215.55 mm

Depth of rectangular portion of compressive stress = $\frac{3}{7} x_{u\,lim} = \frac{3}{7}(215.55)\ \text{mm} = 92.38\ \text{mm}$

Divide the area above the balanced neutral axis into 3 parts with

portion ABFE $\rightarrow$ Part I

portion BCKJ $\rightarrow$ Part II

and portion CDHG $\rightarrow$ Part III

Part I and III are each of width 125 mm and are under the influence of complete compressive stress block over a depth of $x_{u\,lim}$ (= 215.55 mm). Part II is of width 100 mm which is under the influence of compressive stress block for a depth of h = 75 mm. This part II is within the rectangular stress block because h = 75 mm depth $< \frac{3}{7} x_{u\,lim}$ (= 92.38 mm)

Let, C_1 = Compressive force due to Part I

$\therefore$ $C_1 = C_3 = 0.36\ f_{ck}\ b\ x_{u\,lim}$

$= 0.36\,(20)(125)\ 215.55\ \text{N} = 193.995\ \text{kN}$

C_2 = Compressive force due to Part II

$C_2 = 0.45\, f_{ck}\, bh = 0.45\,(20)\,(100)\,75\ \text{N} = 67.5\ \text{kN}$

C_3 = Compressive force due to Part III

From statical equilibrium,

$C_1 + C_2 + C_3 = T$

$\Rightarrow \quad 195.995 + 67.5 + 193.99 = T$

$\Rightarrow \quad T = 455.49\ \text{kN}$

At balanced stage, $f_{st} = 0.87\, f_y$

$\therefore \quad T = 0.87\, f_y\, A_{st} = 455.49 \times 10^3\ \text{N}$

$\Rightarrow \quad A_{st} = \dfrac{457.046 \times 10^3}{0.87(415)} = 1261.57\ \text{mm}^2$

Moment of resistance at balanced stage (MOR)

$= (C_1 + C_3)\,(d - 0.42\, x_{u\,lim}) + C_2\,(d - 75/2)$

$= 2 \times 193.995 \times 10^3 (450 - 0.42 \times 215.55) + 67.5 \times 10^3 (450 - 37.5)\text{Nmm}$

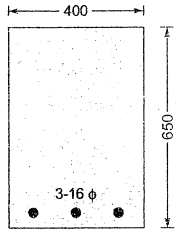
$= 167.314\ \text{kNm}$

Example 4.7 Calculate MR of a rectangular beam of size 400 x 650 mm. Which is reinforced with: (a) 3-16 mmϕ (b) 6-25 mmϕ (c) 8-25 mmϕ

Calculate area of steel required for limiting section also. Use M 20 concrete and Fe 415 steel. Assume effective cover is 50 mm.

Solution:

Case (a) 3-16 mm ϕ bars



$D = 650\ \text{mm}$

$d = 650 - 50 = 600\ \text{mm}$

Area of steel $= 3 \times \dfrac{\pi}{4} \times 16^2 = 603.2\ \text{mm}^2$

Limiting depth of NA

$x_{u\,lim} = 0.48 \times d$ (for Fe 415)

$= 0.48 \times 600 = 288\ \text{mm}$

Actual depth of NA

$C = T$

$0.36 f_{ck}\, b x_u = 0.87 f_y\, A_{st}$

(Assuming under reinforced section so that $f_{st} = 0.87 f_y$)

$0.36 \times 30 \times 400 \times x_u = 0.87 \times 415 \times 603.2$

$x_u = 50.41\ \text{mm} < x_{u\,lim} (= 288\ \text{mm})$

Thus assumption of under-reinforced section is correct.

i.e., $x_u < x_{u\,lim}$

Moment of resistance

$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 30 \times 400 \times 50.41\,(600 - 0.42 \times 50.41)$

$= 126.05\ \text{kNm}$

$\Rightarrow$ Unfactored moment/working moment $= \dfrac{126.05}{1.5} = 84.03\ \text{kNm}$

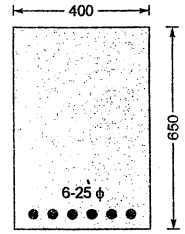
From tension side

$M_R = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 603.2\,(600 - 0.42 \times 50.41)$

$= 126.06\ \text{kNm}$

Unfactored $M_R = \dfrac{126.05}{1.5} = 84.04\ \text{kNm}$



Case (b) 6-25 mm ϕ bars

Given: $A_{st} = 6 \times \dfrac{\pi}{4} \times 25^2 = 2945.24\ \text{mm}^2$

Limiting depth of NA $= 0.48\text{d}$

$\Rightarrow \quad x_{u\,lim} = 0.48(650 - 50)\ \text{mm} = 288\ \text{mm}$

Actual depth of NA

$C = T$

$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$ (assuming under reinforced section)

$0.36 \times 30 \times 400 \times x_u = 0.87 \times 415 \times 2945.24$

$x_u = 246.15\ \text{mm} < x_{u\,lim} (= 288\ \text{mm})$

So section is under reinforced section

Moment of resistance

$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 30 \times 400 \times 246.15(600 - 0.42 \times 246.15)$

$= 528.09\ \text{kNm}$

Unfactored moment/working moment $= \dfrac{528.09}{1.5} = 352.06\ \text{kNm}$

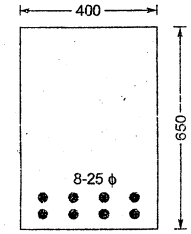
Moment of resistance from tension side

$M_R = 0.87 f_y A_{st} (d - 0.42 x_u) = 528.09\ \text{kNm}$

$0.87 \times 415 \times 2945.24\,[600 - 0.42 \times 246.15]$

Unfactored moment $= \dfrac{528.09}{1.5} = 352.06\ \text{kNm}$

Case (c) 8-25 mm ϕ bars



Given: Area of steel $= 8 \times \dfrac{\pi}{4} \times 25^2 = 3927\ \text{mm}^2$

Limiting depth of NA $x_{u\,lim} = 0.48d = 0.48 \times 600 = 288\ \text{mm}$

Actual depth of NA

$x_u = 0.36 f_{ck} b x_u = 0.87\, f_y\, A_{st}$

$x_u = 328\ \text{mm} > x_{u\,lim} (= 288\ \text{mm})$

Thus section is over reinforced and $f_{st} \neq 0.87 f_y$. So limit x_u to $x_{u\,lim}$

$\therefore \quad M_u = 0.36 f_{ck}\, b x_{u\,lim} (d - 0.42\, x_{u\,lim})$

$= 0.36 \times 30 \times 400 \times 288\,(d - 0.42 \times 288) = 596\ \text{kNm}$

Alternatively $M_u = 0.138 f_{ck}\, bd^2 = 596\ \text{kNm}$ (for Fe 415)

$$\therefore \quad \text{Unfactored } M_R = \frac{596}{1.5} = 397 \text{ kNm}$$

Area of steel for limiting section:

$$A_{st\,lim} = \frac{M_{u\,lim}}{0.87 f_y \left(d - 0.42 x_{u\,lim}\right)}$$

$$= \frac{596 \times 10^6}{0.87 \times 415 (600 - 0.42 \times 288)} = 3445.9 \text{ mm}^2$$

Alternatively $$p_{t\,lim} = 41.61 \left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61 \left(\frac{30}{415}\right)(0.48) = 1.44\%$$

$$\therefore \quad A_{st\,lim} = \frac{1.44}{100} \times 400 \times 600 = 3465.6 \text{ mm}^2$$

4.5 Requirements of Flexural Design

1. **Concrete Cover to Reinforcement:** Concrete cover is provided to protect the reinforcing bars from corrosion, fire and also provides sufficient bond between steel and concrete to prevent slipping out of reinforcing bars from concrete. **Clear cover** is defined as the distance of exposed surface of concrete to the nearest surface of reinforcing steel. **Cl. 26.4.1 of IS 456: 2000** defines **nominal cover** as **"The design depth of concrete cover to all steel reinforcement, including links."** **IS 456: 2000** specifies the deviation in concrete cover from 0 mm to +10 mm. The tolerance '0 mm' indicates that no reduction in the prescribed nominal cover is allowed.

Table 4.4: *Nominal cover requirements based on exposure conditions*

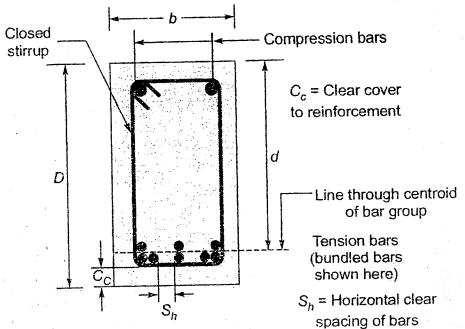
Exposure condition	Minimum Grade of concrete	Nominal Cover (mm)	Permitted allowance
Mild	M 20	20	Can be reduced by 5 mm for bars less than 12 mm dia
Moderate	M 25	30	
Severe	M 30	45	Can by reduced by 5 mm if concrete grade is M 35 or higher
Very Severe	M 35	50	
Extreme	M 40	75	

NOTE

The earlier version of code i.e. **IS 456: 1978** specified clear cover requirements based on the type of structural elements (like 15 mm in slabs, 25 mm in beams, 40 mm in columns etc.), but in the latest version i.e. **IS 456: 2000**, clear cover requirements are now made applicable for all types of structural elements. Cl. 26.4.2.1 of **IS 456: 2000** specifies certain minimum clear cover requirements for columns (generally 40 mm) and Cl. 26.4.2.2 of IS **456: 2000** specifies a certain minimum clear cover requirements for footings (generally 50 mm).

Apart from the above stated clear cover requirements, **Cl. 26.4.3 of IS 456: 2000** specifies clear cover requirements based on **fire resistance (expressed in terms of hours).** For 1 hour fire occurrence, a nominal cover of 20 mm for beams and 40 mm for columns has been specified in the code. Larger cover is required if the member has to be specifically designed for fire resistance.

2. **Spacing of Reinforcing Bars:** Minimum spacing of parallel reinforcing bars are specified in order to ensure that the concrete can be easily placed and compacted during casting. Maximum limit on spacing of parallel reinforcing bars are specified for controlling the cracks in concrete i.e. crack widths.

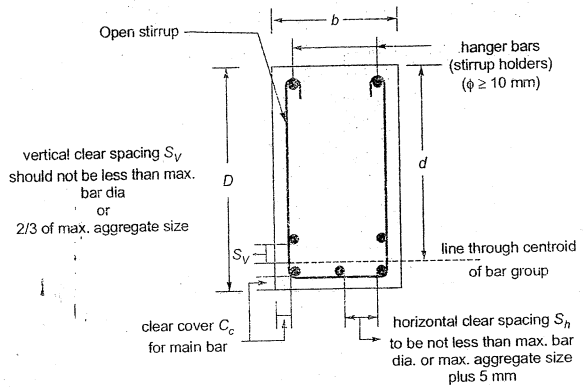


(c) Doubly reinforced beam

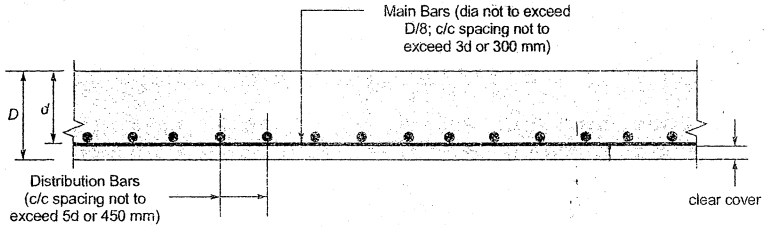
Fig. 4.8 *IS 456: 2000 requirements for placement of reinforcing bars*

In beams, where the width of the member (i.e. beam) is fixed, often it is the minimum spacing requirement that dictates the selection of bar diameter. When all the reinforcing bars cannot be accommodated in one layer conforming to the requirements of minimum spacing of reinforcing bars and the clear cover, then either we have to increase the width of the beam, or place bars in more than one layer or bundle the group of parallel bars. This problem does not occur in slabs where the width of slab is large enough to accommodate the bars and also the percentage of reinforcement in slabs is low as compared to beams in general.

Cl. 26.3 of IS 456: 2000 specifies the requirements of placement of reinforcing bars in concrete members (i.e. beams, slabs) which is explained in Fig. 4.9.



(a) Singly reinforced beam



(b) One-way slab

Fig. 4.9 *Placement of reinfocing bars in beam and slab*

For a given area of tension reinforcement, it is **beneficial to use large number of small diameter bars (distributed uniformly in one or more layers)** rather than small number of large diameter bars to have efficient control on cracks.

In case of slab, Cl. 26.5.2.2 and Cl. 26.3.3b of IS 456: 2000 limits the maximum diameter of reinforcing bars to **1/8th of the total slab thickness** with maximum spacing of these bars being limited to 3d or 300 mm (whichever is less).

In very deep structural members (like *deep beams*), a considerable portion of the web is under tension. Wider cracks may develop in the higher regions of web. Also, in regions of large exposed unreinforced concrete surfaces, cracks appear more frequently due to shrinkage (because of temperature variations). **Cl. 26.5.1.3 of IS 456: 2000** states that side face reinforcement has to be provided on the two faces of the beam if overall depth of beam exceeds 750 mm (or 450 mm for beams subjected to torsion) such that:

"The total area of web reinforcement is NOT less than 0.1% of web area and shall be distributed equally on the two faces at a spacing not exceeding 300 mm or web thickness, whichever is small."

3. **Maximum and Minimum Areas of Reinforcement:** A minimum area of tension steel is required in flexural members (like beams) in order to resist the effect of loads and also control the cracking in concrete due to shrinkage and temperature variations.

   Minimum flexural steel reinforcement in beams: Cl. 26.5.1.1 of IS 456: 2000 specifies minimum area of tension reinforcing steel as:

$$\frac{A_{st\,min}}{bd} \geq \frac{0.85}{f_y}$$

or

$$p_{t\,min} \geq \frac{85}{f_y}$$

$= 0.34\%$ for Fe 250

$= 0.205\%$ for Fe 415

$= 0.17\%$ for Fe 500

For flanged beams, replace 'b' by width of the web 'b_w'.

Minimum flexural steel reinforcement in slabs: Minimum steel requirements in slab are based on ***shrinkage and temperature considerations*** and not on strength consideration because in slabs, there occurs a better distribution of load effects unlike in beams, where minimum steel requirement is based on *strength consideration*. **Cl. 26.5.2 of IS 456: 2000** limits minimum steel reinforcement requirement in slabs in the either direction as:

$A_{st\,min}$ = 0.15% of gross area for Fe 250

= 0.12% of gross area for Fe 415

where, gross area (A_g) is $b \times D$. For one way slabs also, this minimum area of reinforcement has to be provided in the transverse direction as *secondary or distributor reinforcement.* The **spacing** of distribution bars shall NOT exceed **5d or 450 mm**, whichever is less.

Maximum flexural steel reinforcement in beams: Excessive reinforcement leads to congestion of bars which adversely affects the placement and compaction of concrete. **Cl. 26.5.1 of IS 456: 2000** limits the maximum area of steel reinforcement in tension ($A_{st\,max}$) and compression ($A_{sc\,max}$) to 4% of gross area i.e. $0.04bD$.

4. **Control of Deflection:** Excessive deflection in slabs or beams is not desirable as it leads to discomfort to the occupants and also adversely affects the flooring and finishes. Cross sectional sizes of the flexural elements (particularly depth of the section) governs the criterion of deflection control. It is the ratio of span/depth that needs to be controlled in order to control the deflection. **Cl. 23.2a of IS 456: 2000** limits this span/depth ratio to **250** as final deflection due to all types of loads (including the long term effects of creep and shrinkage).

4.6 Deflection Control by Limiting the Span/Depth Ratio

Let a uniform rectangular simply supported beam of size $b \times D$ and made up of a linear elastic material is subjected to a uniformly distributed load 'w' per unit length over the entire span 'l'. The central deflection (Δ) in the beam is given by

$$\Delta = \frac{5}{384}\cdot\frac{wl^4}{EI} \quad \text{...(i)}$$

Also maximum mid-span moment, $M_{max} = \frac{wl^2}{8}$

$$\Rightarrow \quad w = \frac{8M_{max}}{l^2} = (\sigma Z)\frac{8}{l^2} = \left(\sigma\frac{bD^2}{6}\right)\frac{8}{l^2} \quad \text{...(ii)}$$

Substituting eq. (ii) in (i) and writing $I = \frac{bD^3}{12}$, it comes out that:

$$\frac{\Delta}{l} = \text{constant} \times \frac{l}{D}$$

Where, the constant is $\frac{5\sigma}{24E}$. Thus by limiting the $\left(\frac{l}{D}\right)$ ratio, the deflection in the member can be controlled for any type of loading and support conditions. The above result does not apply to reinforced concrete as such because reinforced concrete is a non-homogeneous and composite material. However IS 456: 2000 adopts this concept and prescribes limiting values of $\frac{l}{D}$ ratios.

Cl. 23.2.1 of IS 456: 2000 prescribes limits for limiting l/D ratios for prismatic rectangular beams of span upto 10 m as:

$$\left(\frac{l}{D}\right)_{max} = \left(\frac{l}{D}\right)_{basic} . k_t . k_c$$

here D is effective depth

where, $\left(\frac{l}{D}\right)_{basic} = 7$ for cantilever spans

$= 20$ for simply supported spans

$= 26$ for continuous spans

For spans above 10 m, the $(l/D)_{basic}$ values may be multiplied by [10/span (m)], except for cantilever.

Here the modification factor k_t varies with percentage of tension reinforcement (p_t) and f_{st} as given in **Fig. 4 of IS 456: 2000** and the modification factor k_c varies with percentage of compression reinforcement (p_c) as given in **Fig. 5 of IS 456: 2000**.

The factor 'f_{st}' is defined as:

$$f_{st} = 0.58\, f_y \frac{\text{Area of steel required}}{\text{Area of steel provided}}$$

For flanged beams, **Cl. 23.2.1e of IS 456: 2000** recommends the values of p_t and p_c to be based on the area equal to '$b_f.d$' and the calculated $(l/D)_{max}$ should be multiplied by a suitable reduction factor, the value of which depends on the ratio b_w/b_f. Alternatively, for deflection control in flanged beams, the overhang portions are ignored and the flanged beam is considered as a rectangular beam of effective depth 'd' and width 'b_w' (width of the web).

4.7 Selection of Member Sizes

The ratio of overall depth (D) to width (b) of rectangular beam section should in general lie in the range of **1.5 to 2**. This ratio can go up to **3 for beams carrying heavy loads**. The size of the beam is also governed by shear force at the section of the beam. Many a times, architectural requirements also dictate the sizes of beams and columns. If these architectural requirements are too stringent then required strength of the beam is achieved by making the beam doubly reinforced and using higher grade of concrete and steel.

For building frames, the width of beams should, in general, be less than or equal to the lateral dimension of the columns into which they frame in. Generally adopted beam widths are of 200 mm, 250 mm and 300 mm.

Often the beam is required to support a masonry wall and in such cases, the width of the beam is often made such that the beam sides are flushed with the finished surfaces of the wall and therefore beam widths of 230 mm are quite common in India.

In designs, the beam depth is often taken as 1/10th to 1/15th of span for simply supported and continuous beams. For cantilever beams, lower ratios are adopted and depth of cantilever beam is usually tapered.

1. **Preliminary proportioning of slab thickness:** For *slabs*, the thicknesses are very small as compared to depths of beams and thus limiting the span /depth ratios will generally govern the proportioning.

   For determining the thickness of the slab, the clear cover (based on exposure condition) plus half the diameter of the main reinforcement (usually along the shorter span) have to be added to the effective depth. The calculated value of the thickness should be rounded off to the nearest multiple of 5 mm or 10 mm.

2. **Deep beams and slender beams:**

   Deep Beam: When span to depth (l/d) ratio is less than **2 for simply supported** and 2.5 for **continuous beams**, then such beams are called as deep beams.

   Slender Beam: When span of the beam is very large as compared to cross sectional dimension of the beam, then such beams are called as slender beams. There is always possibility of instability in slender beams and thus **Cl. 23.3 of IS 456: 2000** specify 'slenderness limits' to ensure lateral

stability. The clear distance between the lateral restraints must be minimum of **60b** or **250b^2/d** for simply supported and continuous beams and minimum of **25b** or **100b^2/d** for cantilever beams.

4.8 Design of Reinforced Concrete Rectangular Beams

The design of reinforced concrete rectangular beams consists of determination of the following:

1. Cross sectional dimensions of the beam i.e. width (b), effective depth (d) and thus the overall depth (D).
2. Area of tension reinforcement required to resist the factored bending moment (M_u).

Apart from the above, the material properties viz. grade of concrete (f_{ck}) and the grade of steel (f_y) are fixed at the prior stage itself.

Do You Know ?

In under-reinforced sections, the effect of concrete grade on the ultimate moment of resistance of the beam section is NOT significant and thus it is not so much economical to use higher grades of concrete but at the same time higher grades of concrete is beneficial from the durability point of view.

The basic requirements for the safe design of beam in flexure are:

1. Factored moment (M_u) should be less than or at most equal to the limiting moment of resistance of the beam section i.e.

   $$M_u \le M_{u\,lim}$$

2. The failure at the ultimate limit state should be **ductile** and not brittle.

   Thus

   $$M_u \le M_{u\,lim} \quad \text{such that } x_u \le x_{u\,lim} \text{ so that } f_{st} = 0.87 f_y$$

But for under-reinforced sections, moment of resistance is given from tension side as steel yields earlier than the attainment of ultimate strain (i.e. 0.0035) by concrete.

Therefore,

$$M_u = 0.87 f_y A_{st}(d - 0.416x_u) \text{ for all } x_u \le x_{u\,lim}$$

From statical equilibrium, $C = T$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

Substituting this value of x_u in M_u,

$$M_u = 0.87 f_y A_{st}\left(d - 0.42\frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right) = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad \frac{M_u}{bd^2} = \frac{0.87 f_y A_{st}}{bd^2}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad \frac{M_u}{bd^2} = \frac{0.87 f_y p_t}{100}\left(1 - \frac{f_y}{f_{ck}}\frac{p_t}{100}\right) \qquad \text{Provided } p_t \le p_{t\,lim}$$

where $p_t = \frac{A_{st}}{bd} \times 100$

The expression for $p_{t\,lim}$ as derived in previous sections of this chapter is:

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right)$$

$= 0.0882f_{ck}$ for Fe 250
$= 0.048f_{ck}$ for Fe 415
$= 0.038f_{ck}$ for Fe 500

Thus $\dfrac{M_{u\,lim}}{f_{ck}bd^2} = 0.148$ for Fe 250
$= 0.138$ for Fe 415
$= 0.133$ for Fe 500

Arriving at the Dimensions of Rectangular Beam

Once we know the factored moment (M_u) from the principles of *Structural Analysis*, this factored moment is equated to the moment of resistance of the beam section depending on the grade of steel (viz. $0.148f_{ck}bd^2$, $0.138f_{ck}bd^2$ or $0.133f_{ck}bd^2$ for Fe 250, Fe 415 and Fe 500 respectively). From there, effective depth of the beam section can be computed as:

$$M_u = Rbd^2$$

$$\Rightarrow \quad d = \sqrt{\frac{M_u}{Rb}}$$

Where $R = 0.148f_{ck}$ for Fe 250
$= 0.138f_{ck}$ for Fe 415
$= 0.133f_{ck}$ for Fe500

The overall depth of the beam (D) is then computed as:

$D = d$ + Clear cover + (1/2) × Dia. of reinforcing bar + Dia. of stirrup

The value of D so calculated must be **rounded up (and NOT rounded down)** to the nearest multiple of 10 or 5.

Determining the Area of Tension Steel Required

As stated earlier, for design purpose

$$M_u = M_{u\,lim} \qquad \text{so that } x_u \le x_{u\,lim} \text{ and } f_{st} = 0.87f_y$$

Now $$A_{st} = \frac{M_u}{0.87f_y(d-0.42x_u)} = \frac{M_u}{0.87f_yd\left(1-0.42\frac{x_u}{d}\right)}$$

Here the question arises:

"How to compute $\dfrac{x_u}{d}$ for determination of area of steel required A_{st}?"

Design the beam as balanced section by equating factored moment (M_u) with the ultimate moment of resistance of the beam section ($M_{u\,lim}$) and in that case, $M_{u\,lim}$ can also be computed from the compression side of concrete as:

$$M_u = 0.36f_{ck}bx_u(d-0.42x_u)$$

$$\Rightarrow \quad 0.42\left(\frac{x_u}{d}\right)^2 - \left(\frac{x_u}{d}\right) + \frac{M_u}{0.36f_{ck}bd^2} = 0$$

This is a quadratic equation in $\left(\dfrac{x_u}{d}\right)$ the solution of which comes out to be:

$$\frac{x_u}{d} = 1.202\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

Alternate Solution

Earlier we arrived at the following expression:

$$\frac{M_u}{bd^2} = \frac{0.87f_yp_t}{100}\left(1-\frac{f_y}{f_{ck}}\frac{p_t}{100}\right)$$

The above equation can be rearranged as:

$$\left(\frac{f_y}{f_{ck}}\right)\left(\frac{p_t}{100}\right)^2 - \left(\frac{p_t}{100}\right) + \frac{M_u}{0.87f_ybd^2} = 0$$

Solution of the above quadratic equation gives,

$$\frac{p_t}{100}\left(=\frac{A_{st\ required}}{bd}\right) = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

(As derived in **section 4.4.3**)

From the above expression, we can directly compute the percentage of steel required and hence the area of steel required to resist the factored moment M_u. **Alternatively,** we can use ***Design Aid SP: 16*** wherein different charts are given for different values of f_{ck}, f_y and $\dfrac{M_u}{bd^2}$.

Computing the Number of Bars Required in Beams

Once we have $A_{st\ required}$, number of bars required of particular dia (say ϕ) is calculated as:

$$\text{No. of bars} = \frac{A_{st\,required}}{\frac{\pi}{4}\phi^2}$$

The computed number of bars must be **rounded up** to the nearest whole number.

Computing the Spacing of Bars Required in Slabs

In case of slabs (where b is taken as 1000 mm), instead of computing the number of bars required, we compute the **spacing of bars required (s)** of particular dia (say ϕ) which is given as:

$$\text{Spacing of bars of dia } \phi = \frac{1000A_\phi}{A_{st\ required}}$$

where, $A_\phi = \dfrac{\pi}{4}\phi^2$ = Area of one bar

4.9 Reinforcement Arrangement in Different Types of Beams

The figures given below depict the arrangement / placement of reinforcing bars in various types of beams commonly encountered in day to day life.

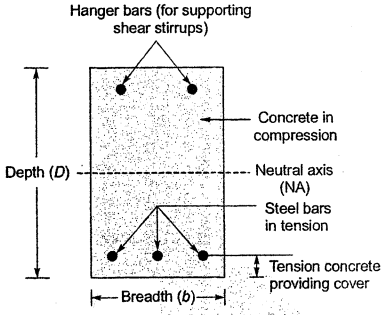


Fig. 4.10 *Singly reinforced rectangular beam under positive bending moment (near midspan)*

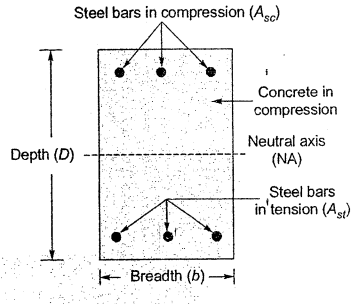


Fig. 4.11 *Doubly reinforced rectangular beam under positive bending moment (near midspan)*

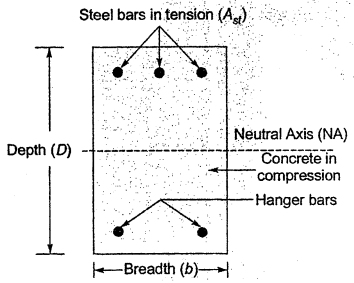


Fig. 4.12 *Singly reinforced rectangular beam under negative bending moment (over the support).*

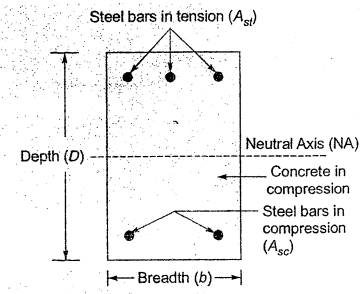


Fig. 4.13 *Doubly reinforced rectangular beam under negative bending moment (over the support).*

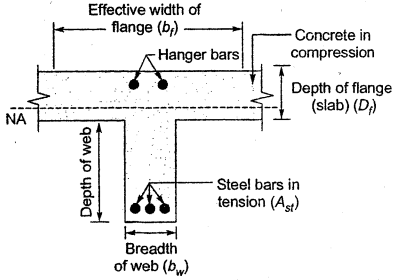


Fig. 4.14 *Singly reinforced T-beam under positive bending moment (near midspan)*

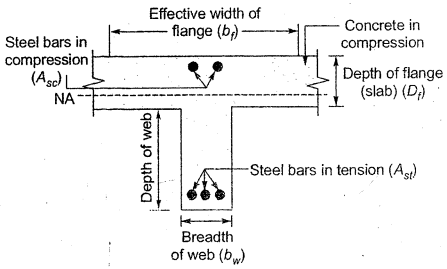
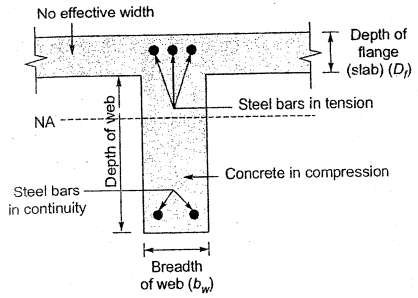
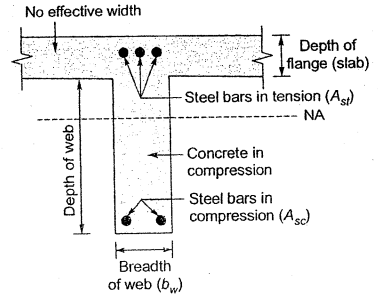


Fig. 4.15 *Doubly reinforced T-beam under positive bending moment (near midspan).*



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig.4.16 *Singly reinforced T-beam under negative bending moment (over the support). (concrete above neutral axis is in tension)*



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.17 *Doubly reinforced T-beam under negative bending moment (over the support).*

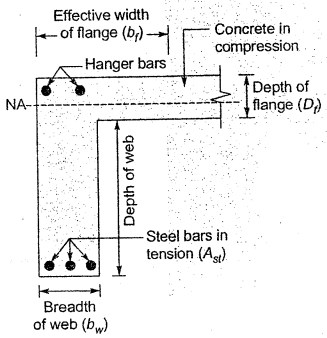


Fig. 4.18 *Singly reinforced L-beam under positive bending moment (near midspan).*

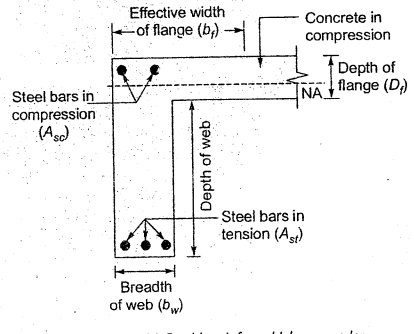
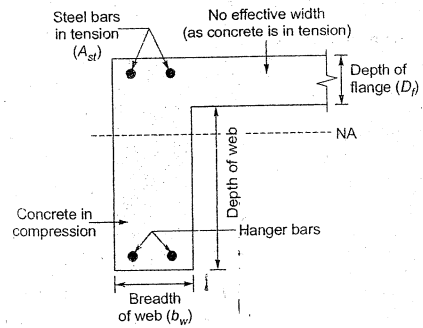
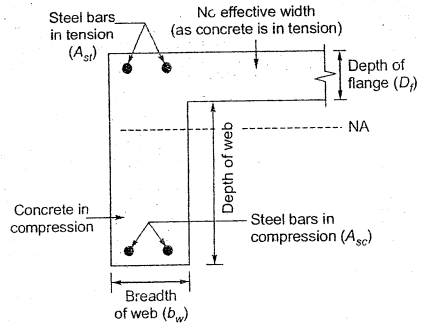


Fig. 4.19 *Doubly reinforced L-beam under positive bending moment (near midspan).*



L-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.20 *Singly reinforced L-beam under negative bending moment (over the support). (Concrete above neutral axis is in tension)*



L-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.21 *Doubly reinforced L-beam under negative bending moment (over the support). (Concrete above neutral axis in in tension)*

Do You Know?

Why is it advantageous to use Design Aid SP: 16?

Use of Design Aids like SP: 16 helps a designer in the following ways:

1. The numerous sets of values of width (b), depth (d) and reinforcing steel (A_{st}) for particular design problems are obtained very quickly.
2. The values given in SP: 16 already take into account the values of steel reinforcement where it is not admissible.

4.10 Comparison between WSM and LSM of Design

Table 4.5: *WSM versus LSM of design*

	WSM	LSM
Concrete	Stress factor of safety $= \frac{\text{Characteristic Stress}}{\text{Allowable stress}} = 3$ For M15, 15/5 = 3 For M20, 20/7 = 2.86 For M25, 25/8.5 = 2.94 For M30, 30/10 = 3.0	Stress factor of safety $= \frac{\text{Characteristic Stress}}{\text{Allowable stress}} = \frac{f_{ck}}{0.45 f_{ck}} = 2.22$
Loads	Load factor of safety = 1.0	Load factor of safety = $\gamma_f = 1.5$
	Total FOS = Stress FOS × Load FOS = 3.0 × 1.0 = 3.0	Total FOS = Stress FOS × Load FOS = 2.22 × 1.5 = 3.33

Example 4.8 Design a rectangular beam of effective span 4.3 m subjected to a live load of 12.5 kN/m use M 25 concrete and Fe 415 steel.

Solution:

Effective span (l) = 4.3 m = 4300 mm

For preliminary proportioning of beam section, take

$$D = \frac{l}{10} = \frac{4300}{10} \text{ mm} = 430 \text{ mm}$$

Adopt overall depth, D = 500 mm

So that effective depth, d = 500 – 35 = 465 mm (Assuming 35 mm effective cover)

b = 300 mm

Loads

Self weight of beam = 0.3 × 0.5 × 25 = 3.75 kN/m

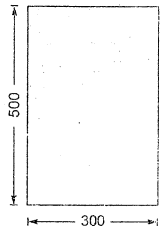
Live load = 12.5 kN/m

Total load = 3.75 + 12.5 = 16.25 kN/m

Factored load (w_u) = 1.5 × 16.25 = 24.375 kN/m

$$\text{Factored moment } (M_u) = \frac{w_u l^2}{8} = 24.375 \times \frac{4.3^2}{8}$$

= 56.34 kNm



Calculation of depth required

Ultimate moment of resistance for balanced section for Fe 415 = $0.138 f_{ck} bd^2$

$\Rightarrow$ $56.34 \times 10^6 = 0.138(25)(300)d^2$

$\Rightarrow$ $d = 233.31$ mm

$\therefore$ $D = 233.31 + 35$ (Assuming 35 mm effective cover)

$= 268.31 \text{ mm} \simeq 300 \text{ mm}$ (say)

Revise cross-section size to 250 mm × 300 mm

$\therefore$ Self weight of beam = 0.25 × 0.3 × 25 = 1.875 kN/m

Live load = 12.5 kN/m

$\therefore$ Total load = 14.375 kN/m

$\therefore$ Factored load = 1.5 × 14.375 = 21.56 kN/m

$$\text{Factored moment } (M_u) = 21.56 \times \frac{4.3^2}{8} = 49.83 \text{ kNm}$$

Now, $49.83 \times 10^6 = 0.138 f_{ck} bd^2$

$= 0.138(25)\,250\,d^2$

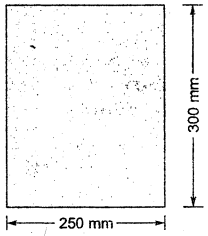
$\therefore$ $d = 240.36$ mm

$\therefore$ $D = 240.36 + 35$

$= 275.36 \text{ mm} < 300 \text{ mm}$ (assumed D) (OK)

$\therefore$ $d = 300 - 35 = 265$ mm

Thus adopt beam size as 250 × 300



Reinforcement requirement

$$R = \frac{M_u}{bd^2} = \frac{49.83 \times 10^6}{250 \times 265^2} = 2.838$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right]$$

$$= \frac{25}{2(415)}\left[1 - \sqrt{1 - 4.598 \times \frac{2.838}{25}}\right] = 9.295 \times 10^{-3}$$

$\therefore$ $p_t = 0.93\%$

Limiting percentage of tensile reinforcement

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61\left(\frac{25}{415}\right)(0.48)$$

$= 1.203\% > p_t (= 0.93\%)$ (OK)

$\Rightarrow$ $A_{st} = \frac{0.9295}{100} \times 250 \times 265 = 615.79 \text{ mm}^2$

$\therefore$ No. of 20 ϕ bars required $= \frac{615.79}{\frac{\pi}{4}(20)^2} = 1.96 \simeq 3$ (say)

$\therefore$ As per IS : 456, minimum two number of bars is to be provided so provide 3 – 20 ϕ bars.

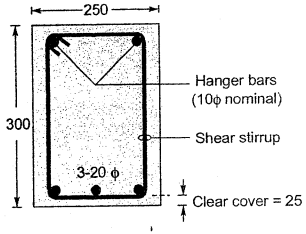
Minimum reinforcement

$$A_{st\,min} = 0.85\frac{bd}{f_y}$$

$$= \frac{0.85\times250\times265}{415}$$

$$= 135.7\ mm^2$$

$$< 615.79\ mm^2 \quad (OK)$$



Example 4.9 An RCC beam of size 380 x 700 mm overall depth is designed for an effective span 6 m and is subjected to a live load of 20 kN/m. Design the beam using LSM. Use M30 concrete and Fe500 steel.

Solution:

Given: Width of beam (b) = 380 mm

Overall depth of the beam (D) = 700 mm

Span (L) = 6 m

Assuming an effective cover of 50 mm,

Effective depth of the beam (d) = 700 – 50 mm = 650 mm

Loads and bending moment

Live load = 20 kN/m

Self weight of the beam = 0.38 x 0.7 x 25 kN/m = 6.65 kN/m

Total load = Dead load + Live load

= Self weight of the beam + Live load

= 6.65 + 20 kN/m = 26.65 kN/m

Factored load (w_u) = 1.5 x 26.65 kN/m = 39.975 kN/m

Factored bending moment $(M_u) = \frac{w_u L^2}{8} = \frac{39.975\times6^2}{8}$ kNm = 179.89 kNm

Limiting depth of neutral axis

$x_{u\,lim}$ = 0.46d (For Fe 500 steel)

= 0.46 x 650 mm = 299 mm

Actual depth of neutral axis

Compression force = Tension force

$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

and $M_u = 0.87 f_y A_{st}(d - 0.42x_u)$

$M_{u\,lim} = 0.36 f_{ck}\, b x_{u\,lim}(d - 0.42\, x_{u\,lim})$

= 0.36 x 30 x 380 x 299 (650 – 0.42 x 299) = 643.5 kNm

Alternatively

$M_{u\,lim} = 0.133\, f_{ck}\, bd^2$ (for Fe 500)

= 640.6 kNm (which is very close to 643.5 kNm)

$\therefore$ $M_u < M_{u\,lim}$ and thus we require an under reinforced section

Calculation of x_u for M_u

$M_u = 0.87 f_y A_{st}(d - 0.42x_u)$

$179.89\times10^6 = 0.36\times30\times x_u \times 380(650 - 0.42\, x_u)$

$43832.846 = 650\, x_u - 0.42\, x_u^2$

$x_u = 70.7$ mm

Area of tension steel required

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42x_u)} = \frac{179.89\times10^6}{0.87\times500(650 - 0.42\times70.7)}$$

$= 666.7\ mm^2$

Alternatively

$$R = \frac{M_u}{bd^2} = \frac{179.89\times10^6}{380\times650^2} = 1.120461$$

$\therefore$ $$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{30}{2(500)}\left[1 - \sqrt{1 - 4.598\times\frac{1.120461}{30}}\right]$$

$\Rightarrow$ $p_t = 0.2679\%$

$\Rightarrow$ $$A_{st} = \frac{0.2697}{100}\times380\times650$$

$= 666.16\ mm^2$ (same as above)

Example 4.10 Design a RCC beam for an effective span of 8.5 m subjected to a live load of 45 kN/m. Width of the beam is 400 mm. Use M 30 concrete and Fe 500 steel. Design by both LSM and WSM and compare the results.

Solution:

By LSM

Let overall depth of the beam is 850 mm

Assuming an effective cover of 50 mm

Live load = 45 kN/m

Self weight of the beam = 0.85 x 0.4 x 25 kN/m = 8.5 kN/m

Total load = 45 + 8.5 kN/m = 53.5 kN/m

Factored load (w_u) = 1.5 x 53.5 kN/m = 80.25 kN/m

Factored bending moment $(M_u) = \frac{w_u L^2}{8} = \frac{80.25\times8.5^2}{8}$ kNm = 724.76 kNm

Design of beam as balanced section

$M_u = M_{u\,lim} = Qbd^2$

Where $Q = 0.36\, f_{ck}\, k(1 - 0.42\, k)$

$= 0.36\times30\times0.46(1 - 0.42\times0.46)$ (for Fe 500, $k = 0.46$)

$= 4.008$

$\therefore$ $724.76\times10^6 = Qbd^2$

$724.76\times10^6 = 4.008\times400\times d^2$

$d = 672.36$ mm

Alternatively

$M_{u\,lim} = 0.133\, f_{ck}\, bd^2$ (for Fe 500)

$\Rightarrow$ $724.76\times10^6 = 0.133(30)(400)\, d^2$

$d = 673.87$ mm

$\therefore$ $D = 673.87 + 50$ mm = 723.87 mm

Provide D = 725 mm, d = 675 mm

$\therefore$ self weight of the beam = 0.725 × 0.4 × 25 = 7.25 kN/m

Total load = 45 + 7.25 = 52.25 kN/m

Factored load = 1.5 × 52.25 = 78.375 kN/m

$$\text{Factored bending moment } (M_u) = \frac{78.375 \times 8.5^2}{8} = 707.82 \text{ kNm}$$

Area of steel required

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right) bd$$

$$= \frac{0.5 \times 30}{500}\left(1 - \sqrt{1 - \frac{4.6 \times 707.82 \times 10^6}{30 \times 400 \times 675^2}}\right) \times 400 \times 675$$

$$= 2948.485 \text{ mm}^2$$

Using 25 mm dia. bars, no. of bars required $= \frac{2948.485}{\frac{\pi}{4} \times 25^2} = 6$ bars

By WSM

Total load = 52.25 kN/m

$$\text{Bending moment } (M) = \frac{52.25 \times 8.5^2}{8} \text{ kNm} = 471.88 \text{ kNm}$$

Beam depth required

$$d = \sqrt{\frac{M}{Qb}}$$

$$Q = \frac{1}{2} \times cjk$$

$$k = \frac{mc}{mc + t} = \frac{9 \times 10}{9 \times 10 + 275} = 0.246$$

$$j = 0.918$$

$$Q = \frac{1}{2} \times 10 \times 0.246 \times 0.918 = 1.13$$

$$d = \sqrt{\frac{483.17 \times 10^6}{1.13 \times 400}} = 1021.76 \approx 1022 \text{ mm}$$

D = 1022 + 50 mm = 1072 mm = 1100 mm

$\therefore$ d = 1100 − 50 mm = 1050 mm

Self weight of the beam = 0.4 × 1.1 × 25 kN/m = 11 kN/m

Total load = 11 + 45 kN/m = 56 kN/m

$$\text{Bending moment } (M) = \frac{wL^2}{8} = \frac{56 \times 8.5^2}{8} \text{ kNm} = 505.75 \text{ kNm}$$

$$d = \sqrt{\frac{505.75 \times 10^6}{1.13 \times 400}} \text{ mm} = 1058 \text{ mm} > 1050 \text{ mm} \quad \text{(Not OK)}$$

Assume D = 1110 mm

$\therefore$ d = 1110 − 50 mm = 1060 mm

Self weight of the beam = 1.11 × 0.4 × 25 kN/m = 11.10 kN/m

Total load = 11.1 + 45 kN/m = 56.1 kN/m

$$\text{Bending moment } (M) = \frac{wL^2}{8} = \frac{56.1 \times 8.5^2}{8} \text{ kNm} = 506.65 \text{ kNm}$$

$$d = \sqrt{\frac{506.65 \times 10^6}{1.129 \times 400}} \text{ mm} = 1059 \text{ mm} < 1060 \text{ mm} \quad \text{(OK)}$$

Area of steel required

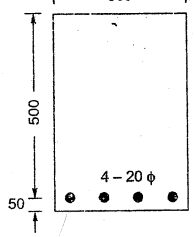
$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{506.65 \times 10^6}{275 \times 0.918 \times 1060} \text{ mm} = 1893 \text{ mm}^2$$

Comparison of results

LSM	WSM
Width (b) = 400 mm Depth (D) = 725 mm d = 675 mm A_{st} = 2948.485 mm^2	Width (b) = 400 mm Depth (D) = 1110 mm d = 1060 mm A_{st} = 1893 mm^2

Remember: Thus with LSM, section comes out to be smaller than that of WSM but in LSM, steel comes out to be larger than that coming out from WSM.

Example 4.11 Determine the permissible value of service load that can be imposed on a beam of effective span 7 m. The beam size is 300 mm × 550 mm with an effective cover of 50 mm. Use M 25 concrete and Fe 500 steel. The beam is reinforced with 4–20 ϕ bars on tension side.



Solution:

Given: f_{ck} = 25 N/mm^2, f_y = 500 N/mm^2, b = 300 mm

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

D = 550 mm

$\therefore$ d = 550 − 50 (effective cover)

Let $x_u < x_{u\,lim}$

Where, $x_{u\,lim}$ = 0.46 d = 0.46 (500) = 230 mm

$$\therefore \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87(500)(1256.64)}{0.36(25)(300)} = 201.34 \text{ mm} < x_{u\,lim}$$

$\therefore$ Assumption is true and f_{st} = 0.87 f_y is correct

$\therefore$ Ultimate moment of resistance of the beam (MOR)

$$= 0.87 f_y A_{st}(d - 0.42 x_u)$$

$$= 0.87(500)(1256.64)(500 - 0.42 \times 201.34) = 227.094 \text{ kNm}$$

Now $\text{MOR} = \frac{w_u l^2}{8}$ (for simply supported beam)

$$\Rightarrow \quad w_u = \frac{8(227.094)}{7^2} = 37.077 \text{ kN/m} = \text{Factored load}$$

$$\therefore \quad \text{Service load} = \frac{\text{Factored load}}{1.5} = \frac{37.077}{1.5} = 24.718 \text{ kN/m}$$

Self weight of beam = 0.3 × 0.55 × 25 kN/m = 4.125 kN/m

$\therefore$ Maximum service imposed loads that the beam is able to take

$$w_L = (24.718 - 4.125)\ \text{kN/m}$$
$$= 20.593\ \text{kN/m} \simeq 20\ \text{kN/m}$$

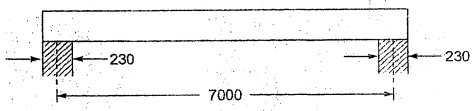
Example 4.12 Design a simply supported rectangular beam whose centre to centre distance between the supports is 7 m and is supported on 230 mm thick walls. Assume live load 7 kN/m length of beam. Use M25 concrete and Fe415 steel.

Solution:

Step: 1 Unknown Parameters : b, d (and thus D), A_{st}

Step: 2 Effective Span : As per Cl. 22.2 (a) of IS 456: 2000, effective span is taken as :

(i) Clear span + effective depth
(ii) Centre to centre distance between the two supports
} whichever is less



$\therefore$ Clear span $= 7000 - \frac{230}{2} - \frac{230}{2} = 6770$ mm

Now Cl. 23.2.1 of IS : 456 – 2000 gives basic ratios of span to effective depth which is 20 for simply supported beam for beam upto 10 m span.

Here, Span = 7 m < 10 m

$\therefore \quad \frac{l}{d} \le 20$

$\Rightarrow \quad d \ge \frac{l}{20} = \frac{7000}{20} = 350$ mm

$\therefore$ Adopt $d = 350$ mm

$\therefore$ Effective span is

(i) Clear span + d = 6770 + 350 = 7120 mm
(ii) c/c distance between supports = 7000 mm
} whichever is less

Hence, $L_{eff} = 7000$ mm = Effective span of beam

Step: 3 Assume percentage of steel reinforcement for calculation of actual effective depth

For Fe 415, $\frac{x_{u\,lim}}{d} = 0.48$

$\therefore \quad p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61\left(\frac{25}{415}\right)0.48 = 1.203\ \%$

= Maximum percentage of tension steel reinforcement

Also, as per Cl. 26.5 . 1. 1(a) of IS 456: 2000, minimum percentage of steel reinforcement is given by,

$$\frac{A_{st\,lim}}{bd} = \frac{0.85}{f_y}$$

Let $p_t = 0.9\ \% < p_{t\,lim}$ (OK)

Step: 4 Effective Depth (d) : The basic value of span/ depth ratio is multiplied by modification factor (k_t) to account for tension reinforcement.

$$f_s = 0.58 f_y \left(\frac{\text{Area of steel required}}{\text{Area of steel provided}}\right) = 0.58\,(415)$$

Assuming area of steel required = Area of steel provided = 240.7 N/mm^2

From Fig. 4 of IS 456: 2000,

$k_t \simeq 1$ for $f_s = 240.7$ N/mm^2 and $p_t = 0.9$ % (assumed)

$\therefore$ Actual span/depth ratio $= \left(\frac{l}{d}\right)_{basic} k_t \times k_c$ Where, $k_c = 1$ for $p_c = 0$ %

$= 20\,(1) = 20$

$\therefore \quad d = \frac{\text{span}}{20} = \frac{7000}{20} = 350$ mm

Which is same as assumed earlier

Let Effective cover = 50 mm

$\therefore$ Overall depth (D) = 350 + 50 = 400 mm

Width of Beam (b): The ratio $\frac{b}{D}$ should preferably lie between 0.5 and 0.7

Let $b = 250$ mm

$\therefore \quad \frac{b}{D} = \frac{250}{400} = 0.625$ (between 0.5 and 0.7) (OK)

Step: 5 Design loads and design bending moment

Cl. 19.2.1 of IS 456: 2000 specifies unit weight of reinforced concrete as 25 kN/m^3

$\therefore$ Self weight of beam = 0.25 × 0.4 × 25 kN/m = 2.5 kN/m

Live load = 7 kN/m

$\therefore$ Total load = 2.5 + 7 = 9.5 kN/m

$\therefore$ Factored load (w_u) = 1.5 × 9.5 kN/m = 14.25 kN/m

(Partial safety factor for dead and live loads is 1.5 for limit state of collapse)

$\therefore$ Factored bending moment (M_u) $= \frac{w_u l^2_{eff.}}{8} = 14.25 \times \frac{7^2}{8} = 87.28$ kNm

Step: 6 Effective depth of beam required from design moment consideration

For Fe 415, $M_{u\,lim} = 0.138\, f_{ck}\, bd^2$

$\Rightarrow \quad 87.28 \times 10^6 = (0.138)(25)(250)d^2$

$\Rightarrow \quad d = 318.11$ mm < 350 mm as calculated above (OK)

Step: 7 Area of steel required

Since $d = 350$ mm and effective depth required from design moment consideration is 318.11 mm which is less than adopted effective depth (= 350 mm).

$\therefore$ Design moment is on conservative side and need not to be revised.

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right]$$

Where $R = \frac{M_u}{bd^2} = \frac{87.28 \times 10^6}{250 \times 350^2} = 2.85$ N/mm^2

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1-\sqrt{1-4.598\times\frac{2.85}{25}}\right]$$

$$= 9.343\times10^{-3}$$

$$\therefore \quad p_t = 0.9343\,\%$$

$$\simeq p_t \text{ assumed } (=0.9\,\%)$$

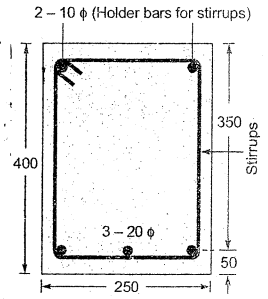
$$\therefore \quad A_{st} = \frac{0.9343}{100}\times250\times350$$

$$= 817.51\ \text{mm}^2$$

Using 20 mm ϕ bars, number of bars reqd. $= \dfrac{817.51}{\frac{\pi}{4}(20)^2}$

$= 2.6 \simeq 3$ nos.

Provide 3-20 ϕ bars at bottom and 2-10 ϕ bars at top for holding the stirrups as hanger bars.



Reinforcement Detailing

Example 4.13 Design a cantilever beam of clear span 2.7 m carrying a superimposed live load of 17.5 kN/m. Use M 25 concrete and Fe 415 steel.

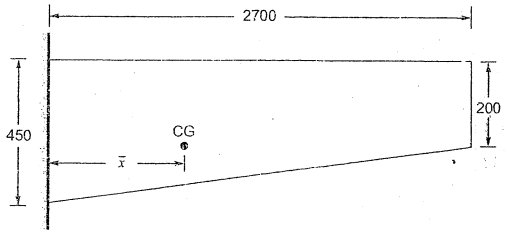
Solution:

Given Clear span = 2700 mm

$$d = \frac{\text{Clear span}}{7} = \frac{2700}{7} = 385.7\ \text{mm}$$

Adopt overall depth as 450 mm at the fixed end with constant width $b = 0.5(D) = 225$ mm. Depth of the beam is reduced to 200 mm at the free end with linear variation of depth along the span.

$$\text{Self weight of beam} = \frac{(0.45+0.2)}{2}\times2.7\times0.225\times25 = 4.94\ \text{kN} \simeq 5\ \text{kN}$$



Distance of centre of gravity (CG) of beam from fixed end

$$= \bar{x}$$

$$= \left(\frac{2\times200+450}{200+450}\right)\frac{2700}{3} = 1176.92\ \text{mm}$$

$$\therefore \quad \text{Factored } BM\,(M_u) = 1.5\left[5\times1.17692+17.5\times\frac{2.7^2}{2}\right] = 104.51\ \text{kNm}$$

$$\therefore \quad M_u = 0.138\,f_{ck}\,bd^2$$

$$\Rightarrow \quad 104.51\times10^6 = 0.138\,(25)\,(225)\,d^2$$

$$\Rightarrow \quad d = 366.93\ \text{mm at fixed end.}$$

$$\therefore \quad \text{Overall depth, } D = 366.91 + 25 + \frac{20}{2} + 8(\text{for shear stirrups}) = 409.91\ \text{mm}$$

Assuming clear cover 25 mm, dia. of main bar 20 mm and stirrups 8 mm.

Thus adopt $D = 450$ mm at fixed end

$$\therefore \quad d = 450 - 25 - \frac{20}{2} - 8 = 407\ \text{mm}$$

Limiting depth of neutral axis $(x_{u\,lim}) = 0.48\,d = 0.48(407) = 195.36$ mm

Limiting reinforcement

$$A_{st\,lim} = \frac{0.362\,f_{ck}\,b\,x_u}{0.87\,f_y} = \frac{0.362(25)(225)195.36}{0.87(415)}$$

$$= 1101.79\ \text{mm}^2 \text{ (corresponding to balanced section)}$$

Minimum reinforcement

$$A_{st\,min} = 0.85\frac{bd}{f_y} = \frac{0.85(225)(407)}{415}$$

$$= 187.56\ \text{mm}^2 < 1101.79\ \text{mm}^2$$

Provide 3 – 20 ϕ bars at top $\left(A_{st} = 3\times\frac{\pi}{4}\times20^2 = 942.5\,\text{mm}^2\right)$

Providing $A_{st} > 1101.79$ mm² will make the section over-reinforced

Shear Check (is covered in more detail in later chapters)

Factored *SF* at fixed end $(V_u) = 1.5[5 + 17.5\times2.7] = 78.375$ kN

Nominal shear stress $(\tau_v) = \dfrac{V_u - \frac{M_u}{d}\tan\beta}{bd}$

$$\tan\beta = \frac{450-200}{2700} = 0.09259$$

$$\therefore \quad \tau_v = \frac{78.375\times10^3 - \frac{104.51\times10^6\,(0.09259)}{407}}{225(407)} = 1.115\ \text{N/m}^2$$

$$p_t = \frac{3\times\frac{\pi}{4}\times20^2}{225\times407}\times100 = 1.029\,\%$$

Shear strength of M 25 concrete for $p_t = 1.029\,\%$ as per table 19 of IS : 456 $(\tau_c) = 0.65$ N/m²

$< \tau_v (= 1.115\ \text{N/mm}^2)$

So, shear reinforcement is required for a design shear stress of

$$\tau_{vs} = \tau_v - \tau_c$$

$$= 1.115 - 0.65 = 0.465\ \text{N/mm}^2$$

Using 2 legged 8 ϕ stirrups, $A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.53\ \text{mm}^2$

Spacing of shear stirrups $(S_v) \le \dfrac{0.87 f_y A_{sv}}{\tau_{us} b} \le \dfrac{0.87(415)100.53}{0.465(225)} \le 346.9$ mm c/c

$\not> 0.75\, d\,(= 0.75 \times 407 \text{ mm} = 305.25 \text{ mm})$

But spacing of shear stirrups $(S_v) < \begin{cases} 300\text{mm} \\ 0.75\ \text{d} \end{cases}$ (whichever is less)

∴Provide 2 legged 8 ϕ stirrups @ 250 c/c

$$A_{sv\,min} \ge \frac{0.4 b S_v}{0.87 f_y} = \frac{0.4(225)250}{0.87(415)}$$

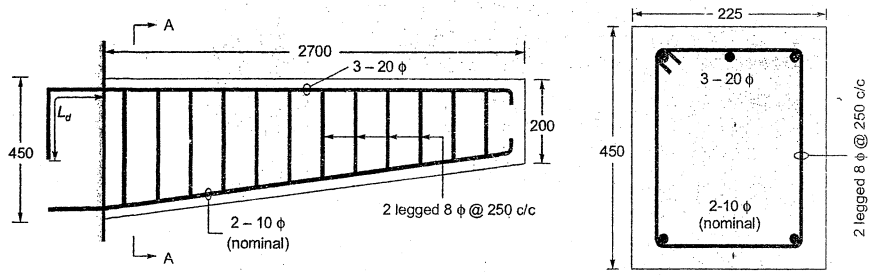
$= 62.32\ \text{mm}^2 < 100.53\ \text{mm}^2$ (OK)

Development Length (is covered in more detail in chapters)

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

where $\tau_{bd} = 1.6 \times 1.4 = 2.24\ \text{N/mm}^2$

∴ $L_d = \dfrac{0.87(415)20}{4(2.24)} = 805.92\ \text{mm} = 810\ \text{mm (say)}$

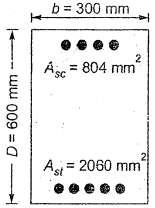


Reinforcement Detailing

Example 4.14 Determine the moment of resistance of a beam of size 300 × 600 mm (overall depth)? It is reinforced with 804 sq. mm compression reinforcement and 2060 sq.mm tension reinforcement. Take effective cover as 50 mm. Use M20 concrete and Fe 415 steel.

Solution:

MOR of over reinforcement section



Effective cover = 50 mm

f_{ck} = 20 N/mm² (M20 concrete)

f_y = 415 N/mm² (Fe 415 steel)

Effective depth (d) = 600 – 50 = 550 mm

$x_{u\,lim}$ = 0.479 d = 0.479 × 550 = 263.45 mm

Let $x_u < x_{u\,lim}$

∴ $0.362\, f_{ck}\, b x_u = 0.87 f_y A_{st}$

$$\Rightarrow x_u = \frac{0.87 \times 415 \times 2060}{0.362(20)300}$$

$= 342.43\ \text{mm} > x_{u\,lim}$

∴ $f_{st} \ne 0.87\, f_y$

I[st] iteration

Let $x_u = \dfrac{342.43 + 263.45}{2} = 302.94$ mm

$$\epsilon_{sc} = 0.0035\left(\frac{550}{302.94} - 1\right) = 0.0028544$$

∴ $f_{st} = 351.817\ \text{N/mm}^2$

∴ $x_u = \dfrac{f_{st} A_{st}}{0.362 f_{ck} b} = \dfrac{351.817 \times 2060}{0.362(20)300} = 333.675$ mm

II[nd] iteration

Let $x_u = \dfrac{302.94 + 333.675}{2} = 318.3075$ mm

$$\therefore = 0.0035\left(\frac{550}{318.3075} - 1\right) = 0.0025476$$

∴ $f_{st} = 345.538\ \text{N/mm}^2$

∴ $x_u = \dfrac{f_{st} A_{st}}{0.362 f_{ck} b} = \dfrac{345.538 \times 2060}{0.362(20)300} = 327.72$ mm

III[rd] iteration

Let $x_u = \dfrac{318.3075 + 327.72}{2} = 323.01375$ mm

$$\therefore = 0.0035\left(\frac{500}{323.01375} - 1\right) = 0.0024595$$

∴ $f_{st} = 343.273\ \text{N/mm}^2$

∴ $x_u = \dfrac{343.273 \times 2060}{0.362(20)300} = 325.572$ mm

IV[th] iteration

Let $x_u = \dfrac{323.01375 + 325.572}{2} = 324.293$ mm

$$\therefore = 0.0035\left(\frac{550}{324.293} - 1\right) = 0.00243599$$

∴ $f_{st} = 342.6683\ \text{N/mm}^2$

$$\therefore \quad x_u = \frac{342.6683 \times 2060}{0.362(20)300}$$

$$= 324.99 \text{ mm (converged)} \simeq 325 \text{ mm}$$

Thus $x_u > x_{u\,lim}$

$\therefore$ Moment of resistance (MOR) $= 0.362\, f_{ck}\, b x_u (d - 0.42\, x_u)$

$= 0.362\,(20)\,300\,(325)\,(550 - 0.42 \times 325)$

$= 291.89$ kNm

Alternatively

$$\text{MOR} = f_{st} A_{st} (d - 0.42\, x_u)$$

$$= 342.6683\,(2060)\,(550 - 0.42 \times 325)$$

$$= 291.89 \text{ kNm}$$

4.11 Slabs as Rectangular Beams

Slabs under uniaxial flexure behave in a same way as that of beams. A slab of constant thickness which is subjected to a bending moment uniformly distributed over its entire width can be treated as wide shallow beam. In such type of slabs, the reinforcing bars are placed uniformly over the width of the slab. As stated above, computations are generally done by considering 1m (=1000 mm) wide strip of the slab. The spacing of bars is given by:

$$s = \frac{1000 \cdot A_\phi}{A_{st}}$$

Here, A_ϕ is the area of one reinforcing bar and A_{st} is the area of tensile reinforcement required for 1 m wide strip. Reinforced concrete slabs are generally singly reinforced.

4.12 Transverse Moments in One Way Slabs

In the bending of a normal beam under the influence of gravity loads like dead and live loads, as the beam bends about the neutral axis, the top portion comes under compression and bottom portion under tension. In the top portion under compression, due to Poisson's effect, a lateral expansion/elongation takes place. Similarly due to Poisson's effect, lateral contraction also takes place in the bottom tensile portion of the beam. Thus the cross section of the beam acquires a trapezoidal shape rather than a rectangular shape.

However in the case of one way slab, such lateral displacements are invariably prevented due to the remaining part of the slab on either side of the 1000 mm wide strip of the beam considered except at the two extreme edges. The remaining part of the slab restraints the transverse displacement of the 1000 mm strip by inducing lateral stresses. This is called as plain strain condition. These restraints induce secondary moments in the slab in the transverse direction.

Thus one way slab requires secondary transverse reinforcements in the transverse direction to resist these secondary moments. Shrinkage and temperature also induce secondary moments in slab which is to be resisted by secondary transverse reinforcement.

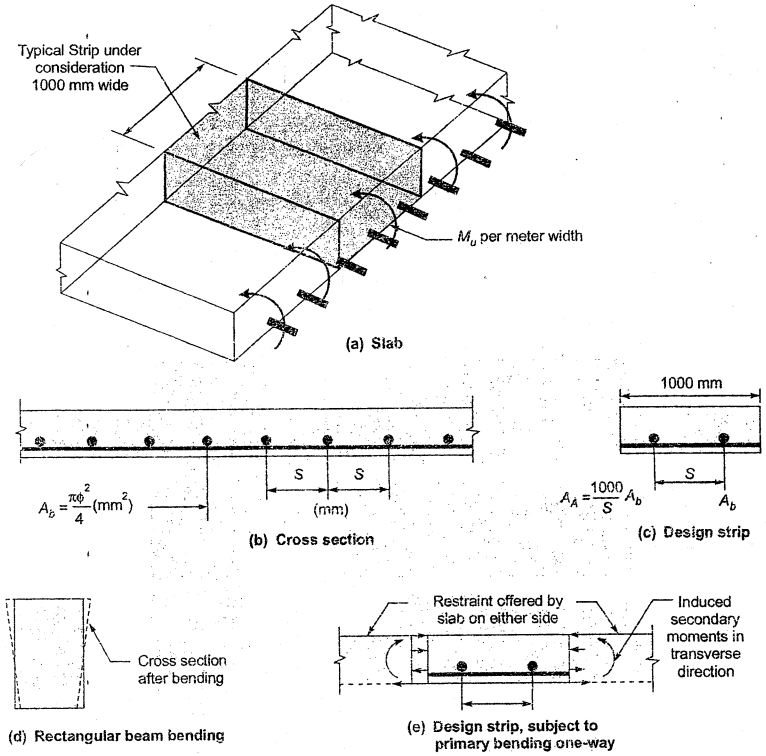
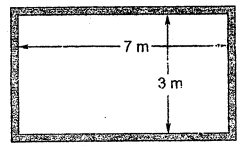


Fig. 4.22 *Transverse moments in one way slabs*

Example 4.15 A simply supported roof slab of clear size 7 m x 3 m subjected to a live load of 4 kN/m^2. Use M25 concrete and Fe415 steel. The slab rest on 230 mm thick (one brick wall) masonry walls all around.

Solution:



$$\frac{l_y}{l_x} = \frac{7}{3} > 2 \Rightarrow \text{One way slab}$$

Let effective depth of slab $= \frac{3000}{20} = 150$ mm

Assuming clear cover 15 mm and dia. of main reinforcement is 20 mm.

$\therefore$ Overall depth of slab (D) $= 150 + 15 + \frac{20}{2} = 175 \text{ mm} \simeq 190 \text{ mm (say)}$

$\therefore \quad d = 190 - 15 - \frac{20}{2} = 165$ mm

$$\text{Effective span } (l_e) = \begin{Bmatrix}\text{Clear span} + d \\ \text{c/c distance between the supports}\end{Bmatrix} \text{whichever is less}$$

$$= \begin{Bmatrix}3000 + 165 = 3165 \text{ mm} \\ 3000 + \frac{230}{2} + \frac{230}{2} = 3230 \text{ mm}\end{Bmatrix} \text{whichever is less}$$

$$= 3165 \text{ mm}$$

Loads

Assuming 1 m strip of slab

Self weight of slab = $0.19 \times 25 = 4.75$ kN/m

Live load = 4 kN/m

Assume miscellaneous load = 1 kN/m

Total load = $4.75 + 4 + 1$ kN/m = 9.75 kN/m

Factored load (w_u) = 1.5×9.75 kN/m = 14.625 kN/m

$$\text{Factored moment } (M_u) = \frac{w_u l^2}{8} = \frac{14.625}{8}(3.165)^2 = 18.31 \text{ kNm}$$

Reinforcement requirement in slab

$$R = \frac{M_u}{bd^2} = \frac{18.31 \times 10^6}{1000 \times 165^2} = 0.6725$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - \frac{4.598R}{f_{ck}}}\right] = 1.92 \times 10^{-3}$$

$$\Rightarrow \quad p_t = 0.192\%$$

$$\Rightarrow \quad A_{st} = \frac{0.192}{100} \times 1000 \times 165 = 317.493 \text{ mm}^2/\text{m}$$

Using 20 mm diameter bars, spacing $= \dfrac{1000 \times \frac{\pi}{4} \times 20^2}{317.493} = 989.5 \text{mm} > 300 \text{ mm}$ (Not OK)

Using 12 mm diameter bars, spacing $= \dfrac{1000 \times \frac{\pi}{4} \times 12^2}{317.493} = 356.2 \text{mm} > 300 \text{ mm}$ (Not OK)

Using 10 mm diameter bars, spacing $= \dfrac{1000 \times \frac{\pi}{4} \times 10^2}{317.493} = 247.4 \text{mm} < 300 \text{ mm}$ (OK)

Maximum bar spacing $\not>$ $\begin{Bmatrix}3d = 3 \times 165 = 495 \text{mm} \\ 300 \text{mm}\end{Bmatrix}$ (whichever is small)

Provide 10ϕ bars @ 230 mm c/c.

Distribution reinforcement

Distribution reinforcement @ 0.12% $= \dfrac{0.12}{100} \times 1000 \times 190 = 228 \text{mm}^2/\text{m}$

Using 10ϕ bars, spacing $= \dfrac{1000 \times \frac{\pi}{4} \times 10^2}{228} = 344.5 \text{mm c/c}$

Maximum bar spacing $\not>$ $\begin{Bmatrix}5d = 5 \times 165 = 825 \text{mm} \\ 450 \text{mm}\end{Bmatrix}$ (whichever is less) = 450 mm

Provide 10ϕ bars @ 300 mm c/c.

Check for deflection

$A_{st\,reqd.} = 317.493 \text{ mm}^2/\text{m}$

$$A_{st\,provided} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{230} = 341.48 \text{mm}^2/\text{m}$$

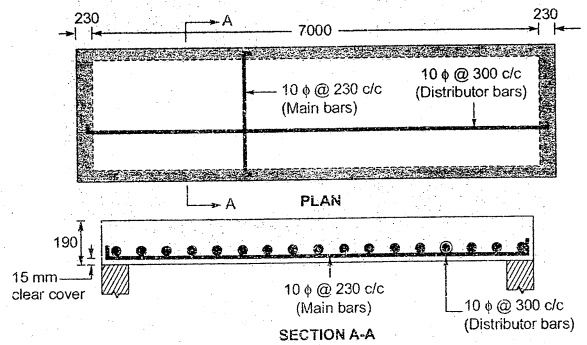
$$f_{st} = 0.58 \times \frac{317.493}{341.48} \times 415 = 223.79 \text{ N/mm}^2$$

$$p_t = \frac{341.48}{1000 \times 165} = 0.207\%$$

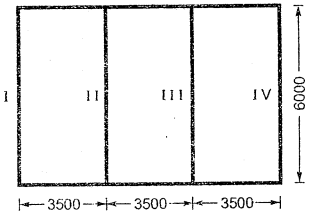
$k_t \simeq 1.8$ (from fig. 4 of IS 456: 2000)

$$\left(\frac{l}{d}\right)_{max} = k_t \cdot k_c \left(\frac{l}{d}\right)_{basic} = 1.8\,(1)\,(20) = 36 \text{ where } k_c = 1 \text{ for } p_c = 0$$

$$\left(\frac{l}{d}\right)_{provided} = \frac{3165}{165} = 19.18 < \left(\frac{l}{d}\right)_{max}$$ (OK)



Example 4.16 Design a one-way slab as shown in figure below which is subjected to imposed load of 5 kN/m². Use M 20 and Fe 415. Take dead load of finish as 0.5 kN/m². Width of beams at support is 300 mm.



Solution:

Step 1: Preliminary depth of slab

∵ The slab is simply supported on edges I and IV and continuous on edges II and III.

$$\therefore \quad \text{Span/depth ratio} = \frac{20 + 26}{2}$$

Average of simply supported and continuous limiting span/depth values.

$= 23$ (Cl. 23.2.1 of IS 456: 2000)

Now for $p_t = 0.5\%$ and $f_y = 240 \text{ N/mm}^2$, $k_t = 1.2$ (Fig. 4 of IS 456: 2000)

Effective depth required from deflection criterion = $d = \frac{3500}{23 \times 1.2} = 126.8$ mm

Let effective cover = 25 mm

∴ Overall depth of slab, D = 126.8 + 25 = 151.8 mm ≈ 155 mm (say)

∴ d = 155 – 25 = 130 mm

Step 2: Design loads, bending moment and shear force

DL of slab per meter width = 0.155 × 25 = 3.875 kN/m

DL of floor finish given as 0.5 kN/m² = 0.5 kN/m per meter width of stab.

∴ Total DL = 3.875 + 0.5 = 4.375 kN/m

LL = 5 kN/m² = 5 kN/m per meter width of slab.

∴ Total load = 4.375 + 5 = 9.375 kN/m

∴ Factored load = 1.5 × 9.375 = 14.06 kN/m

Maximum positive B.M. = $(14.06)\frac{3.5^2}{12} = 14.4$ kNm/m

Maximum shear force (V_u) = (14.06) 3.5 (0.4) = 19.7 kN

Step 3: Determining the effective depth required

For Fe 415, $M_{u\,min} = 0.138 f_{ck} bd^2$

⇒ $17.2 \times 10^6 = 0.138 (20) (1000) d^2$

⇒ d = 78.9 mm < 130 mm (= $d_{provided}$) (OK)

Step 4: Check for slab depth w.r.t. shear

Design shear strength of M 20 concrete for

$p_t \le 0.15$ %, τ_c = 0.28 N/mm² (Table 19 of IS 456: 2000)

k = 1.29 (As per Cl. 40.2.1 of IS 456: 2000)

∴ $k\tau_c$ = 1.29 (0.28) = 0.36 N/mm²

$\tau_{c\,max}$ (M 20 concrete) = 2.8 N/m² (Table 20 of IS 456: 2000)

$\tau_v = \frac{V_u}{bd} = \frac{19.7 \times 1000}{1000 \times 130} = 0.15$ N/mm²

Thus $\tau_v < \tau_c < \tau_{c\,max}$

∴ Depth adopted is OK and no shear reinforcement is required.

Step 5: Flexural reinforcement requirement

Negative reinforcement

$M_u^- = 17.2$ kNm/m

∴ $R = \frac{17.2 \times 10^6}{10^3 \times 130^2} = 1.018$

∴ $\frac{p_t^-}{100} = \frac{A_{st}^-}{bd} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - \frac{4.598 \times 1.018}{20}}\right] = 0.003$

∴ p_t^- = 0.3 %

and A_{st}^- = 390 mm²/m

Using 10 mm ϕ bars, spacing = $\frac{1000 \times \frac{\pi}{4} \times 10^2}{390} = 201.4$ mm = 200 mm c/c (say)

Provide 10 ϕ bars @ 200 c/c

M_u^+ = 14.4 kNm/m

∴ $R = \frac{14.4 \times 10^6}{10^3 \times 130^2} = 0.852$

∴ $\frac{p_t^+}{100} = \frac{A_{st}^+}{bd} = \frac{20}{2(415)}\left[1 - \sqrt{1 - \frac{4.598 \times 0.852}{20}}\right] = 0.0025$

∴ p_t^+ = 0.25 %

A_{st}^+ = 325 mm²/m

Using 10 mm ϕ bars, spacing = $\frac{1000 \times \frac{\pi}{4} \times 10^2}{325} = 241.7$ mm c/c = 240 mm c/c (say)

Provide 10ϕ bars @ 200 c/c

Distribution reinforcement @ 0.12 % along long span

= $\frac{0.12}{100} \times 1000 \times 130 = 156$ mm²/ m

Also, minimum reinforcement ($A_{st\,min}$) = 0.12% of bd

= $\frac{0.12}{100} \times 1000 \times 130 = 156$ mm²/m

Using 8 mm ϕ bars for distribution reinforcement,

Spacing = $\frac{1000 \times \frac{\pi}{4}(8)^2}{156} = 322.2$ mm c/c = 280 mm c/c (say)

Maximum spacing for main bars ≯ 3d or 300 mm which ever is less.

≯ 3 × 130 or 300 mm

≯ 390 or 300 mm

= 300 mm

Maximum spacing of distribution bars ≯ 5d or 450 mm which even is less

≯ 5(130) or 450

≯ 650 or 450

= 450 mm

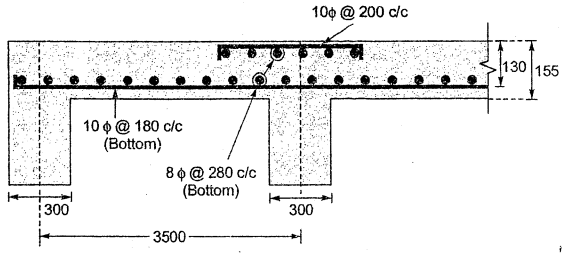
≯ 280 mm (OK)

Maximum main bar dia. ≯ $\frac{D}{8} = \frac{155}{8} = 19.375$ mm

> 10 mm (OK) (main bars)

> 8 mm (OK) (distribution bars)

Provide 8ϕ bars @ 280 c/c as distribution reinforcement.



Example 4.17 A simply supported slab of size 3.6 m × 9.0 m (clear) is subjected to a live load of 8 kN/m². Floor finishing is 60 mm thick of cement concrete flooring. The slab is supported over 300 mm thick masonry wall supports. Design the slab using LSM. Use M 20 concrete and Fe 500 steel.

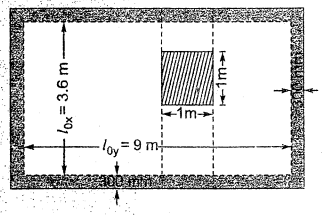
Solution:

Effective span:

Effective depth as per deflection criterion

$$d = \frac{\text{span}}{20}$$

$$\Rightarrow \quad d = \frac{3600}{20} = 180 \text{ mm}$$



Assuming effective cover of 30 mm,

Overall depth of the slab (D) = 180 mm + 30 mm = 210 mm

Effective span along x, l_x = clear span + d = 3.6 + 0.18 m
= clear span + c/c distance between the supports } whichever is less

= 3.78 m
= 3.6 + 0.3 m = 3.9 m } whichever is less

= 3.78 m

Effective span along, y l_y = clear span + d = 9.0 + 0.18 m
= clear span + c/c distance between the supports } whichever is less

= 9.18 m
= 9.0 + 0.3 m = 9.3 m } whichever is less

= 9.18 m

$$\frac{l_y}{l_x} = \frac{9.18}{3.78} = 2.42 > 2$$

∴ it is a one way slab.

Load Calculation

Live load = 8 kN/m² (Given)

Live load for 1 m × 1 m slab area = 8 × 1 × 1 kN = 8 kN

Dead load

Self weight of the slab = 0.21 × 1 × 1 × 25 kN
= 5.25 kN

Dead load of floor finishing = 0.06 × 1 × 1 × 24 kN = 1.44 kN

Total load, w = 8 + 5.25 + 1.44 kN
= 14.69 kN = 14.69 kN/m

Factored load, w_u = 1.5 × 14.69 kN/m = 22.04 kN/m

Design Bending Moment

Factored bending moment, $M_u = \frac{w_u l_x^2}{8} = \frac{22.04 \times 3.78^2}{8}$ kNm = 39.36 kN

Slab depth required

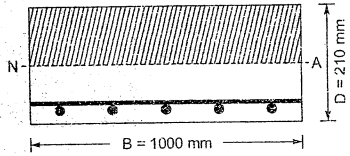
$$Q = 0.36 f_{ck} k(1 - 0.42k)$$
$$= 0.36 \times 20 \times 0.46 (1 - 0.42 \times 0.46) = 2.67 \qquad (k = 0.46 \text{ for Fe } 500)$$

$$d = \sqrt{\frac{M_u}{QB}} = \sqrt{\frac{39.36 \times 10^6}{2.67 \times 1000}} = 121.41 \text{ mm}$$

But depth required from deflection criterion = 180 mm

∴ Adopt effective depth of 180 mm.

∴ Overall depth of the slab (D) = 180 + 30 mm
= 210 mm



Area of steel required

$$A_{st} = \frac{M_u}{0.87 f_y jd}$$

$$= \frac{39.36 \times 10^6}{0.87 \times 500 \times 0.8 \times 180} \text{ mm}^2 = 628.35 \text{ mm}^2$$

Alternatively, area of steel can be calculated as,

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.598 M_u}{f_{ck} B d^2}}\right] bd$$

$$= \frac{0.5 \times 20}{500}\left[1 - \sqrt{1 - \frac{4.598 \times 39.36 \times 10^6}{20 \times 1000 \times 180^2}}\right] \times 1000 \times 180 \text{ mm}^2$$

$$= 544.04 \text{ mm}^2$$

Using 10 mm dia. bars, spacing of bars required

$$= \frac{1000 A_\phi}{A_{st}} = \frac{1000}{A_{st}} \times \frac{\pi}{4} \times \phi^2 = \frac{1000}{544.04} \times \frac{\pi}{4} \times 10^2$$

= 144.36 mm c/c ≈ 140 mm c/c

$\ngtr$ 3d (3 × 180 mm = 540 mm)

$\ngtr$ 400 mm (OK)

Distribution reinforcement

Distribution reinforcement = 0.12% of $bD = \frac{0.12}{100} \times 1000 \times 210 = 252 \text{ mm}^2$

Using 8 mm dia. bars, spacing of bars required

$$= \frac{1000A_\phi}{A_{st}} = \frac{1000}{A_{st}} \times \frac{\pi}{4} \times \phi^2 = \frac{1000}{252} \times \frac{\pi}{4} \times 8^2$$

$$= 199.46 \text{ mm c/c} \approx 190 \text{ mm c/c}$$

Check for shear

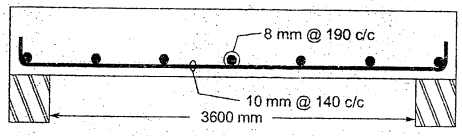
Factored shear force $V_u = \frac{w_u l}{2} = \frac{22.04 \times 37.5}{2} \text{kN} = 41.66 \text{ kN}$

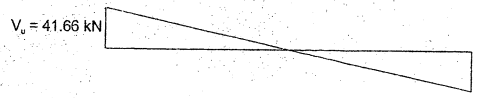
Nominal shear stress, $\tau_{ve} = \frac{V_u}{Bd} = \frac{41.66 \times 10^3}{1000 \times 180} \text{N/mm}^2 = 0.23 \text{ N/mm}^2$

$< \tau_c (= 0.28 \text{ N/mm}^2)$

So the slab is safe in shear.

Check for bond (is covered in more detail later chapters)





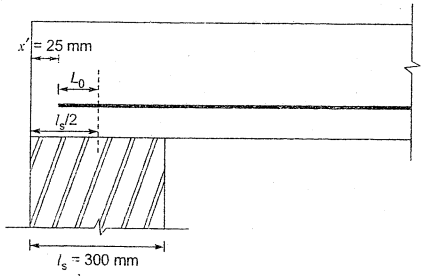
$$\tau_{bd(dev)} = \frac{V_u}{\Sigma o j d} = \frac{V_u}{n\phi\pi jd}$$

$$= \frac{41.66 \times 10^3}{\left(\frac{1000}{140}\right) \times \pi \times 10 \times 0.8 \times 180} \text{N/mm}^2 = 1.29 \text{ N/mm}^2$$

$\tau_{bd(per)} = 1.2 \times 1.60 \text{ N/mm}^2$ (for tension in LSM) $= 1.92 \text{ N/mm}^2$

$> 1.29 \text{ N/mm}^2$ (OK)

Check for development length (covered in more detail in later chapters)



$$L_0 = \frac{l_s}{2} - x' = \frac{300}{2} - 25 \text{ mm} = 125 \text{ mm}$$

$$M_{u_1} = 0.87 f_y A_{st} jd = \left(\frac{1000}{140}\right) \times \frac{\pi}{4} \times (10)^2 \times 0.87 \times 500 \times 0.8 \times 180 \text{ Nmm}$$

$$= 35.14 \text{ kNm}$$

$V_u = 41.66 \text{ kN}$

Development length, $L_d = \frac{0.87 f_y \phi}{4\tau_{bd(per)}} = \frac{0.87 \times 500 \times 10}{4 \times 1.92} \text{ mm} = 566.4 \text{ mm}$

Now, $1.3\frac{M_{u_1}}{V_u} + L_0 = 1.3 \times \frac{35.14 \times 10^6}{41.66 \times 10^3} + 125\text{mm} = 1221.57 \text{ mm}$

$\therefore \quad L_d \not> 1.3\frac{M_{u_1}}{V_u} + L_0$ (OK)

Objective Brain Teasers

Q.1 When the neutral axis depth (x_u) of a beam section is more than the limiting depth ($x_{u\,lim}$), then stress in tension steel reinforcement:
(a) $> 0.87 f_y$ (b) $< 0.87 f_y$
(c) $= 0.87 f_y$ (d) $0.67 f_y$

Q.2 The limiting percentage of tension reinforcement in a beam made up of M35 concrete and reinforced with Fe 415 steel is:
(a) 3.09% (b) 1.53%
(c) 1.41% (d) 1.68%

Q.3 An inverted T-beam subjected to gravity loading will act as:
(a) T beam (b) Rectangular beam
(c) L beam (d) Data insufficient

Q.4 The moment of resistance of a beam with minimum amount of tension reinforcement ($A_{st\,min}$) is approximately same as:
(a) Cracking torque of the beam
(b) Cracking moment of the beam
(c) Modulus of rupture of concrete
(d) None of the above

Q.5 The minimum grade of concrete recommended for severe climatic condition as per IS 456: 2000 is
(a) M40 (b) M30
(c) M35 (d) M20

Q.6 The minimum nominal cover to reinforcement recommended for very severe climatic condition as per IS 456: 2000 is:
(a) 30 mm (b) 75 mm
(c) 45 mm (d) 50 mm

Q.7 The minimum diameter of hanger bars recommended for holding the stirrups is:
(a) 6 mm (b) 8 mm
(c) 10 mm (d) 12 mm

Q.8 A beam of width 300 mm and effective depth 500 mm of M25 concrete is subjected to a factored flexural moment of 100 kNm. The depth of neutral axis of the beam is:
(a) 79 mm (b) 265.5 mm
(c) 240 mm (d) 50 mm

Q.9 The limiting *l*/D ratio for the end beam of a continuous span whose one end is simply supported and other continuous is:
(a) 7 (b) 23
(c) 20 (d) 10

Q.10 For an under-reinforced beam, the moment of resistance of the beam can be computed from:
(a) Tension side formula
(b) Compression side formula
(c) Both tension and compression side formula
(d) None of (a) or (b)

Q.11 A continuous beam of span 3 m is considered as a deep beam if:
(a) Beam depth < 1200 mm
(b) Beam depth < 1500 mm
(c) Beam depth > 1200 mm
(d) Beam depth > 1350 mm

Q.12 The limit on span to depth ratio is specified in IS 456: 2000 because
(a) it ensures the limit or tensile crack width
(b) it ensures safety against shear failure
(c) it limits the maximum deflection of beam
(d) it limits the maximum tensile stress in steel

Q.13 The depth of rectangular portion of stress block of concrete in compression is
(a) $\frac{3}{7}x_u$ (b) $\frac{4}{7}x_u$
(c) $\frac{2}{7}x_u$ (d) x_u

Q.14 The size of the test specimen to determine the modulus of rupture of concrete is
(a) 150 mm × 150 mm × 700 mm
(b) 100 mm × 100 mm × 700 mm
(c) 150 mm × 150 mm × 150 mm
(d) 100 mm × 100 mm × 100 mm

Q.15 With increase in the rate of loading during testing, compressive strength of concrete
(a) decrease
(b) increase
(c) may increase or decrease depending on concrete grade
(d) data insufficient

Q.16 In order to ensure durability for similar conditions of exposure, water cement ratio in reinforced concrete as compared to plain concrete is
(a) less (b) more
(c) equal (d) data insufficient

Q.17 Consider the following statements:
(i) Compressive strength of concrete decreases with increase in w/c ratio of the concrete
(ii) Water is needed in concrete for hydration of cement and workability
(iii) Creep and shrinkage of concrete are independent of w/c ratio of concrete
of the above statement, true statement(s) is are:
(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (i), (ii) and (iii)

Q.18 Maximum strain in extreme fibre of concrete and in tension steel of Fe 415 grade ($E_s = 2 \times 10^5$ N/mm^2) in a balance section in limit state of flexure are respectively
(a) 0.0035 and 0.0031
(b) 0.0035 and 0.0038
(c) 0.002 and 0.0042
(d) 0.002 and 0.0038

Q.19 The modulus of elasticity of M30 concrete as per IS 456: 1978 and IS 456: 2000 respectively are
(a) 31220 N/mm^2, 27386 N/mm^2
(b) 27386 N/mm^2, 31220 N/mm^2
(c) 27386 N/mm^2, 27386 N/mm^2
(d) None of these

Q.20 The main reinforcement in RC slab consists of 10 mm diameter bars at 100 mm c/c spacing. When 10 mm bars are replaced by 12 mm diameter bars then spacing would be
(a) 175 mm (b) 160 mm
(c) 150 mm (d) 140 mm

Q.21 A RC beam is subjected to following flexural moments
DL moment = 30 kNm
LL moment = 40 kNm
Seismic moment = 20 kNm
The design flexural moment for limit state of collapse is
(a) 105 kNm (b) 108 kNm
(c) 135 kNm (d) 90 kNm

Q.22 The assumption that "plane sections remain plane before and after the bending" leads to
(a) uniform stress over the beam section
(b) uniform strain over the beam section
(c) linearly varying stress over the beam section
(d) linearly varying strain over the beam section

Q.23 The basic principle of structural design is based on
(i) strong column weak beam concept
(ii) strong footing weak column concept
Which of the above statement(s) is/are true?
(a) (i) only (b) (ii) only
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

Q.24 A RC beam is of effective span 5500 mm. The permissible deflection in beam including the effect of all heads along with temperature, creep and shrinkage is
(a) 15.71 mm (b) 22 mm
(c) 2.75 cm (d) 1.1 cm

Q.25 The direct tensile strength of concrete is determined from
(a) modulus of elasticity of concrete
(b) modulus of rupture of concrete
(c) creep coefficient of concrete
(d) shrinkage coefficient of concrete

Q.26 By controlling the span to depth ratio of a beam, the following can be controlled
(a) shear stress in beam
(b) flexural stress in beam
(c) deflection of beam
(d) All of the above

Answers

1. (b)	2. (d)	3 (b)	4. (b)	5. (b)
6. (d)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (c)	13. (a)	14. (a)	15. (b)
16. (a)	17. (d)	18. (b)	19. (a)	20. (d)
21. (b)	22. (d)	23. (c)	24. (b)	25. (b)
26. (c)				

Hints:

2. (d)

$$p_{tlim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\lim}}{d}\right)$$

$$= 41.61\left(\frac{35}{415}\right)(0.48) = 1.68\%$$

8. (a)

Use $\frac{x_u}{d} = \left(1.2021 - \sqrt{1 - 4.598\frac{M_u}{bd^2 f_{ck}}}\right)$

or $M_u = 0.36\, f_{ck}\, B\, x_u (d - 0.42\, x_u)$

9. (b)
It will be average of 20 and 26 i.e. simply supported and continuous spans.

11. (c)
For continuous deep beams, $L/D < 2.5$ and for simply supported beams, this ratio should be < 2.

18. (b)

$$\epsilon_s = 0.002 + \frac{f_y}{1.15\,\epsilon_s}$$

$$= 0.002 + \frac{415}{1.15 \times 2 \times 10^5}$$

$$= 0.0038$$

19. (a)

$$\epsilon_c = 5700\sqrt{f_{ck}} \quad \text{(IS 456: 1978)}$$

$$= 5000\sqrt{f_{ck}} \quad \text{(IS 456: 2000)}$$

20. (d)

$$A_{st} = \frac{1000 A_\phi}{S}$$

$$\therefore \quad \frac{1000 \times \frac{\pi}{4} \times 10^2}{100} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{S}$$

$$\Rightarrow \quad S = 100 \times \frac{12^2}{10^2} = 144 \simeq 140 \text{ mm (say)}$$

21. (b)
1.5 (DL + LL) = 1.5(30 + 40) = 105 kNm
1.5 (DL + EQL) = 1.5(30 + 20) = 75 kNm
1.2(DL + LL + EQL)= 1.2(30 + 40+20)=108 kNm

24. (b)
Permissible deflection
$= \frac{5500}{250} = 22$ mm

Conventional Practice Questions

Q.1 A RC rectangular beam is one brick wide (i.e., 230 mm wide) and overall 400 mm deep. It is reinforced with 4 nos. of 12 mm dia. bars of Fe415 at an effective cover of 31 mm. The beam is simply supported over a span of 3.5 m. Calculate the moment of resistance of the beam and the maximum super-imposed uniformly distributed load that the beam can carry. Use M 20 concrete.

Ans. [Moment of resistance = 53.5 kNm, Maximum super-imposed load = 21 kN/m]

Q.2 Find the amount of tension steel required in a reinforced concrete beam of size 230 mm × 390 mm to resist an ultimate moment of 50 kNm. Assume effective cover as 40 mm and use M 20 concrete and Fe 500 steel.

Ans. [A_{st} = 386 mm^2]

Q.3 Determine the ultimate moment capacity of a rectangular beam of size 250 mm × 450 mm with 50 mm effective cover. The total area of tension steel provided is 3600 mm^2. Use M20 concrete and Fe415 steel.

Ans. [M_{ulim} = 110 kNm]

Q.4 A 150 mm thick one way reinforced concrete slab is reinforced with 10 mm dia. bars provided at 200 mm c/c. Assuming 25 mm effective cover, determine the ultimate moment of resistance of the section. Use M20 concrete and Fe415 steel.

Ans. [M_{ulim} = 16.6 kNm]

Q.5 A reinforced concrete beam of size 500 mm × 800 mm is provided with a duct of size 260 mm × 560 mm located concentrically with the beam. The centroid of tension steel is at a distance of 60 mm from the bottom face of beam. Determine the balanced moment of resistance of the section and amount of reinforcement required. Use M 20 concrete and Fe 415 steel.

Ans. [$M_{u\,lim}$ = 553 kNm, A_{st} = 2478 mm^2]

CHAPTER 05

Design of Doubly Reinforced Beam by Limit State Method

5.1 Introduction

After understanding the concept of singly reinforced beam sections wherein reinforcement is provided only on one side i.e. on tension side, we will now look at doubly reinforced beam sections wherein, apart from tension reinforcement, additional reinforcement is also provided in the compression zone of the beam. This increases the moment of resistance (or in other words, the moment carrying capacity) of the beam. But now the question arises:

"Why to provide compression reinforcement for enhancing the moment carrying capacity of a beam while we can always do this by increasing the effective depth (thereby overali depth) of the beam?"

The answer to above question lies in the fact that it is NOT always possible to increase the depth of the beam due to architectural considerations, head room restrictions etc. Moreover, increasing the depth of beam adds significantly to self-weight of the beam (as compared to weight of compression reinforcement).

Above all, the MOST important reason for providing doubly reinforced beams is that they provide safety against reversal of stresses in a structure due to wind, seismic forces, temperature stresses and many other reasons.

5.2 Doubly Reinforced Beam Section

Doubly reinforced beam sections are usually provided where there are restrictions in the beam depth due to architectural considerations or other restrictions like availability of head room etc. and where singly reinforced beam sections are not adequate/strong enough to carry the design moment coming over the beam.

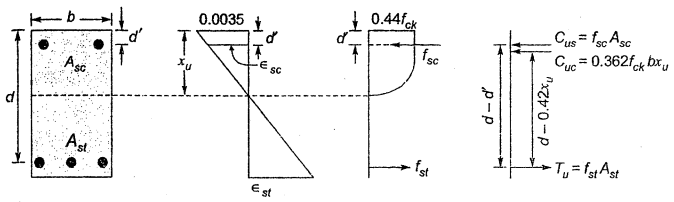
Doubly reinforced beam sections are also used where there is a possibility of occasional reversal of stresses due to lateral forces like wind or earthquake.

Provision of compression reinforcement is advantageous because it reduces the long term deflections due to shrinkage. All compression reinforcement is enclosed by ties to prevent their possible bucking under compression.

5.3 Hanger Bars v/s Compression Reinforcement

Hanger bars are of nominal diameter provided in the compression zone of a beam for holding the shear stirrups. Hanger bars are generally not considered as compression reinforcement. But in case, the area of hanger bars is significant (greater than 0.2%), then hanger bars are treated as compression reinforcement.

5.4 Analysis of Doubly Reinforced Rectangular Beam Sections



(a) Beam section (b) Strains (c) Stresses (d) Resultant force

Fig. 5.1 *Stress Strain distribution in doubly reinforced rectangular beam*

From the above figure, it can be seen that the stress strain distribution in doubly reinforced beams are similar to those of singly reinforced beams. The only difference in the present case is that there is additional stress (f_{sc}) in compression steel (A_{sc}) which also needs to be considered.

Now, the important thing which is to be noted that the stress in compression steel (f_{sc}) may or may not reach the design yield strength of steel ($0.87f_y$). The stress in compression steel (f_{sc}) depends on the corresponding compression strain (ϵ_{sc}).

From the above strain diagram

$$\frac{0.0035}{x_u} = \frac{\epsilon_{sc}}{x_u - d'}$$

$$\Rightarrow \quad \epsilon_{sc} = 0.0035 \times \left(1 - \frac{d'}{x_u}\right)$$

where, d' = Distance of centroid of compression steel from the extreme compression fiber of concrete

In practical situations, d' is expressed in terms of the ratio (d'/d) which generally varies from 0.05 to 0.2.

Remember: In most cases, when Fe 250 is provided as compression reinforcement, then it will attain its design yield point ($0.87f_y$) but in case of Fe 415 and Fe 500 steel, it is not possible.

From static equilibrium,

$$C_{uc} + C_{us} = T_u$$

where, C_{uc} and C_{us} denote the compressive force in concrete and compression steel respectively.

Now, $C_{uc} = 0.362f_{ck}bx_u$

and $C_{us} = (f_{sc} - 0.45f_{ck})A_{sc}$

and $T_u = f_{st}A_{st}$

where, $f_{st} = 0.87f_y$ if $x_u \le x_{u\,lim}$

Thus, $0.362f_{ck}bx_u + (f_{sc} - 0.45f_{ck})A_{sc} = f_{st}A_{st}$

$$\Rightarrow \quad x_u = \frac{f_{st}A_{st} - (f_{sc} - 0.45f_{ck})A_{sc}}{0.362f_{ck}b}$$

The above equation can be solved **explicitly** for x_u if $f_{st} = f_{sc} = 0.87f_y$.

For all other cases when f_{st} and f_{sc} are NOT equal to $0.87f_y$, the above equation has to be solved **implicitly.** Since in that case f_{st} and f_{sc} will in fact depend on x_u only (which is unknown and yet to be determined). This involves a trial and error procedure.

After having the value of x_u, the moment of resistance M_R can be determined as:

$$M_R = C_{uc}(d - 0.42x_u) + C_{us}(d' - d)$$

5.5 Limiting Moment of Resistance

The limiting moment of resistance can be obtained *when* x_u reaches $x_{u\,lim}$ and in that case, limiting moment of resistance of doubly reinforced beam section is given by:

$$M_{u\,lim} = 0.362f_{ck}bx_{u\,lim}(d - 0.42x_{u\,lim}) + (f_{sc} - 0.45f_{ck})A_{sc}(d - d')$$

Table 5.1: *Values of f_{sc}, when $x_u = x_{u\,lim}$ for different d'/d ratios*

Grade of steel	d'/d			
	0.05	0.1	0.15	0.2
Fe 250	217.5	217.5	217.5	217.5
Fe 415	355.1	351.9	342.4	329.2
Fe 500	423.9	411.3	395.1	370.3

Here the value of f_{sc} depends on the strain in compression steel (ϵ_{sc}). When $x_u = x_{u\,lim}$, f_{sc} can be calculated by interpolation from the above table for different $\frac{d'}{d}$ ratios.

A close look at the above values indicates that 217.5 N/mm^2 (=0.87 × 250 N/mm^2) is the design yield stress of Fe 250 which confines "Rememer" tip of art. 5.4.

5.6 Balanced Doubly Reinforced Sections

Due to IS 456: 2000 recommendation and also due to obvious reasons, over reinforced sections are undesirable in structural members. Thus the depth of neutral axis cannot exceed the limiting depth of neutral axis ($x_{u\,lim}$) for different grades of steel.

For singly reinforced sections which are obviously under reinforced, $x_u \le x_{u\,lim}$ and $p_t \le p_{t\,lim}$. In this case, if we further increase the compression resistance by introducing compression steel (doubly reinforced), then x_u will decrease further and section remains under reinforced.

But if the beam section is doubly reinforced with $p_t > p_{t\,lim}$ and $p_c > 0$, then the condition to avoid over reinforced sections i.e. $x_u \le x_{u\,lim}$ can be met by limiting $(p_t - p_{t\,lim})$ to a value which is in congruence to the value of p_c (percentage compression reinforcement).

Thus, $p_t = p_{t\,lim} + (p_t - p_{t\,lim})$

Here p_{tlim} corresponds to percentage tension steel for balanced singly reinforced sections and the corresponding tensile force in reinforcement steel is balanced completely by the ultimate compressive force in concrete i.e. $0.362f_{ck}bx_u$.

The remaining steel i.e. $(p_t - p_{t\,lim})$ and the associated tensile force is balanced by additional compressive force in compression steel C_{us}.

Let p_c^* = Percentage compression steel corresponding to balanced doubly reinforced section.

Thus, tensile force due to additional $(p_t - p_{t\,lim})$ steel = Compressive force due to additional compression steel

$$0.87f_y\frac{(p_t - p_{t\,lim})bd}{100} = (f_{sc} - 0.45f_{ck})\frac{p_c^*}{100}bd$$

$$p_c^* = \frac{0.87f_y}{(f_{sc} - 0.45f_{ck})}(p_t - p_{t\,lim})$$

The above expression gives the percentage compression reinforcement corresponding to balanced doubly reinforced section.

Remember: Thus p_c provided in beams MUST NOT be less than p_c^*. It implies that when $p_c > p_c^*$ then $x_u \le x_{u\,lim}$ and the section is under reinforced.

If however $p_c < p_c^*$ then section becomes over reinforced.

The moment of resistance of balanced doubly reinforced beam section is given by:

$$M_{u\,lim\text{-}DR} = 0.87f_yA_{st}(d - 0.42x_{u\,lim}) + (f_{sc} - 0.45f_{ck})A_{sc} \times (d - d')$$

$$\Rightarrow \quad M_{u\,lim\text{-}DR} = 0.87f_y\left[\frac{p_{t\,lim}}{100}bd(d - 0.42x_{u\,lim}) + \frac{(p_t - p_{t\,lim})bd}{100}(d - d')\right]$$

5.7 Design of Doubly Reinforced Rectangular Beam Section

Due to restrictions/limitations in the beam depth, the dimensions of the beam section are more or less already fixed/pre-determined and their design involves only the determination of tension reinforcement (A_{st}) and compression reinforcement (A_{sc}).

It is always to be kept in the mind that the depth of neutral axis (x_u) cannot exceed the limiting depth of neutral axis ($x_{u\,lim}$). This is achieved by resolving the design moment (M_u) into $M_{u\,lim}$ and ΔM_u.

Thus, $$M_u = M_{u\,lim\text{-}DR} = M_{u\,lim\text{-}SR} + \Delta M_u$$

Here, $M_{u\,lim\text{-}SR}$ is the limiting moment capacity of the singly reinforced beam section and ΔM_u is the additional moment which is required to be resisted by the compression steel (A_{sc}) and the corresponding additional tension steel ΔA_{st} to be provided in the tension zone.

The contribution of concrete in compression is entirely accounted for in $M_{u\,lim\text{-}SR}$ and thus it does not contribute to ΔM_u.

Alternatively, it can be expressed as,

$$p_t = p_{t\,lim} + \Delta p_t$$

$$\Rightarrow \quad A_{st} = A_{st\,lim} + \Delta A_{st}$$

Here $A_{st\,lim}$ is the tension steel corresponding to $M_{u\,lim\text{-}SR}$ and ΔA_{st} is corresponding to ΔM_u

Here ΔM_u is obtained from the couple formed by the compressive force in compression steel $(f_{sc} - 0.45f_{ck})A_{sc}$ and an equal (and opposite) tensile force of additional tensile steel i.e., $0.87f_y\Delta A_{st}$.

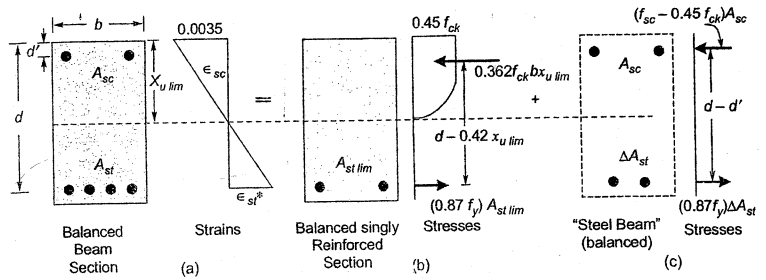


Fig. 5.2 *Balanced doubly reinforced beam section design*

The lever arm of this couple is $(d - d')$. The stress f_{sc} in the compression steel at ultimate limit state obviously depends on the strain in compression steel (ε_{sc}) at ultimate limit state.

Now, $$\Delta M_u = M_u - M_{u\,lim}$$

$$\Rightarrow \quad 0.87f_y\Delta A_{st}(d - d') = M_u - M_{u\,lim}$$

$$\Rightarrow \quad \Delta A_{st} = \frac{M_u - M_{u\,lim}}{0.87f_y(d - d')}$$

$$\Rightarrow \quad \frac{\Delta p_t}{100} = \frac{R_u - R_{u\,lim}}{0.87f_y\left(1 - \frac{d'}{d}\right)} \qquad \text{Where } R_u = \frac{M_u}{bd^2} \text{ and } R_{u\,lim} = \frac{M_{u\,lim}}{bd^2}$$

Thus, $$\Delta p_t = \frac{100(R_u - R_{u\,lim})}{0.87f_y\left(1 - \frac{d'}{d}\right)}$$

Also, $$A_{sc} = \frac{0.87f_y\Delta A_{st}}{f_{sc} - 0.447f_{ck}}$$

$$\Rightarrow \quad p_c^* = \frac{0.87f_y(p_t - p_{t\,lim})}{f_{sc} - 0.447f_{ck}} = \text{Percentage compression reinforcement.}$$

Table 5.2 *: Values of stress (in N/mm²) in compression steel (f_{sc}) at various strain levels*

Fe415		Fe500		Stress Level
STRAIN	STRESS (N/mm²)	STRAIN	STRESS (N/mm²)	(N/mm²)
0.00144	288.7	0.00174	347.8	$0.8f_{yd}$
0.00163	306.7	0.00195	369.6	$0.85f_{yd}$
0.00192	324.8	0.00226	391.3	$0.9f_{yd}$
0.00241	342.8	0.00277	413	$0.95f_{yd}$
0.00276	351.8	0.00312	423.9	$0.975f_{yd}$
0.00380	360.9(= 0.87 × 415)	0.00417	434.8(= 0.87 × 500)	$1.0f_{yd}$

f_{yd} = Design yield stress of steel = $0.87f_y$.

If value of f_{sc} is not given then take f_{sc} = 350 N/mm² for Fe415 and f_{sc} = 420 N/mm² for Fe500.

5.8 Design Steps for a Given Factored Moment (M_u)

1. **Determine A_{st}:** For a given rectangular beam section (i.e. given b, d, f_{ck}, f_y), determine the limiting moment of resistance of the singly reinforced rectangular section (which is $0.148f_{ck}bd^2$, $0.138f_{ck}bd^2$ and $0.133f_{ck}bd^2$ for Fe250, Fe415 and Fe500 respectively). If $M_{u\,lim}$ is greater than or equal to the factored moment M_u then section is designed as singly reinforced balanced section. If $M_{u\,lim} < M_u$ then section is designed as doubly reinforced section. Assume a suitable value of d′ and determine ΔA_{st} as derived in above steps. The total $A_{st\,reqd.}$ can be obtained as $(A_{st\,lim} + \Delta A_{st})$.
2. **Determine A_{sc}:** Using the value of ΔA_{st} actually provided, determine the value of $A_{sc\,reqd}$ from the expression of A_{sc} as derived above. The value of stress in compression steel (f_{sc}) is obtained from the value of strain in compression steel (ε_{sc}). The compression bars should be so selected that the area of compression steel provided is as close (but in any case must not be less than) to the compression steel required as possible.

5.9 Deflection Control in Doubly Reinforced Beams

For doubly reinforced beams, deflection check is normally not required because the limiting (l/d) ratio is usually satisfied due to the modification factor (k_c) for compression steel for deflection control. However, when beam is very shallow, check for deflection becomes necessary.

Example 5.1 Calculate the moment of resistance of doubly reinforced beam section of size 420 × 750 mm. Reinforcement provided on the tension side is 6 nos. 25 mm dia. bars and on the compression side is 5 nos. 20 mm dia. bars. Use M 25 concrete and Fe 500 steel.

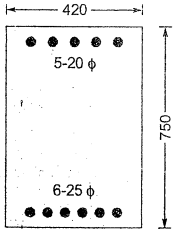
Solution:

Width of the beam (b) = 420 mm

Overall depth of the beam (D) = 750 mm

Assuming an effective cover of 50 mm,

Effective depth of the beam (d) = 750 – 50 mm = 700 mm



Area of tension reinforcement $(A_{st}) = 6\times\frac{\pi}{4}\times 25^2 = 2945\ mm^2$

Area of compression reinforcement $(A_{sc}) = 5\times\frac{\pi}{4}\times 20^2$

$= 1570.8\ mm^2 \approx 1571\ mm^2$

Limiting depth of neutral axis

$x_{u\,lim} = 0.46\,d$

$= 0.46 \times 700$ mm $= 322$ mm

Actual depth of neutral axis

$C = T$

$0.36f_{ck}bx_u + (f_{sc} - 0.45f_{ck})A_{sc} = 0.87f_yA_{st}$

$\Rightarrow 0.36 \times 25 \times 420 \times x_u + (f_{sc} - 0.45 \times 25) \times 1571 = 0.87 \times 500 \times 2945$

$\Rightarrow 3780x_u + 1571(f_{sc}) = 1298749$ ...(i)

Let us solve by trial and error

Assume $f_{sc} = 420\ N/mm^2$

$\therefore x_u = 169$ mm ... I[st] trial

$$\epsilon_{sc} = \frac{x_u - d_c}{x_u}\times 0.035 = \frac{169-50}{169}\times 0.0035$$

$$\epsilon_{sc} = 0.00246$$

$$f_{sc} = 391.3 + \frac{(413-391.3)}{(0.00277-0.00226)}\times(0.00246-0.00226) = 399.8\ N/mm^2$$

In eq. (i), put the value of f_{sc}

$\therefore x_u = 177.34$ mm ...II[nd] trial

$$\epsilon_{sc} = \frac{177.34-56}{177.34}\times 0.0035 = 0.00251$$

$$\therefore f_{sc} = 391.3 + \frac{(413-391.3)}{(0.00277-0.00226)}\times(0.00251-0.00226)$$

$f_{sc} = 402\ N/mm^2$

Put this value of f_{sc} in eq. (i)

$\therefore x_u = 176.5$ mm $< x_{u\,lim}$ (= 322 mm)

Take $x_u = 177$ mm and $f_{sc} = 402\ N/mm^2$

Moment of resistance

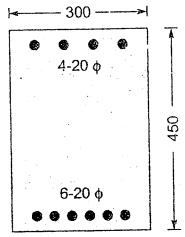
$M_R = 0.36\,f_{ck}bx_u(d - 0.42x_u) + (f_{sc} - 0.45f_{ck})\,A_{sc}\,(d - d_c)$

$= 0.36 \times 25 \times 420 \times 177(700 - 0.42 \times 177) + (402 - 0.45 \times 25)(700 - 50)$

$= 817.6$ kNm

Example 5.2 Calculate the moment of resistance of doubly reinforced beam section of size 300 × 450 mm. It is reinforced with 6-20ϕ bars in tension side and 4-20ϕ bars in compression side. Use M 20 concrete and Fe 250 steel.

Solution:



Width of the beam, $b = 300$ mm

Overall depth of the beam, $D = 450$ mm

$f_{ck} = 20\ N/mm^2$

$f_y = 250\ N/mm^2$

Let effective cover = 50 mm

∴ Effective depth of the beam, $d = 450 - 50$ mm $= 400$ mm

Tension reinforcement, $A_{st} = 6\times\frac{\pi}{4}\times 20^2 = 1885\ mm^2$

Compression reinforcement, $A_{sc} = 4\times\frac{\pi}{4}\times 20^2 = 1256\ mm^2$

$C = T$

$\Rightarrow 0.36f_{ck}bx_u + (f_{sc} - f_{cc})A_{sc} = f_{st}A_{st}$

Let $f_{st} = f_{sc} = 0.87f_y$ and neglecting 'f_{cc}', we have

$$x_u = \frac{0.87f_y(A_{st}-A_{sc})}{0.36f_{ck}b} = \frac{0.87(250)(1885-1256)}{0.36(20)(300)}$$

$$= 63.34 \text{ mm}$$

Limiting depth of neutral axis for Fe 250 $x_{u\,lim} = 0.53d = 0.53 \times 400 = 212$ mm $> x_u$

$\because$ $x_u < x_{u\,lim}$

So secion is under reinforced and assumption of $f_{sc} = f_{st} = 0.87f_y$ is correct.

Moment of resistance, $MOR = 0.36f_{ck}\,bx_u(d - 0.42\,x_u) + f_{sc}A_{sc}\,(d-d')$

$= 0.36 \times 20 \times 300 \times 63.34(400 - 0.42 \times 63.34)$

$+0.87 \times 250 \times 1256\,(400-50)$ Nmm

$= 146.7$ kNm

$\therefore$ MOR of doubly reinforced beam section is 146.7 kNm $\approx$ 146 kNm.

Example 5.3 Design a reinforced concrete beam of span 8 m which is being subjected to a live load of 30 kN/m. Overall depth of the beam is limited to 650 mm. Use M20 concrete and Fe 415 steel.

Solution:

Step-1: Loads

Let overall depth of beam, $D = 650$ mm

and width of beam, $B = 300$ mm

Self weight of beam, $= 25(0.3)(0.65) = 4.875$ kN/m

Live load $= 30$ kN/m (Given)

$\therefore$ Total load $= 34.875$ kN/m

$\therefore$ Factored load $w = 1.5 \times 34.875$

$= 52.3125$ kN/m

Step-2: Calculate design moment and design shear

Design moment $M_u = \frac{wl^2}{8} = 52.3125 \times \frac{(8)^2}{8} = 418.5$ kNm

Design shear force $V_u = \frac{wl}{2} = 52.3125 \times \frac{8}{2} = 209.25$ kN

Step-3: Calculate limiting MOR for singly reinforced beam section

For Fe 415, limiting moment of resistance

$M_{u\,lim} = 0.138\,f_{ck}bd^2$

Let effective cover $= 50$ mm

Effective depth of beam $d = 650 - 50$ mm $= 600$ mm

$\therefore$ $M_{u\,lim} = 0.138\,(20)\,(300)\,(600)^2$

$= 298.08$ kNm $< M_u\,(= 418.5$ kNm)

Thus doubly reinforced beam section is required.

Step-4: Calculate amount of compression reinforcement required.

$\Delta M_u = M_u - M_{u\,lim}$

$= 418.5 - 298.08$ kNm $= 120.42$ kNm

$\therefore$ $f_{sc} = \frac{0.0035(x_{ulim}-d')}{x_{ulim}} \times E_s$

$= \frac{0.0035(0.48\times600-50)}{0.48\times600} \times 2\times10^5$ N/mm^2

$= 578.47$ N/mm^2

But $f_{sc} \not> 0.87f_y = 0.87(415) = 361.05$ N/mm^2

$\therefore$ $A_{sc} = \frac{M_u - M_{u\,lim}}{f_{sc}(d-d')} = \frac{120.42\times10^6}{361.05(600-50)} = 606.413$ mm^2

Provide 2-20ϕ bars on compression side so that

$A_{sc\,provided} = 628.3$ mm$^2 > 606.413$ mm^2 (OK)

Step-5: Calculate amount of tension reinforcement required

A_{st2} = Area of tension steel to balance A_{sc}

$= \frac{f_{sc}A_{sc}}{0.87f_y} = 606.413$ mm^2

$A_{st1} = A_{st\,lim} = \frac{0.362f_{ck}bx_{ulim}}{0.87f_y}$

$= \frac{0.362(20)(300)(0.48\times600)}{0.87(415)} = 1732.5$ mm^2

Alternatively

$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\frac{x_{ulim}}{d} = 41.61\left(\frac{20}{415}\right)(0.48) = 0.9625\%$

$\therefore$ $A_{st\,lim} = \frac{0.9625}{100} \times 300 \times 600 = 1732.5$ mm^2 (same as above)

$\therefore$ $A_{st} = A_{st1} + A_{st2}$

$= 1732.5 + 606.413$ mm^2

$= 2338.913$ mm$^2 \approx 2339$ mm^2

Provide 5-25ϕ bars so that $A_{st\,provided} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.4$ mm$^2 > 2339$ mm^2 (OK)

$p_{t\,provided} = 1.3636\%$

Step-6: Design of shear reinforcement (more details are covered in chapter 6)

Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{209.25\times1000}{300\times600} = 1.1625$ N/mm^2

For M 20 concrete and 1.3636% p_t, design shear strength of concrete as per table 19 of IS: 456-2000 $(\tau_c) = 0.7$ N/mm$^2 < \tau_v\,(= 1.1625$ N/mm$^2)$

$\therefore$ Shear reinforcement is required.

$V_{us} = (\tau_v - \tau_c)\,bd$

$$= (1.1625 - 0.7)\,300 \times 600\ \text{N} = 83.25\ \text{kN}$$

Using 2-legged 8 mm dia. stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\ \text{mm}^2$$

$$\therefore \quad S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87(415)100.53(600)}{83.25 \times 1000} = 261.6\ \text{mm c/c}$$

Maximum spacing of stirrups $\not> \begin{cases} 0.75d = 0.75(600)\text{mm} = 450\ \text{mm c/c} \\ 300\ \text{mm} \end{cases}$ (whichever is less)

∴ Provided 2-legged 8 mm diameter stirrups @ 250 c/c near the supports and spacing can be increased gradually towards the mid span of beam.

Step-7: Deflection control

$$\left(\frac{l}{d}\right)_{actual} = \frac{8000}{600} = 13.33$$

$$\left(\frac{l}{d}\right)_{maximum} = \left(\frac{l}{d}\right)_{basic} k_t k_c$$

$$A_{sc} = 628.3\ \text{mm}^2$$

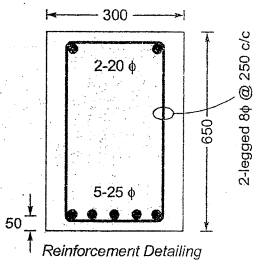
$$\therefore \quad p_c = \frac{628.3}{300 \times 600} \times 100 = 0.35\%$$

$$\therefore \quad k_t = 0.9 \quad \text{(fig. 4 of IS : 456-2000)}$$

$$k_c = 1.1 \quad \text{(fig. 5 of IS : 456-2000)}$$

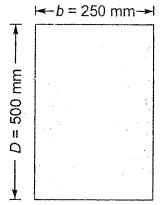
$$\therefore \quad \left(\frac{l}{d}\right)_{maximum} = 20 \times 0.9 \times 1.1$$

$$= 19.8 > 13.33 \quad \text{(OK)}$$



Reinforcement Detailing

Example 5.4 Design a RC beam of effective span 6 m to carry a superimposed load of 25 kN/m. The beam size is restricted to 250 × 500 mm. Use M20 and Fe415.



Solution:

Span (l) = 6 m

Effective cover = 40 mm

f_{ck} = 20 N/mm² (M20 concrete)

f_y = 415 N/mm² (Fe415 steel)

Self weight of beam = 0.25 × 0.5 × 25 = 3.125 kN/m

Effective depth (d) = 500 − 40 = 460 mm

Superimposed load = 25 kN/m

∴ Total load = 3.125 + 25 = 28.125 kN/m

∴ Factored load = 1.5 × 28.125 = 42.1875 kN/m

∴ Factored moment = Design moment = $M_u = 42.1875 \times \frac{6^2}{8} = 189.84$ kNm

$$M_{u\,lim} = 0.138\, f_{ck}\, bd^2 = 0.138\,(20)\,250\,(460)^2 = 146\ \text{kNm}$$

Thus, $M_u > M_{u\,lim}$

∴ Doubly reinforced beam is required since section can not be increased i.e. b and D are fixed.

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61\left(\frac{20}{415}0.479\right) = 0.961\%$$

$$\therefore \quad A_{st\,lim} = \frac{0.961}{100} \times 250 \times 460 = 1105.15\ \text{mm}^2$$

Using 20 mm diameter bars in compression, effective cover to compression reinforcement (d') = 30 mm clear cover + 8 mm dia shear stirrups + $\frac{20}{2}$

= 30 + 8 + 10 mm = 48 mm

$$\therefore \quad \Delta A_{st\,reqd.} = \frac{M_u - M_{u\,lim}}{0.87 f_y (d - d')} = \frac{(189.84 - 146)10^6}{0.87(415)}(460 - 48) = 294.72\ \text{mm}^2$$

∴ No. of 20 mm of bars required = $\frac{294.75}{\frac{\pi}{2} \times 20^2} = 0.94 \simeq 1\,(\text{say})$

∴ Total tension steel (A_{st}) = $A_{st\,lim} + \Delta A_{st\,reqd.} = 1105.15 + 294.72 = 1399.87\ \text{mm}^2$

∴ No. of 20 mm of bars required = $\frac{1399.87}{\frac{\pi}{4} \times 20^2} = 4.5 = 5$ Nos. (say)

Determining A_{sc}

Let $x = x_{u\,lim}$ (for balanced doubly reinforced section)

$$\frac{d'}{d} = \frac{48}{460} = 0.104 \simeq 0.1$$

∴ Stress in compression steel (f_{sc}) = 351.9 N/mm² ≠ 0.87 f_y = (0.87 × 415 = 361.05 N/mm²)

$$\therefore \quad A_{sc\,reqd.} = \frac{0.87 f_y \Delta A_{st}}{f_{sc} - 0.447 f_{ck}} = \frac{0.87(415)\frac{\pi}{4} \times 20^2 \times 1}{351.9 - 0.447 \times 20} = 330.73\ \text{mm}^2$$

∴ No. of 20 mm dia. bars required = $\frac{330.73}{\frac{\pi}{4} \times 20^2} = 1.05 \simeq 2$ Nos. (say)

Check

Actual effective depth (d) = 460 mm (as computed above)

Effective cover to compression reinforcement (d') = $30 + 8 + \frac{20}{2} = 48$ mm

$$A_{st\ provided} = 5-20\phi \text{ bars} = 5\times\frac{\pi}{4}\times 20^2 = 1570.8\ mm^2$$

$$\therefore\quad p_{t\ provided} = \frac{1570.8}{250\times 460}\times 100 = 1.37\%$$

$$A_{sc\ provided} = 2-20\phi \text{ bars} = 2\times\frac{\pi}{4}\times 20^2 = 628.3\ mm^2$$

Minimum % of compression reinforcement required (p_c*)

$$= \frac{0.87 f_y\left(p_t - p_{t\,lim}\right)}{\left(f_{sc} - 0.447 f_{cu}\right)} = \frac{0.87(415)(1.37-0.961)}{(351.9-0.447\times 20)} = 0.431\%$$

$$p_{c\ provided} = \frac{328.3}{250\times 460}\times 100 = 0.546\% > p_c\,^*(=0.431\%) \qquad \text{(OK)}$$

Deflection check

$p_t = 1.37\%$

$$\therefore\quad f_{st} = 0.58 f_y \frac{p_{t\,reqd.}}{p_{t\,provided}} = \frac{0.58(415)\dfrac{1399.87}{250\times 460}\times 100}{1.37}$$

$= 213.87\ N/m^2$

$k_t = 1.009$ (fig. 4 of IS 465: 2000)

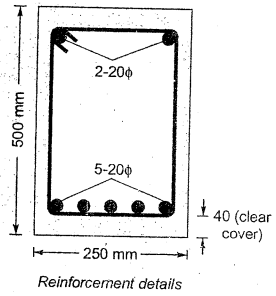
$p_c = 0.546\%$

$\therefore\quad k_c = 1.15104$ (fig. 5 of IS 456: 2000)

$$\therefore\quad \left(\frac{l}{d}\right)_{max} = \left(\frac{l}{d}\right)_{basic} k_t . k_c$$

$$= 20\times 1.009\times 1.15104 = 23.23$$

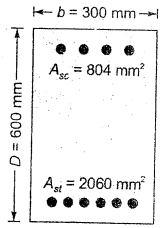
$$\left(\frac{l}{d}\right)_{provided} = \frac{6000}{460} = 13.04 < 23.23 \text{ (OK)}$$



Reinforcement details

Example 5.5 Find the MOR of a beam section 300 x 600 mm (overall depth) reinforcement with 804 sq. mm compression steel and 2060 sq. mm tension steel. Use M20 and Fe415. Take effective cover as 50 mm.

Solution:



Effective cover = 50 mm

$\therefore$ Effective depth (d) = 600 – 50 = 550 mm

$f_{ck} = 20$ MPa

$f_y = 415$ MPa

$d' = 50$ mm

$x_{u\,lim} = 0.479d = 0.479\,(550) = 263.45$ mm

Let $f_{st} = f_{sc} = 0.87\ f_y$

$\therefore\quad C_{us} + C_{uc} = T_u$

But $C_{uc} = 0.362\, f_{ck}\, b x_u = 0.362\,(20)\,(300)\, x_u = 2172\, x_u$

$C_{us} = (f_{sc} - 0.447\, f_{cu})\, A_{sc} = (0.87\times 415 - 0.447\times 20)\,804$

$= 283096.44$ N

$\therefore\quad C_{us} + C_{uc} = T_u$

$\Rightarrow\quad 283096.44 + 2172\, x_u = 743763$

$\Rightarrow\quad x_u = 212.09\ mm < x_{u\,lim}\ (= 263.45\ mm)$

$\therefore$ Assumption of $f_{st} = 0.87\ f_y$ is correct.

$$\text{Strain in compression steel } (\epsilon_{sc}) = 0.0035\left(1-\frac{d'}{x_u}\right) = 0.0035\left(1-\frac{50}{212.09} = 0.0026749\right)$$

$$\text{Yield strain } (\epsilon_y) = \frac{0.87 f_y}{\epsilon_s} + 0.002 = \frac{0.87\times 415}{200000} + 0.002 = 0.00380525$$

Thus, $\epsilon_y > \epsilon_{sc}$

$\Rightarrow$ Compression steel has not yielded.

$\Rightarrow\quad f_{sc} \neq 0.87\ f_y$

$\therefore\quad x_u \neq 212.09$ mm

I[st] Iteration:

Let $\epsilon_{sc} = 0.0026749$

$\therefore\quad f_{sc} = 348.8\ N/mm^2$

$\therefore\quad C_{us} = (f_{sc} - 0.447\, f_{ck})\, A_{sc}$

$= (348.8 - 0.447\times 20)\,804$

$= 273247.44$ N

$$\therefore\quad x_u = \frac{743763 - 273247.44}{2172} = 216.63\ mm$$

$$\therefore\quad \epsilon_{sc} = 0.0035\left(1-\frac{50}{216.63}\right) = 0.002692$$

II[nd] Iteration:

Let $\epsilon_{sc} = \dfrac{0.0026749 + 0.002692}{2} = 0.00268345$

$\therefore\quad f_{sc} = 349.032\ N/mm^2$

$\therefore\quad C_{us} = (349.032 - 0.447\times 20)\,804$

$= 273433.968$ N

$$\therefore\quad x_u = \frac{743763 - 273433.968}{2172} = 216.54\ mm$$

$$\therefore\quad \epsilon_{sc} = 0.0035\left(1-\frac{50}{216.54}\right) = 0.0026918 \simeq \epsilon_{sc} \text{ assumed}$$

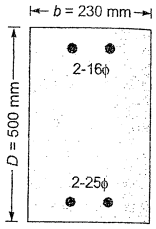
$\therefore\quad x_u = 216.54$ mm

$$\therefore \quad MOR = C_{uc}(d - 0.42x_u) + C_{us}(d - d')$$
$$= 2172 \times 216.54 (550 - 0.42 \times 216.54) + 273433.968 (550 - 50)$$
$$= 352.62 \text{ kNm}$$

Example 5.6 A RC beam of size 230 x 500 mm overall depth is having effective of 40 mm both in tension and compression side. 2 nos. 16 mm diameter bars are provided in compression zone and 2 nos. 25 mm diameter bars are provided in tension zone of the beam. Find the moment of resistance of this beam using M20 concrete and Fe415 steel.

Table: Stress in compression

Fe415	d'/d			
	0.05	0.1	0.15	0.2
f_{sc} (MPa)	355	353	342	329



Solution:

Effective cover = 40 mm (for both tension and compression)

M20 concrete and Fe 415 steel

Effective depth (d) = 500 – 40 = 460 mm

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$A_{st} = 2 \times \frac{\pi}{4} \times 25^2 = 981.75 \text{ mm}^2$$

$$x_{u\,lim} = 0.479\,d = 0.479\,(460) = 220.34 \text{ mm}$$

Let $f_{st} = f_{sc} = 0.87 f_y$

$\therefore$ $C_{uc} = 0.362 f_{ck} b x_u = 0.362\,(20)\,230\,(x_u) = 1665.2 x_u$

$C_{us} = (0.87 f_y - 0.447 f_{ck}) A_{sc}$

$= (0.67 \times 415 - 0.447 \times 20)\,402.12 = 141590.47$ N

$T_u = 0.87 f_y A_{st} = 0.87\,(415)\,981.75 = 354460.84$ N

$C_{uc} + C_{us} = T_u$

$\Rightarrow$ $1665.2 x_u + 141590.47 = 354460.84$

$\therefore$ $x_u = 127.83 \text{ mm} < x_{u\,lim} (= 220.34 \text{ mm})$

$\therefore$ Assumption of $f_{st} = 0.87 f_y$ is correct.

Strain in compression steel $(\epsilon_{sc}) = 0.0035\left(1 - \frac{40}{127.83}\right) = 0.002404795$

Yield strain $(\epsilon_y) = \frac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.00380525 > \epsilon_s$

$\therefore$ Compression steel has not yielded.

$\Rightarrow$ $f_{sc} \neq 0.87 f_y$

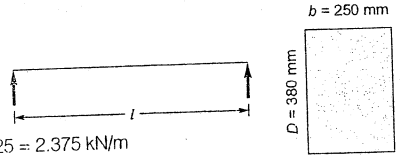
Thus, $x_u \neq 127.83$ mm

$$\frac{d'}{d} = \frac{40}{460} = 0.0869$$

$\therefore$ For $x_u = x_{u\,lim}$, $f_{sc} = 353.524$ N/mm^2

$$\therefore \quad M_{u\,lim} = 0.362 f_{ck} b x_{u\,lim} (d - 0.42 x_{u\,lim}) + (f_{sc} - 0.447 f_{ck}) A_{sc} (d - d')$$
$$= 0.362\,(20)\,230\,(220.34)\,(460 - 0.42 \times 220.34)$$
$$+ (353.524 - 0.447 \times 20)\,402.12\,(460 - 40)$$
$$= 193 \text{ kNm}$$

Example 5.7 A simply supported beam of 4.5 m effective span is carrying an imposed load of 25 kN/m. The size of the beam is restricted to 250 x 380 mm. Design the beam using M20 and Fe415.



Solution:

$l = 4.5$ m

$LL = 25$ kNm

M20 concrete and Fe415 steel

Self weight of beam = $0.38 \times 0.25 \times 25 = 2.375$ kN/m

$\therefore$ Total load = $2.375 + 25 = 27.375$ kN/m

$\therefore$ Factored load (w_u) = $1.5 \times 27.375 = 41.0625$ kN/m

Assume an effective cover of 50 mm both at the top and bottom

$$\frac{d'}{d} = \frac{50}{330} = 0.15$$

$\therefore$ $\epsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{u\,lim}}\right) = 0.0035\left(1 - \frac{50}{0.479 \times 330}\right) = 0.002393$

$$\epsilon_y = \frac{0.87 \times 415}{200000} + 0.002 = 0.00380525 > \epsilon_{sc}$$

$\therefore$ Compression steel has not yielded.

Let $x_u = x_{u\,lim}$ and for $\frac{d'}{d} = 0.15$, $f_{sc} = 342$ N/mm^2

$\therefore$ $A_{sc\,reqd.} = \frac{0.87 f_y \Delta A_{st}}{(f_{sc} - 0.447 f_{ck})} = \frac{0.87(415)\frac{\pi}{4} \times 20^2}{(342 - 0.447 \times 20)} = 340.56 \text{ mm}^2$

$\therefore$ No. of 20 mm ϕ bars required = $\frac{340.56}{\frac{\pi}{4} \times 20^2} = 1.08 = 2$ (say)

$\therefore$ Factored moment (M_u) = $w_u \frac{l^2}{8} = 41.0625 \times \frac{4.5^2}{8} = 103.94$ kNm

Now, $M_{u\,lim} = 0.138 f_{ck} b d^2$

Let Effective cover = 50 mm

$\therefore$ $d = 380 - 50 = 330$ mm

$\therefore$ $M_{u\,lim} = 0.138\,(20)\,250\,(330)^2 = 75.141 \text{ kNm} < M_u$

$\therefore$ Doubly reinforced beam is required.

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 0.961\%$$

$\therefore$ $A_{st\,lim} = \frac{0.961}{100} \times 250 \times 330 = 792.825 \text{ mm}^2$

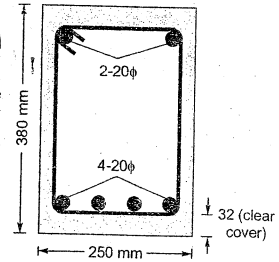
Let effective cover to compression reinforcement (d') = 50 mm

$$\therefore \quad \Delta A_{st\,reqd.} = \frac{M_u - M_{ulim}}{0.87 f_y (d-d')} = \frac{(103.94 - 75.141)10^6}{0.87(415)(330-50)} = 284.87 \text{ mm}^2$$

$$\therefore \quad \text{No. of } 20\phi \text{ bars required} = \frac{284.87}{\frac{\pi}{4}\times 20^2} = 0.9 = 1 \text{ (say)}$$

$$\therefore \quad A_{st} = A_{st\,lim} + \Delta A_{st\,reqd.} = 792.825 + 284.87 = 1077.695 \text{ mm}^2$$

$$\therefore \quad \text{No. of } 20\,\phi \text{ bars required} = \frac{1077.695}{\frac{\pi}{4}\times 20^2} = 3.43 = 4 \text{ (say)}$$



Reinforcement details

Objective Brain Teasers

Q.1 Providing doubly reinforced beam is a better recourse than to increase the depth of beam because:
(i) Self-weight of the beam does not increase much by providing doubly reinforced beam.
(ii) It takes care of occasional reversal of stresses in framed structures.
Which of the above statement(s) is/are false?
(a) (i) only (b) (ii) only
(c) Both (i) and (ii) (d) None of (i) or (ii)

Q.2 Hanger bars are considered as compression reinforcement in beams when the area of hanger bars is:
(a) Less than 0.1% (b) Greater than 0.2%
(c) Greater than 0.5% (d) Less than 0.05%

Q.3 Which of the following grade of steel generally attains its yield value when provided as compression reinforcement?
(a) Fe 250 (b) Fe 415
(c) Fe 500 (d) All of the above

Q.4 Consider the following for a rectangular beam:
p_t = Percentage tension reinforcement
p_{tlim} = Limiting percentage of tension reinforcement corresponding to balanced singly reinforced beam section
p_c = Percentage of compression reinforcement
p_c^* = Percentage of compression reinforcement corresponding to balanced doubly reinforced beam section
It is given that $p_t > p_{t\,lim}$ and $p_c > 0$ then in order to avoid over reinforced section, we must have:
(a) $p_c < p_c^*$ (b) $p_c > p_c^*$
(c) $p_c = 0$ (d) $p_c < (p_t - p_{t\,lim})$

Q.5 If it is known that in a doubly reinforced beam, the stress in compression steel (f_{sc}) and stress in tension steel (f_{st}) has attained the yield value then depth of neutral axis for this beam can be computed:
(a) Explicitly
(b) Implicitly
(c) Both (a) and (b) can be used
(d) Data insufficient

Q.6 Deflection check for doubly reinforced beams is generally not required due to the following factor:
(a) Modification factor k_t
(b) Modification factor k_c
(c) Stress in compression steel (f_{sc})
(d) All of the above

Q.7 It is essential to provide percentage compression reinforcement as close as possible (but not less than) the percentage compression reinforcement for balanced doubly reinforced section because otherwise
(a) the beam will fail in deflection
(b) the beam will fail in shear
(c) the beam will become over-reinforced
(d) All of the above

Q.8 In a framed RCC structure, beam sections are generally
(a) Singly reinforced without hanger bars
(b) Singly reinforced with hanger bars
(c) Doubly reinforced
(d) It depends on the type of loading

Q.9 A doubly reinforced concrete beam of size 300 mm × 500 mm effective depth is reinforced with 2200 mm² steel in tension zone and 628 mm² steel in compression zone. The effective cover to compression steel is 50 mm. The steel on both tension and compression side yield and concrete used is of M20 grade and steel of Fe250 grade. The depth of the neutral axis is
(a) 161 mm (b) 205 mm
(c) 185 mm (d) 215 mm

Q.10 The moment of resistance for the above beam is
(a) 298 kNm (b) 270 kNm
(c) 209 kNm (d) 238 kNm

Q.11 In a doubly reinforced RC beam, the maximum strain in concrete at the level of compression steel is
(a) $0.0035\left(1-\frac{d'}{x_u}\right)$ (b) $0.0035\left(1-\frac{x_u}{d'}\right)$
(c) 0.0035 (d) None of these

Q.12 Doubly reinforced beam is uneconomical as compared to singly reinforced beam because
(a) compressive steel is not stressed upto $0.87 f_y$
(b) more steel is required in doubly reinforced beams
(c) compression concrete is not stressed fully
(d) All of the above

Answers

1. (d)	2. (b)	3. (a)	4. (b)	5. (a)
6. (b)	7. (c)	8. (c)	9. (a)	10. (c)
11. (a)	12. (a)			

Hints:

10. (c)

$$C_{uc} = 0.36 f_{ck}\, b x_u$$
$$C_{us} = (f_{sc} - 0.45 f_{ck}) A_{sc}$$
$$T_u = 0.87 f_y A_{st} \quad (\because \text{steel yields})$$
$$\therefore \quad C_{uc} + C_{us} = T_u$$
$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - 0.45 f_{cu}) A_{sc}}{0.36 f_{ck} b}$$

Let $f_{sc} = 0.87 f_y$

$$\therefore \quad x_u = 160.9 \text{ mm}$$
$$x_{u\,lim} = 0.53\, d = 265 \text{ mm}$$

$\therefore$ $x_u < x_{u\,lim}$ and thus $f_{st} = 0.87 f_y$ is true.

Moment of resistance

$$(M_u) = C_{uc}(d - 0.42\, x_u) + C_{us}(d - d')$$
$$= 209 \text{ kNm}$$

Conventional Practice Questions

Q.1 Design a doubly reinforced beam to resist a factored moment of 375 kNm with b = 250 mm, d = 500 m and d' = 50 mm. Use M30 and Fe415. Use LSM.

Q.2 Find the MOR of a doubly reinforced beam of width 300 mm, effective depth 600 mm, d' = 75 mm. Compression reinforcement consists of 5-25 ϕ bars and tension reinforcement consists of 4-32 ϕ + 4-16 ϕ bars. Use M30 and Fe500.

Q.3 Design simply supported beam of effective span 8 m subjected to a live load of 35 kN/m. The beam dimensions are restricted to 300 × 650 mm. Use M 25 and Fe 415.

Q.4 Find the ultimate moment capacity of a doubly reinforced beam of width 350 mm, effective depth 600 mm and d' = 50 mm. 6-25 ϕ bars are provided on tension side and 4-20ϕ bars on compression side. Use M20 and Fe415.

CHAPTER 06

Design for Shear in Reinforced Concrete

6.1 Introduction

Shear is one of the major considerations in the design of reinforced structures. It is one of the **major limit state of collapse** in the *limit state design* of reinforced concrete structures. Here, we will define the nominal shear stress in beams, design shear strength of concrete and maximum permissible shear strength of concrete. We will look into the factors that affect the design shear strength of concrete and its governing mathematical expression.

We will see the critical locations for shear in beams, why we need to provide minimum shear reinforcement in concrete members, the amount of minimum shear reinforcement as per the provisions of IS 456: 2000. At last, we will design the shear reinforcement to be provided in the form of stirrups and bent up bars.

6.2 Shear in Reinforced Concrete

Any reinforced concrete member which is subjected to bending moments is usually accompanied by transverse shear, axial forces and torsion. The shear associated with varying bending moments is called as **flexural shear**.

The simplified method / conventional method of shear design of reinforced concrete members assumes that plane sections remain plane before and after the bending. In this method, the shear design is achieved by transverse shear reinforcement and on the other hand, the longitudinal reinforcement is designed for axial forces and flexure moments.

Flexural shear failure may not lead to an immediate collapse of the member/structure but it significantly reduces the flexural strength and ductility of the member/structure.

The exact analysis of flexural shear alone or in combination with torsion and axial forces is quite complex because of non-homogeneity of the material, presence of cracks, reinforcement etc. The present knowledge of shear is basically based on experimental results and simplified assumptions and not on an exact and universally accepted theory.

6.3 Shear Stress Distribution in Rectangular Homogeneous Section

Shear stress at any section at a distance 'y' from neutral axis is given by,

$$q = \frac{V}{Ib}.A\bar{y}$$

$$= \frac{V}{Ib}.\left[b.\left(\frac{D}{2}-y\right).\left\{y+\frac{1}{2}\left(\frac{D}{2}-y\right)\right\}\right]$$

$$= \frac{V}{I}.\left(\frac{D}{2}-y\right)\left(\frac{D}{4}+\frac{y}{2}\right)$$

$$= \frac{V}{2I}.\left(\frac{D}{2}-y\right)\left(\frac{D}{2}+y\right)$$

$$= \frac{V}{2I}.\left(\frac{D^2}{4}-y^2\right)$$

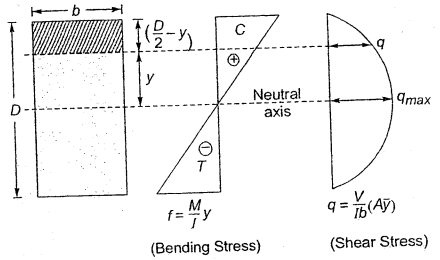


Fig. 6.1 *Bending and Shear Stress Distribution in Homogeneous Rectangular Beam*

Maximum shear stress will occur at $y = 0$ i.e. at NA

$$q_{max} = \frac{V}{2I}.\frac{D^2}{4} = \frac{VD^2}{8I}$$

6.3.1 Shear Stress Distribution in Reinforced Concrete Section

In a reinforced concrete section, the shear stress distribution is parabolic only in the compression area. (uncracked portion).

$$q = \frac{V}{Ib}.A\bar{y}$$

$$= \frac{V}{Ib}.\left\{b.(x_a-y).\left(y+\frac{x_a-y}{2}\right)\right\}$$

$$= \frac{V}{I}.(x_a-y).\left(\frac{2y+x_a-y}{2}\right)$$

$$= \frac{V}{2I}.(x_a-y).(x_a+y)$$

$$= \frac{V}{2I}.(x_a^2-y^2)$$

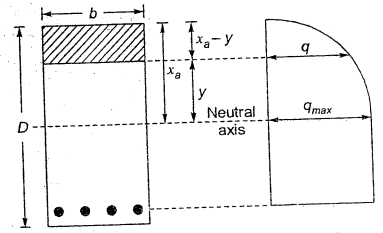


Fig. 6.2 *Shear stress distribution in rectangular reinforced concrete beam*

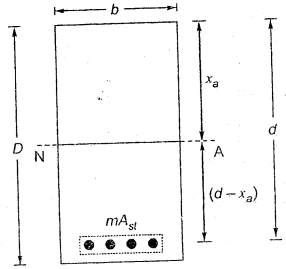
Maximum shear stress will occur at neutral axis i.e. at $y = 0$.

$$q_{max} = \frac{V}{2I}x_a^2$$

$$I = I_{eq}$$

$$I_{base} = \frac{b}{12}x_a^3 + bx_a\left(\frac{x_a}{2}\right)^2$$

$$= bx_a^3\left(\frac{1}{12}+\frac{1}{4}\right) = \frac{b}{3}x_a^3$$



6.3.2 Shear Stress at any Cross-Section

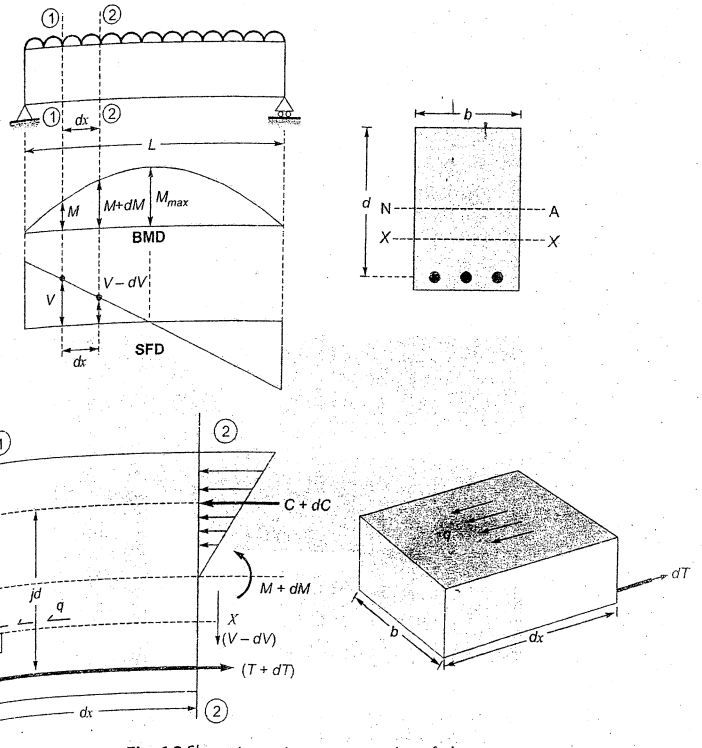


Fig. 6.3 *Shear stress at any cross-section of a beam*

Consider two sections 1-1and 2-2 at a distance 'dx' in a beam which is simply supported and is subjected to a Uniformly Distributed Load (UDL).

At section 1-1,

Moment, $M = C.jd = T.jd$ ...(i)

At section 2-2,

Moment $M + dM = (C+dC)jd = (T + dT)jd$ ...(ii)

From (i) and (ii) $dM = dT.\ jd$

$$dT = \frac{dM}{jd} \quad \text{...(iii)}$$

This unbalanced force dT is responsible for :

1. shear stress between steel and concrete (called as bond stress).
2. shear stress at any section between neutral axis and reinforcement.

Due to this 'dT' force, shear stress 'q' is developed at section X-X of the beam.

Resisting force = $q.bdx$

Now, Disturbing force = Resisting force

$$\therefore \quad dT = \frac{dM}{jd} = q.b.dx$$

Thus shear stress developed at section X-X is,

$$q = \frac{dM}{jdbdx} = \frac{V}{bjd} \Rightarrow q = \frac{V}{bjd}$$

This is the horizontal shear stress at section X-X.

Remember: This shear stress 'q' will be same at any cross-section between neutral axis and reinforcement because the disturbing force dT is same for all cross-section and area of concrete in tension zone is neglected.

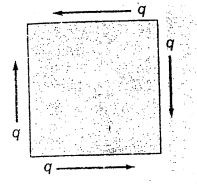


Fig. 6.4 *Complementary Shear Stresses*

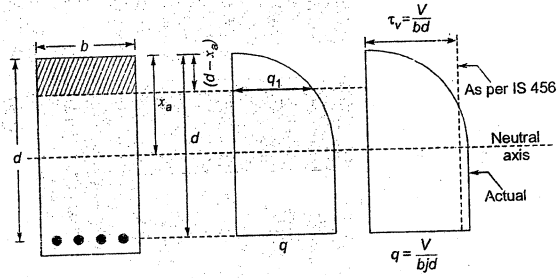


Fig. 6.5 *Shear stress distribution as per IS : 456 in rectangular reinforced concrete beam*

Now analyze the vertical shear stress for the beam cross section.

V = width x (area of shear stress diagram)

Assuming shear stress distribution as shown in Fig. 6.5.

$$V = b\left\{\frac{2}{3}x_a q + q(d - x_a)\right\}$$

$$V = bq\left\{\frac{2}{3}x_a + d - x_a\right\}$$

$$V = bq\left\{d - \frac{x_a}{3}\right\} = bqjd$$

$$q = \frac{V}{bjd}$$

Also as per IS code, nominal shear stress is given as,

$$\tau_v = \frac{V}{bd}$$

6.4 Combined Effect of Bending Stress and Shear Stress in a Beam

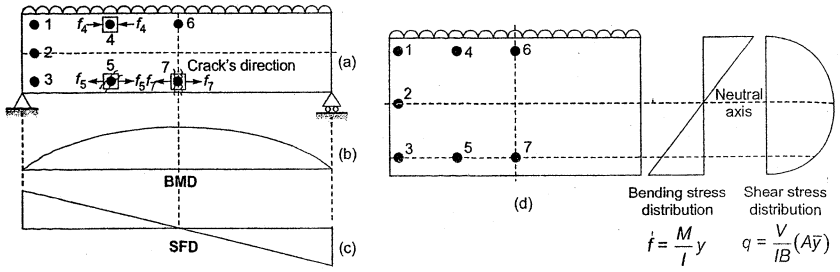


Fig. 6.6 *Explanation of combined effect of bending and shear stresses in a beam*

Consider points 1, 2, ...7 as shown in Fig. 6.6.

At point 5:

Bending stress $f_5 = \frac{M_5}{I}y$

Shear stress $q_5 = \frac{V_5}{IB}A\bar{y}$

Major principal stress, $p_1 = \frac{f_5}{2} + \sqrt{\left(\frac{f_5}{2}\right)^2 + q^2}$ (Positive i.e. tensile)

Minor principal stress, $p_2 = \frac{f_5}{2} - \sqrt{\left(\frac{f_5}{2}\right)^2 + q^2}$ (Negative i.e. comp.)

Cracks will be developed in p_2 direction so shear reinforcement is provided in p_1 direction.

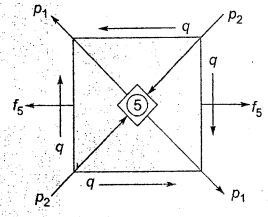
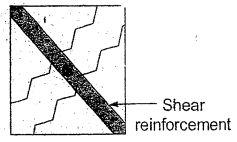


Fig. 6.7 *Stresses at point no. 5 of Fig. 6.6 (d)*



At point 4:

Principal stresses are,

Major principal stress,

$p_1 = \frac{-f_4}{2} + \sqrt{\left(\frac{f_4}{2}\right)^2 + q^2}$ (Positive i.e. tensile, small value)

Minor principal stress,

$p_2 = \frac{-f_4}{2} - \sqrt{\left(\frac{f_4}{2}\right)^2 + q^2}$ (Negative i.e. compression)

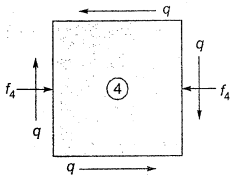


Fig. 6.8 *Stresses at point no. 4 of Fig. 6.6(d)*

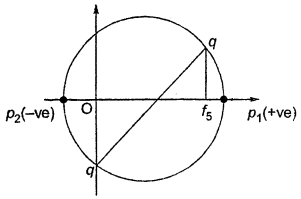


Fig. 6.9 *Mohr's circle for point no. 5 of Fig. 6.6 (d)*

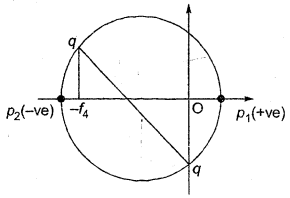
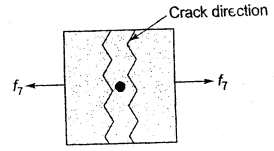


Fig. 6.10 *Mohr's circle for point no. 4 of Fig. 6.6 (d)*

At point 7:

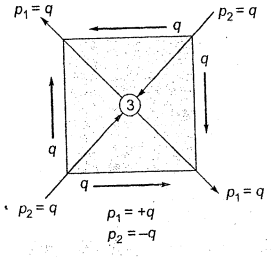
Only stress is the bending stress = f_7

$q = 0$ due to $V = 0$



(So longitudinal reinforcement is provided as shear reinforcement)

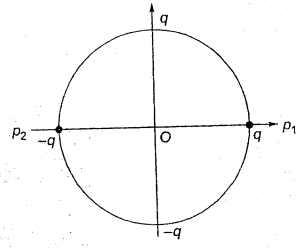
At points 3, 2 and 1:



Since $M = 0$ and therefore $f_3 = 0$

Shear stress, $q_3 = \frac{V}{IB}A\bar{y}$

- This $+q = p_1$ is called as diagonal tension.
- In beams, first crack starts due to this diagonal tension only.
- Direction of this crack is at 45 degrees.



$+q = p_1$ is diagonal tension

Cracking pattern of beam due to shear

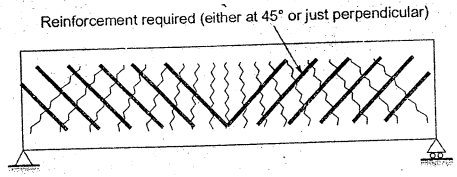


Fig. 6.11 *Shear reinforcement requirement*

6.5 Shear Transfer Mechanism

Shear from one plane to the adjacent plane in concrete gets transferred in many ways. The unbalanced vertical force i.e. shear force at a section is balanced by:

1. Shear resistance of uncracked portion of concrete (V_{cr}).

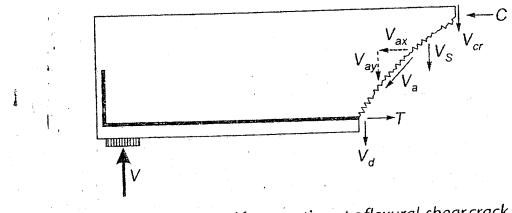


Fig. 6.12 *Internal forces acting at a flexural-shear crack*

2. Vertical component of the force due to aggregate interlocking (V_{ay}).
3. Dowel force in the tension reinforcement (V_d).
4. Shear resistance carried by transverse shear reinforcement (V_s) if shear reinforcement is provided.

Thus, at equilibrium:

$$V = V_{cr} + V_{ay} + V_d + V_s$$

The relative contribution of the various components of the shear resisting forces depends on loading, extent of cracking, material type etc. Before the commencement of flexural cracking, the applied/superimposed shear is resisted entirely by the uncracked concrete (V_{cr}). As the concrete cracking commences, redistribution of stresses take place resulting in development of aggregate interlock force (V_{ay} vertical component) and dowel force V_d. As the commencement of diagonal tension crack occurs, shear reinforcement intercepts the cracks and undergoes a sudden increase in tensile stress and strain. At this stage, all the four major components which resist external shear are very much effective. The resulting behavior of concrete including the ultimate shear failure depends on the mode of shear transfer mechanism and on how the forces in shear resisting elements get redistributed.

Presence of longitudinal reinforcement in the flexural tension zone adds to dowel action (V_d) and also controls the propagation of flexural cracks. It also contributes in increasing the depth of neutral axis and thus the depth of uncracked concrete section increases in compression. This increases the contributions of V_a and V_{cr}. This implies that, higher the percentage of tension reinforcement, greater is the shear resistance.

6.6 Nominal Shear Stress

6.6.1 Prismatic Beam

For prismatic beams of rectangular sections, the nominal shear stress (τ_v) is defined as:

$$\tau_v = \frac{V_u}{bd}$$

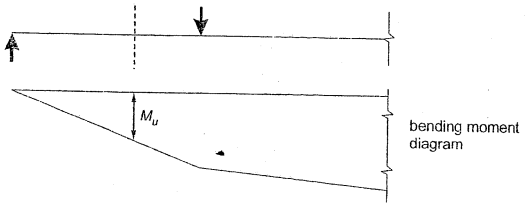
where, V_u is the factored shear force at the section under consideration, b is the width of the rectangular beam (take b_w, width of the web, in case of flanged beams) and d is the effective depth of the rectangular section.

NOTE

This quantity i.e. the nominal shear stress (τ_v) is simply a parameter to aid in calculation and design aspects, and in NO way represents the actual average shear stress across the rectangular section. In fact the actual shear stress distribution in concrete is quite complex.

6.6.2 Non-Prismatic Beam

For beams of varying depth the nominal shear stress incorporates the effect of angle β at which the tensile force is inclined to the horizontal.



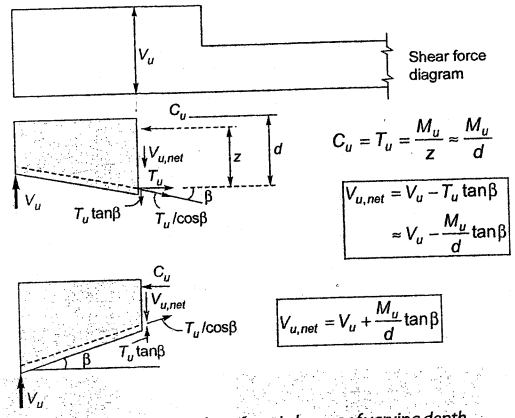


Fig. 6.13 *Design shear force in beams of varying depth*

Thus, nominal shear stress at per **Cl. 40.1.1** of **IS 456: 2000** for beams of varying depth is defined as

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$

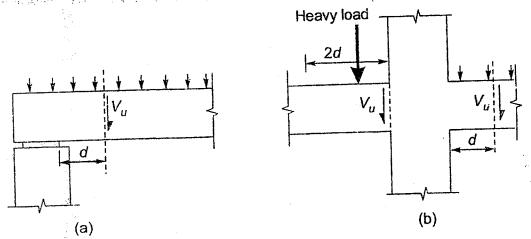
where, V_u = factored shear force
M_u = factored bending moment

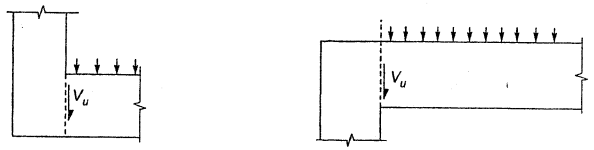
6.7 Critical Sections for Shear

While designing a concrete member for flexural shear, critical section(s) has to be located where the shear force reaches to its maximum value, and/or where the area of cross section is the minimum. In flexural members, maximum shear force generally occurs at the face of the support. **(Cl. 22.6.2 of IS 456: 2000)**. In case of point loads, shear force is very high at locations between the support and the first concentrated load.

It is quite evident that the support reaction gives rise to transverse compression in the end regions of the concrete member. This increases the shear strength of this region (i.e. support region) and the inclined cracks do not develop near the face of the support. In such a case, the critical section is located at a distance '*d*' from the support face. **(Cl. 22.6.2.1 of IS 456: 2000)**. The beam portion between this critical section for shear and the face of the support need to be designed only for the shear force at the critical section.

NOTE: It is to be noted that if a concentrated load acts on the member with in a distance of 2*d* from the support face, then the support face becomes the critical section for shear.





------ Dotted line represents critical section for shear

Fig. 6.14 *Critical sections for shear at supports*

6.8 Design Shear Strength of Concrete without Shear Reinforcement

In reinforced concrete. the margin of strength after the appearance of cracks (diagonal cracking) is quite unpredictable on account of various factors and hence it is ignored in the design process. Thus average design shear strength of concrete (τ_c) of reinforced concrete beams without shear reinforcement is limited to nominal shear stress τ_v corresponding to the load at which first inclined crack develops.

The magnitude of design shear strength of concrete (τ_c) depends on the grade of concrete (f_{ck}), percentage of tension reinforcement (p_t). As per **IS 456: 2000**, the values of τ_c **(table 19 of IS 456: 2000)** are based on the following expression:

$$\tau_c = 0.85\sqrt{0.8f_{ck}}\frac{\left(\sqrt{1+5\beta}-1\right)}{6\beta}$$

Here $\beta = \dfrac{0.8f_{ck}}{6.89p_t}$ or 1 whichever is greater where $p_t = \dfrac{100\,A_{st}}{bd}$

The design shear strength of concrete of various grades is given in **table 19 (for LSM) and table 23 (for WSM) of IS 456: 2000.**

The design shear strength of concrete gets enhanced in regions which are close to supports (from face of support to a distance of 2*d*). **Cl. 40.5 of IS 456: 2000** allows for a increase in the design shear strength of concrete at any section located within a distance 2*d* from the support face.

It is recommended to limit the increase in the design shear strength of concrete (τ_c) within a distance '*d*' from the support face to a value equal to '$2\tau_c$'.

Table 6.1: *Design shear stress of concrete (τ_c in N/mm²)*

Percentage of Steel (%)	WSM		LSM	
	M20	M25	M20	M25
0.15	0.18	0.19	0.28	0.29
0.25	0.22	0.23	0.36	0.36
0.50	0.30	0.31	0.48	0.49
0.75	0.35	0.36	0.56	0.57
1.00	0.39	0.40	0.62	0.64
1.25	0.42	0.44	0.67	0.70
1.50	0.45	0.46	0.72	0.74
1.75	0.47	0.49	0.75	0.78
2.00	0.49	0.51	0.79	0.82
2.25	0.51	0.53	0.81	0.85
2.50	0.51	0.55	0.82	0.88
2.75	0.51	0.56	0.82	0.90
3 and above	0.51	0.57	0.82	0.92

6.8.1 Design Shear Strength of Concrete in Slabs

IS 456: 2000 allows to have an increase in the shear strength of slabs (= $k\tau_c$) for solid slabs (not the ribbed slabs), where the value of factor '*k*' being adopted is as given below:

$k = 1.3$ for all $D \le 150$ mm

$k = 1.6 - 0.002D$ for 150 mm $< D <$ 300 mm

$k = 1.0$ for all $D \ge 300$ mm

Here *D* is the overall depth of the slab in mm.

Thus thinner is the slab, greater is the increase in shear strength.

6.8.2 Maximum Shear Strength of Concrete

In order to ensure ductile failure and to avoid shear compression failure (which is brittle in nature), it is essential that $\tau_c < \tau_{c\,max}$ where, $\tau_{c\,max} = 0.625\sqrt{f_{ck}}$.

For design purposes, the maximum shear strength of concrete ($\tau_{c\,max}$) is as per **table 20 (for LSM) and table 24 (for WSM) of IS 456: 2000.**

	M15	M20	M25	M30	M35	M 40 and above
$\tau_{c\,max}$ (N/mm²) (WSM)	1.6	1.8	1.9	2.2	2.3	2.5
$\tau_{c\,max}$ (N/mm²) (LSM)	2.5	2.8	3.1	3.5	3.7	4.0

Do you know?: The provisions for the design shear strength of concrete are applicable only for flexural shear (one-way shear) and are not applicable for punching shear (two-way shear).

6.8.3 Effect of Axial Force on Design Shear Strength of Concrete

The shear strength of concrete gets improved in presence of axial compression and gets reduced due to axial tension. Axial compressive force delays the appearance of first flexural crack in concrete but axial tensile force accelerates the appearance of first flexural crack in concrete.

Cl. 40.2.2 of IS: 456-2000 recommends that the design shear strength of concrete in presence of axial compression should be taken as $\delta\tau_c$ where the factor δ is defined as:

$$\delta = \begin{cases} 1+\dfrac{3P_u}{A_g f_{ck}} \\ 1.5 \end{cases} \quad \text{(whichever is less)}$$

Here, P_u = Factored compressive force (in N)

A_g = Gross sectional area (mm²)

f_{ck} = Characteristic compressive strength of concrete (N/mm²)

6.9 Shear Strength of Concrete with Shear Reinforcement

Cl. 40.4 of IS 456: 2000 describes following types of shear reinforcement (web reinforcement, as it is sometimes called as):

1. Shear stirrups normal to beam longitudinal axis.
2. Shear stirrups inclined (usually 45° or more) to beam longitudinal axis.
3. Bent-up longitudinal bars at 45° to 60° to the beam axis.

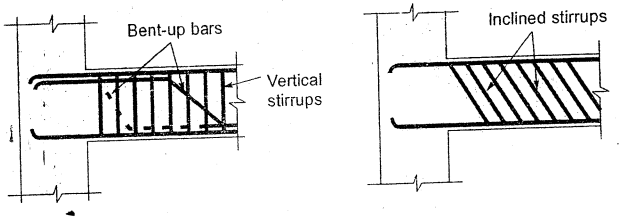
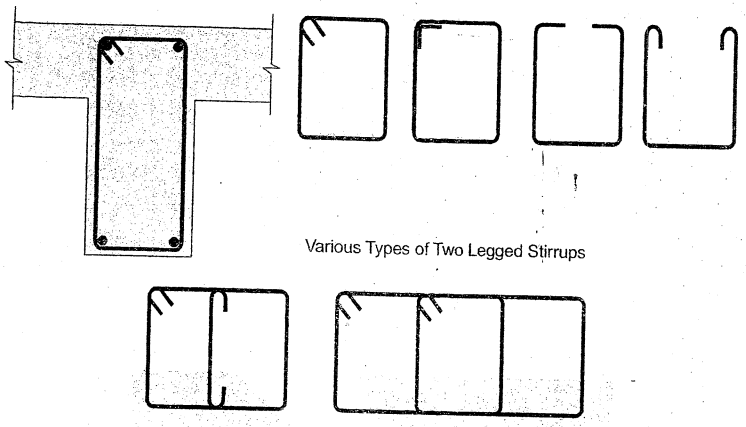


Fig. 6.15 *Types of shear reinforcement*



Various Types of Two Legged Stirrups

Fig. 6.16 *Various types of multi legged stirrups*

6.9.1 Shear Strength of Vertical Shear Reinforcement

It is the most prevalent type of shear reinforcement. Shear strength of vertical shear reinforcement is given by,

V_s = *No. of bars x cross section area of shear reinforcement at one section x strength of bar in shear*

$$= \frac{d}{s_v} A_{sv} \sigma_{sv}$$

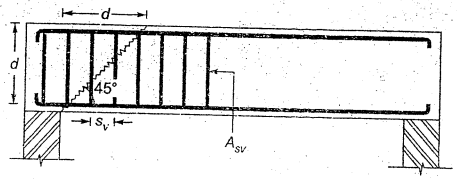


Fig. 6.17 *Explanation of shear strength of vertical shear stirrups*

6.9.2 Shear Reinforcement in Beams

1. **Vertical shear reinforcement:** It is the most common type of shear reinforcement. The first stirrup must be provided at a distance less than 50 mm from the face of support.

   The spacing of vertical shear reinforcement is given by the following expressions:

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} \qquad \text{(Cl. 40.4 (a) of IS 456: 2000 for LSM)}$$

$$s_v = \frac{\sigma_{sv} A_{sv} d}{V_{us}} \qquad \text{(Cl. B 5.4 (a) of IS 456: 2000 for WSM)}$$

- Closed stirrups are much better than open stirrups since the former resist torsion better and also helps in confining the compression reinforcement.

- Hook of a closed stirrup must be located in compression zone since this helps in improving the anchorage and avoid crack initiation.

Area of Shear Reinforcement (A_{sv})

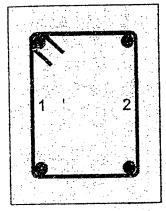


Fig. 6.18 *Two-legged shear reinforcement*

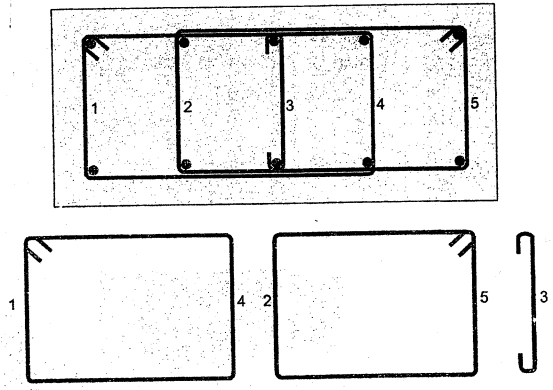


Fig. 6.19 *Five-legged shear reinforcement*

Area of shear reinforcement (A_{sv}) = No. of legs × Cross section area of one reinforcement

$$= n\left(\frac{\pi}{4}\right)\phi^2 \qquad \text{where } \phi = \text{diameter of shear reinforcement bar}$$

2. Inclined Shear Reinforcement

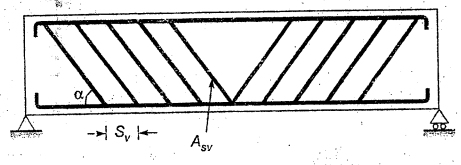


Fig. 6.20 *Inclined shear reinforcement*

From Fig. 6.20

$$\tan\alpha = \frac{d-d'}{x}$$

$$x = (d-d')\cot\alpha$$

Ignoring d', the number of stirrups crossing the crack is given as:

$$n = \frac{d+x}{s_v} = \frac{d(1+\cot\alpha)}{s_v}$$

Tensile force induced in stirrups = $0.87 f_y A_{sv}$

Thus shear force taken by stirrups is given as:

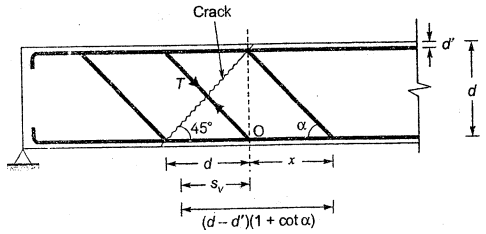


Fig. 6.21 *Inclined stirrups as shear reinforcement*

$$SF = \frac{d(1+\cot\alpha)}{s_v}(0.87 f_y A_{sv})$$

The vertical component of the above shear force is given as:

$$SF_{vertical} = (SF)\sin\alpha = \frac{d(1+\cot\alpha)}{s_v}(0.87 f_y A_{sv})\sin\alpha$$

The above vertical component of shear force must be equal to the shear strength of stirrups required (V_{us}).

$$V_{us} = \frac{d(1+\cot\alpha)}{s_v}(0.87 f_y A_{sv})\sin\alpha = \frac{d}{s_v}(0.87 f_y A_{sv})\left(1+\frac{\cos\alpha}{\sin\alpha}\right)\sin\alpha$$

$$= \frac{d}{s_v}(0.87 f_y A_{sv})(\sin\alpha+\cos\alpha) \quad \text{(Cl. 40.4 (b) of IS 456: 2000 for LSM)}$$

$$= \frac{\sigma_{sv} A_{sv} d}{s_v}(\sin\alpha+\cos\alpha) \quad \text{(Cl. B 5.4 (b) of IS 456: 2000 for WSM)}$$

Do you know? Bent up bars can also be used along with vertical or inclined shear reinforcement. Only the bent up bars cannot be used as shear reinforcement.

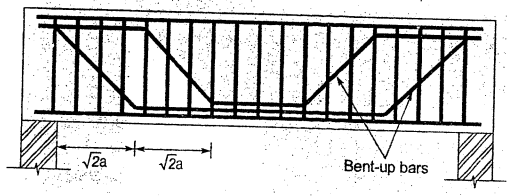


Fig. 6.22 *Shear reinforcement design with bent-up bars*

Remember

- First location of bent up bars = $a\sqrt{2} = jd\sqrt{2}$ (Distance from support)
- Second and subsequent location of bent up bars are at $a\sqrt{2}$ from previous location.
- Bars should not be bent beyond $l/4$ distance from each support.
- Vertical or inclined shear reinforcement shall be designed for maximum of $(V_s - V_{sb})$ and $V_s/2$.
- Contribution of bent up bars cannot be considered more than 50% of remaining shear force.

6.10 Minimum Shear Reinforcement

When nominal shear stress $\left(\tau_v = \frac{V_u}{bd}\right)$ is less than the design shear strength of concrete (τ_c) then a minimum shear reinforcement is provided as per Cl. 26.5.1.6 of IS 456: 2000 which states that when $\tau_v > 0.5\tau_c$, then

$$\frac{A_{sv\,min}}{bs_v} \geq \frac{0.4}{0.87 f_y} \quad \text{(This applies to both LSM and WSM and } f_y \leq 415 \text{ N/mm}^2\text{)}$$

Remember

The value of f_y is limited to 415 N/mm² because the width of diagonal crack is related to strain in the stirrups. Limiting the stress in stirrups will limit the strain and thus the width of diagonal crack.

Special case: When $\tau_v < (1/2)\,\tau_c$ i.e. in members of minor structural importance such as lintels, then as per Cl. 26.5.1.6 of IS 456: 2000, this provision of minimum shear reinforcement need not to be complied with.

In every case, whether $\tau_v < \tau_c$ or $\tau_v > \tau_c$ a minimum shear reinforcement must be provided.

Minimum shear reinforcement is provided for the following reasons:

1. A sudden failure of beam is prevented in case the concrete cover bursts and the bond between steel and concrete is lost.
2. Possibility of brittle shear failure is prevented which would have occurred without any shear reinforcement.
3. During the concreting process at site, it holds the reinforcing bars in position while pouring concrete.
4. Tie effect of compression steel makes the section more effective.
5. It improves down action of longitudinal tension bars.

6.11 Maximum Spacing of Shear Reinforcement

The maximum spacing of shear reinforcement (s_v) shall be as per the following:

$s_v \leq 0.75d$ (for vertical bars)

$s_v \leq d$ (for inclined bars)

In no case, spacing should exceed 300 mm.

6.12 Steps for Shear Reinforcement Design

1. Compute nominal shear stress $\left(\tau_v = \frac{V_u}{bd}\right)$.
2. Find $\tau_{c\,max}$ depending on the concrete grade (table 20 of IS 456: 2000 for LSM and table 24 for WSM)
3. Check if $\tau_v \leq \tau_{c\,max}$. If not then increase the size of section.
4. Find τ_c depending on percentage tension reinforcement and concrete grade (table 19 (for LSM) and table 23 (for WSM) of IS 456: 2000).
5. If $\tau_v < 0.5\,\tau_c$ then no shear reinforcement is required.
6. If $0.5\,\tau_c < \tau_v < \tau_c$ then provide only the minimum shear reinforcement as per Cl. 26.5.1.6 of IS 456: 2000.
7. If $\tau_v > \tau_c$ then provide shear reinforcement for $(\tau_v - \tau_c)$ which should not be less than minimum shear reinforcement.

6.13 Shear Connectors in Members Subjected to Flexure

It is well known that shear stress occurs in pairs. Thus a shear stress on the vertical plane of a beam section is accompanied by a corresponding horizontal shear that acts on every horizontal plane of the beam.

Many a times, a beam is required to be made up of two or more dissimilar materials (like providing reinforced concrete slab on steel joist etc.) as shown in the Fig. 6.22.

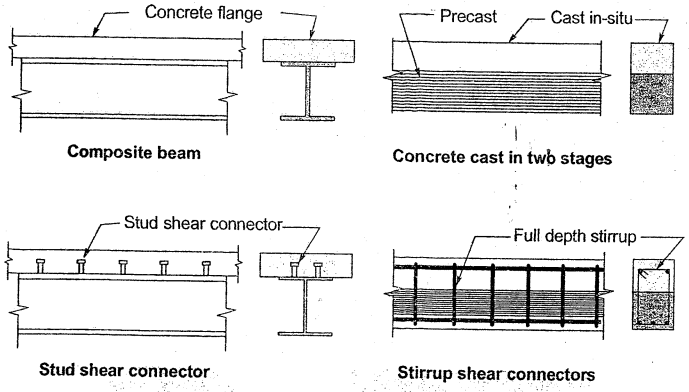


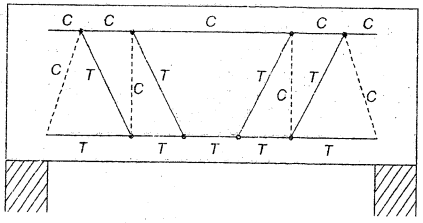
Fig. 6.23 *Provision of shear connectors at interfaces*

If such a composite beam is required to act as a unit then proper provision has to be introduced to transmit the horizontal shear from one material to the other so that it will prevent the relative slip between the interfaces of two dissimilar materials. This is accomplished by providing **shear connectors** in the form of studs, channels etc. These are welded on the top of steel joist/beam.

Where two different surface of concrete acts as interface (like casting of slab on precast concrete beam etc.), there full depth stirrups are provided to act as shear connectors.

6.14 Lattice Girder Effect

The combined behaviour of a RCC beam with bent up bars is similar to a truss as shown in figure. Where steel and concrete act as tension and compression members is called Lattice Girder Effect.



- Top concrete acts as compression members.
- Bottom longitudinal reinforcement acts as tension members.
- Concrete between inclined bent up bars acts as diagonal compression members.
- Bent up bars act as diagonal tension members.

Do You Know?

Many a times, we chip off and clean the old surface (of concrete etc.) and apply bonding material to have another material on the top of existing surface. This is NOT an effective way of shear transfer at interface except at very small localized regions

Example 6.1 A reinforced concrete beam of size 300 mm x 600 mm carries a maximum shear force of 200 kN. The beam is reinforced with 4-20 mm diameter bars for flexural resistance. Calculate the shear reinforcement required for (i) M 20 concrete and Fe 250 steel (ii) M 30 concrete and Fe 500 steel.

Solution: Since the beam is reinforced with 4-20 mm diameter bars and thus area of tension steel provided (A_{st}) is 1256.63 mm^2.

Assuming an effective cover of 50 mm, effective beam depth

$$= 600 \text{ mm} - 50 \text{ mm} = 550 \text{ mm}$$

$$\text{Percentage tension steel } (p_t) = \frac{1256.63 \times 100}{(300 \times 550)} = 0.7616\%$$

Now shear reinforcement is designed for factored shear force.

$$\text{Factored shear force } (V_u) = 1.5 \times 200 \text{ kN} = 300 \text{ kN}$$

$$\text{So shear stress } (\tau_v) = \frac{300 \times 1000}{(300 \times 550)} = 1.818 \text{ N/mm}^2$$

(i) M 20 concrete and Fe 250 steel

For $p_t = 0.7616\%$ and M 20 concrete, shear strength of concrete (τ_c) = 0.56 N/mm^2 (Table 19 of IS 456 : 2000)

Maximum shear strength for M 20 concrete ($\tau_{c\,max}$) = 2.8 N/mm^2 (Table 20 of IS 456: 2000)

Shear force to be resisted by shear reinforcement $(V_{us}) = V_u - \tau_c bd$

$$= 300 \times 10^3 - 0.56 \times 300 \times 550$$

$$= 207140.64 \text{ N} = 207.14 \text{ kN}$$

Using 2 legged 10 mm diameter stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 10^2 = 2 \times 78.54 = 157.08$$

$$\text{Spacing of shear reinforcement, } (s_v) = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 157.08 \times 550}{207140.64} \text{ mm}$$

$$= 90.71 \text{ mm} = 85 \text{ mm (say)}$$

$$< 300 \text{ mm}$$

$$< 0.75d = 0.75 \times 550 \text{ mm} = 412.5 \text{ mm} \quad \text{(OK)}$$

Check for minimum area of shear reinforcement:

$$A_{sv\,min} \geq \frac{0.4 b s_v}{0.87 f_y} = \frac{0.4(300)(85)}{0.87(250)}$$

$$= 46.89 \text{ mm}^2 < 157.08 \text{ mm}^2 \quad \text{(OK)}$$

(ii) M 30 concrete and Fe 500 steel

For $p_t = 0.76\%$ and M 30 concrete, shear strength of concrete (τ_c) = 0.59 N/mm^2 (Table 19 of IS 456: 2000)

Maximum shear strength for M 30 concrete ($\tau_{c\,max}$) = 3.5 N/mm^2 (Table 20 of IS 456: 2000)

Shear force to be resisted by shear reinforcement $(V_{us}) = V_u - \tau_c bd$

$= 300 \times 10^3 - 0.59 \times 300 \times 550$

$= 202650\ \text{N} = 202.65\ \text{kN}$

Using 2-legged 8 mm diameter stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 2 \times 50.27\ \text{mm}^2 = 100.54\ \text{mm}^2$$

$$\text{Spacing of shear reinforcement } (s_v) = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87(500)100.54(550)}{202650}$$

$= 118.69\ \text{mm} = 110\ \text{mm (say)}$

$< 300\ \text{mm}$

$< 0.75d = 0.75 \times 550\ \text{mm} = 412.5\ \text{mm}$ (OK)

Check for minimum area of shear reinforcement:

$$A_{sv\,min} \geq \frac{0.4 b s_v}{0.87 f_y} = \frac{0.4(300)110}{0.87(500)}$$

$= 30.34\ \text{mm}^2 < 100.54\ \text{mm}^2$ (OK)

Example 6.2 **Design the shear reinforcement of a beam of size 300 mm × 500 mm which is subjected to a working shear force of 120 kN. The beam is made up of M25 concrete and is reinforced with 4-20 Φ bars Fe415 steel.**

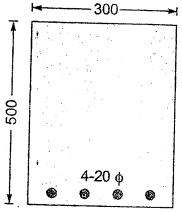
Solution:

Let clear cover = 20 mm

∴ Effective depth of beam (d) = $500 - 20 - \frac{20}{2} = 470$ mm

SF under working load conditions = 120 kN

∴ Factored shear force (V_u) = $1.5 \times 120 = 180$ kN



$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{180 \times 10^3}{300 \times 470}\ \text{N/mm}^2 = 1.277\ \text{N/mm}^2$$

$$\text{Percentage of tension reinforcement provided } (p_t) = \frac{A_{st}}{bd} \times 100$$

where $A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64\ \text{mm}^2$

∴ $p_t = \frac{1256.64}{300 \times 470} \times 100 = 0.89\%$

Design shear strength of concrete for M 25 concrete and $p_t = 0.89\%$ as per table 19 of IS 456: 2000 (τ_c) = 0.6 N/mm².

Maximum shear stress of concrete for M25 (τ_{max}) = 3.1 N/mm² (Table 20 of IS 456: 2000)

Thus, $\tau_v > \tau_c$

⇒ Shear reinforcement is designed for a shear force of $V_{us} = V_u - \tau_c bd$

(Since shear force equal to '$\tau_c bd$' will be taken up by concrete)

∴ $V_{us} = V_u - \tau_c bd = 180 \times 10^3 - 0.60 \times 300 \times 470$

$= 95400 = 95.4\ \text{kN}$

Using 2 legged 8 mm diameter stirrups,

Cross-sectional area of shear stirrups (A_{sv}) = $2 \times \frac{\pi}{4} \times 8^2 = 100.53\ \text{mm}^2$

Spacing of shear reinforcement required (s_v), (Cl. 40. 4(a) of IS 456: 2000)

$$= \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87(415)100.53(470)}{95400} = 178.81\ \text{mm c/c}$$

Also $\frac{A_{sv\,min}}{bs_v} \leq \frac{0.4}{0.87 f_y}$ (Cl. 26.5.1.6 of IS 456: 2000)

when $\tau_v > 0.5\,\tau_c$

i.e. $1.277\ \text{N/mm}^2 > 0.5 \times 0.6092\ \text{N/mm}^2$

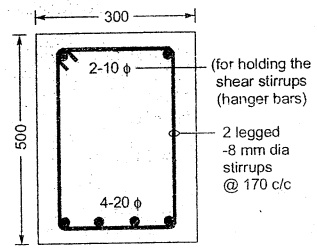
⇒ $1.277\ \text{N/mm}^2 > 0.3046\ \text{N/mm}^2$ (True)

⇒ $s_v \leq \frac{0.87 f_y A_{sv\,min}}{0.4 b}$

Thus $s_v \leq \frac{0.87(415)(100.53)}{0.4(300)}$

$= 302.47\ \text{mm c/c} > 178.81\text{mm c/c}$

Thus provide 2 legged 8 mm diameter stirrups @ 170 mm c/c.



Check for shear spacing

$$s_v \leq \begin{cases} 0.75d = 0.75(470) = 352.5\ \text{mm} \\ 300\ \text{mm} \end{cases}$$

Thus $s_v = 170\ \text{mm} < 300\ \text{mm}$ (OK)

Example 6.3 **A reinforced concrete slab is 200 mm thick. It is reinforced with 10ϕ @ 150 c/c at supports. Check if the slab is safe in shear. Use M20 concrete and Fe415 steel. Assume design factored load as 13 kN/m² and effective span as 4.5 m.**

Solution:

Overall depth of slab (D) = 200 mm

Assuming clear cover of 15 mm,

Effective depth of slab = $d = 200 - 15 - \frac{10}{2} = 180$ mm

Flexural reinforcement at supports = 10ϕ @ 150 mm c/c

$$= \frac{1000 \times A_\phi}{150} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{150} = 523.6\ \text{mm}^2/\text{m}$$

$$\therefore \quad p_t = \frac{523.6}{180\times1000}\times100 = 0.29\%$$

Design factored load (w_u) = 13 kN/m²

Effective span (l) = 4.5 m

Now shear force is predominant at supports.

$$\text{Factored shear force } (V_u) = w_u\frac{l}{2} = 13\times\frac{4.5}{2} = 29.25 \text{ kN/m}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{29.25\times10^3}{10^3\times180} = 0.1625 \text{ N/mm}^2$$

Design shear strength of M20 concrete for p_t = 0.29% as per table 19 of IS:456 (τ_c) = 0.38 N/mm²

Now for D = 200 m,

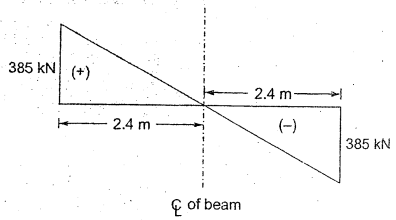
$k = 1.6 - 0.002\,D$

$\Rightarrow \quad k = 1.6 - 0.002\,(200) = 1.2$

$\therefore \quad k\tau_c = 1.2\,(0.38) = 0.46 \text{ N/mm}^2 > \tau_v\ (= 0.1625 \text{ N/mm}^2)$

Thus slab section is safe against shear failure at supports. It implies that it will be safe against shear failure at 'd' distance from support also.

Example 6.4 A reinforced concrete beam of size 350 mm x 600 mm is reinforced with 4-25 mm diameter bars of Fe500 steel. Out of the four bars, two bars are bent up in supports region at 45° to horizontal. The factored shear force diagram for the beam is as shown below. Check the adequacy of shear reinforcement. Assume M25 concrete. The clear span of beam is 4.8 m.



Solution:

At supports, out of the total 4-25ϕ bars, only 2 - 25ϕ bars are continuing.

Thus at supports, $A_{st} = 2\times\frac{\pi}{4}\times25^2 = 981.75 \text{ mm}^2$

Let clear cover = 20 mm

$$\therefore \quad \text{Effective depth of beam } (d) = 600 - 20 - \frac{25}{2} = 567.5 \text{ mm}$$

$$\therefore \quad \text{Percentage reinforce at supports, } p_t = \frac{A_{st}\,(\text{at supports})}{bd}\times100$$

$$= \frac{981.75}{350\times567.5}\times100 = 0.49\%$$

Design shear strength of M 25 concrete for p_t = 0.49% as per table 19 of IS 456: 2000 (τ_c) = 0.487 N/mm²

Maximum shear stress for M 25 concrete ($\tau_{c\,max}$) = 3.1 N/mm² (Table 20 of IS 456: 2000)

$$\text{Nominal shear stress } (\tau_v) = \frac{V_u}{bd} = \frac{385\times10^3}{350\times567.5} = 1.938 \text{ N/mm}^2$$

Thus $\tau_c < \tau_v < \tau_{c\,max}$

Shear resistance provided by bent up bars

Since 2-25ϕ bars are bent up at 45° to the horizontal near the supports.

$\therefore$ Shear resistance of bent up bars (V_{ui}) = $0.87\,f_y\,A_{sv}\sin\alpha$

$$= 0.87(500)\left(2\times\frac{\pi}{4}\times25^2\right)\sin45° = 301977.90 \text{ N}$$

$$\tau_{ui} = \frac{V_{ui}}{bd} = \frac{301977.90}{350\times567.5} = 1.52 \text{ N/mm}^2$$

Thus inclined bars can take a shear stress of 1.52 N/mm² but **as per IS 456: 2000, half of the design shear stress must be carried by vertical shear stirrups.**

Using 2-legged 8 mm diameter bars as stirrups, $A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.53 \text{ mm}^2$

$$\text{Spacing of vertical shear stirrups } (s_v) = \frac{0.87\,f_y\,A_{sv}\,d}{V_{us}} = \frac{0.87(500)100.53(567.5)}{0.5(\tau_v - \tau_c)bd}$$

$$= \frac{0.87(500)100.53(567.5)}{0.5(1.938-0.48)350(567.5)} = 171.39 \text{ mm c/c}$$

Bent-up bars are effective for a length of $d(1 + \cot\alpha)$ from support face. Thus 2-25ϕ bars can be bent-up at a distance of $d(1 + \cot\alpha)$ from support i.e. 567.5 (1 + cot 45°) = 1135 mm from the support face.

Shear at 1 m from the support face

SF at 1 m from the support face (V_u) = 224.583 kN $\left(=\frac{385}{2.4}\times1.4 \text{ kN}\right)$

$$\text{Nominal shear stress } (\tau_v) = \frac{224.583\times10^3}{350\times567.5} = 1.131 \text{ N/mm}^2$$

τ_c = 0.487 N/mm²

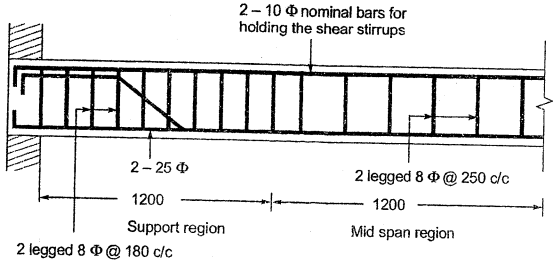
$\therefore$ Design shear stress (τ_{us}) = 1.131 – 0.487 = 0.644 N/mm²

Spacing of 2 legged 8 mm diameter shear stirrups (s_v)

$$= \frac{0.87\,f_y\,A_{sv}}{0.4\,b} = \frac{0.87(500)2\times\frac{\pi}{4}\times8^2}{0.4(350)} = 312.36 \text{ mm c/c}$$

$< 0.75\,d\ (= 0.75\times567.5 \text{ mm} = 425.625 \text{ mm})$ (OK)

Provide 2 legged 8 mm stirrups @ 250 mm c/c in the mid-span region and 2 legged 8 mm stirrups @180 mm c/c in the supports region.



Example 6.5 A simply supported beam of span 7.5 m and size 350 × 650 mm is subjected to a live load of 68 kN/m (excluding self weight of the beam). Tensile reinforcement of 8-25 mm dia. bars has been provided, out of which maximum 4 bars can be used as bent up bars. Design the shear reinforcement for the beam using bent up bars at one location only. (Use M 25 concrete and Fe 415 Steel). (Use LSM)

Solution:

Given: Width of the beam (b) = 350 mm

Overall depth of the beam (D) = 650 mm

Assuming an effective cover of 50 mm,

Effective depth of the beam (d) = 650 – 50 mm = 600 mm

Live load = 68 kN/m

Self weight of the beam = 0.35 × 0.65 × 25 kN/m = 5.68 kN/m

Total load = 68 + 5.68 kN/m = 73.68 kN/m

Bent up bars:

Let us bend only 2 bars

I[st] location = $\sqrt{2}\,a$ distance for the support

= $\sqrt{2}\,jd = \sqrt{2} \times 0.8 \times 600$ mm

= 679 mm < (span/4 = 7500/4 = 1875 mm) (OK)

II[nd] location = $2\sqrt{2}\,a = 2\sqrt{2}\,jd = 2 \times 679$ mm

= 1358 mm < (span/4 = 7500/4 = 1875 mm) (OK)

There is no possibility for III[rd] location because $3 \times \sqrt{2}\,a = 3 \times 679$ mm

= 2037 mm > (span/4 = 7500/4 = 1875 mm) (Not possible)

Calculation of shear force

Total load = 73.68 kN/m

Factored shear force (V_u) = $\frac{wl}{2} = \frac{1.5 \times 73.68 \times 7.5}{2}$ kN = 414.5 kN

Nominal shear stress at support

$$\tau_v = \frac{V_u}{bd} = \frac{414.5 \times 10^3}{350 \times 600} \text{ N/mm}^2$$

= 1.97 N/mm^2 < $\tau_{c\,max.}$ (= 3.1 N/mm^2)

(for M 25 concrete as per table 20 of IS 456: 2000)

Design Shear strength of concrete

Percentage of reinforcement at support (p_t) = $\frac{6 \times \frac{\pi}{4} \times 25^2 \times 100}{350 \times 600} = 1.4\%$

From table 19 of IS 456: 2000, the design shear strength of M 25 concrete,

τ_c = 0.724 N/mm^2

∴ SF resisted by concrete, $V_{cu} = \tau_c bd$

= 0.724 × 350 × 600 N = 152.04 kN

$V_{us} = V_u - V_{cu}$

= 414.5 – 152.04 kN = 262.46 kN

Shear strength of bent up bars

Shear strength of bent up bars ($V_{s\,bu}$) = $0.87 f_y A_{sv} \sin\alpha$

= $0.87(415) 2 \times \frac{\pi}{4} \times 25^2 \times \sin 45^\circ$ = 250.64 kN

Vertical shear reinforcement shall be designed for a shear force of

$V_{us} - V_{s\,bu}$ = 262.46 – 250.64 kN = 11.82 kN

$\frac{V_{us}}{2} = \frac{262.46}{2} = 131.23$ kN

Using 2 legged 8 mm dia. stirrups,

$$s_v = \frac{0.87 f_y A_{sv} d}{\frac{V_{us}}{2}} = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 600}{131.23 \times 10^3} \text{ mm}$$

= 165.95 mm

Spacing as per minimum shear reinforcement requirement

$$s_v \le \frac{0.87 f_y A_{sv}}{0.4b}$$

$$\le \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2}{0.4 \times 350} \text{ mm}$$

≤ 259 mm

Maximum spacing of shear reinforcement

1. $0.75d = 0.75 \times 600$ mm = 450 mm
2. 300 mm

} whichever is less = 300 mm

∴ For portion 1 provide 2 legged 8 mm dia. bars @ 160 mm c/c

Shear reinforcement design for portion 2

Shear force at a section just beyond the bent up bar

$$V_{u1} = \frac{3071}{3750} \times 414.5 = 339.45 \text{ kN}$$

Percentage of steel reinforcement (p_t) = $\frac{8 \times \frac{\pi}{4} \times 25^2}{350 \times 600} \times 100 = 1.87\%$

Design shear strength of M 25 concrete as per table 19 of IS 456: 2000

$= 0.8\ \text{N/mm}^2$

$V_{cu} = \tau_c bd = 0.8 \times 350 \times 600\ \text{N} = 168\ \text{kN}$

$V_{su} = V_u - V_{cu} = 339.45 - 168 = 171.45\ \text{kN}$

Spacing of 2 legged 8 mm dia. shear stirrups

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{2 \times \frac{\pi}{4} \times 8^2 \times 0.87 \times 415 \times 600}{171.45 \times 10^3}\ \text{mm}$$

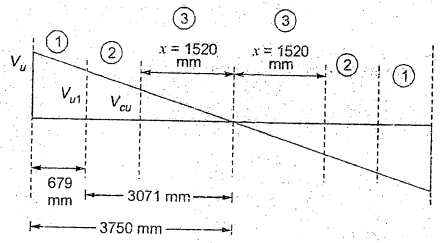
$= 127.02\ \text{mm}$

∴ For portion 2, provide 2-legged 8 mm dia. bars @ 120 mm c/c

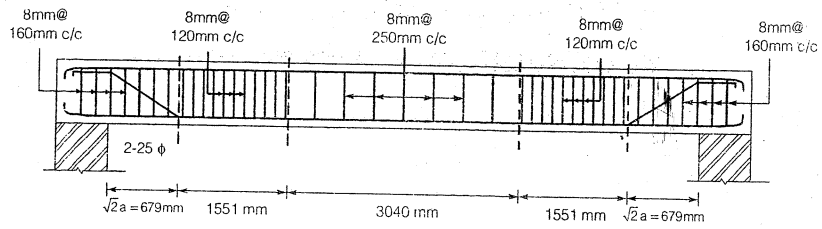
Shear reinforcement design for portion 3

$$\frac{168}{414.5} = \frac{x}{3750}$$

$x = 1520\ \text{mm}$



Length of portion 2 = 3750 – 679 – 1520 mm = 1551 mm



Shear reinforcement detailing

Example 6.6 A rectangular beam of size 300 x 550 mm overall depth is supported over a simply supported span of 6.5 m and is subjected to a live load of 45 kN/m. If reinforcement provided for tension is 6 nos. 25 mm dia. bars and for compression is 4 nos. 20 mm dia. bars, design the beam for shear by LSM and WSM both. Use M 20 concrete Fe 415 steel.

Solution:

Solution by WSM

Width of the beam (b) = 300 mm

Overall depth of the beam (D) = 550 mm

Assuming an effective cover of 50 mm

Effective depth of the beam (d) = 550 – 50 mm = 500 mm

Span of the beam (L) = 6500 mm

Area of tension reinforcement (A_{st}) = $6 \times \frac{\pi}{4} \times 25^2 = 2945.24\ \text{mm}^2$

Area of compression reinforcement (A_{sc}) = $4 \times \frac{\pi}{4} \times 20^2 = 1256.64\ \text{mm}^2$

Loads for the shear force calculation

Live load = 45 kN/m

Self weight of the beam = 0.3 x 0.55 x 25 kN/m = 4.125 kN/m

Total load = Dead load + Live load

= Self weight of the beam + Live load

= 4.125 + 45 = 49.125 kN/m

Maximum shear force (V) = $\frac{49.125 \times 6.5}{2}\ \text{kN} = 159.66\ \text{kN}$

Nominal shear stress (τ_v) = $\frac{V}{bd} = \frac{159.66 \times 10^3}{300 \times 500}\ \text{N/mm}^2 = 1.064\ \text{N/mm}^2 < \tau_{c\,max.}$

(= 1.8 N/mm² for M 20 concrete as per Table 24 of IS 456: 2000) (OK)

Design shear strength of concrete

Percentage tension reinforcement (p_t) = $\frac{A_{st}}{bd} \times 100 = \frac{6 \times \frac{\pi}{4} \times 25^2}{300 \times 500} \times 100 = 1.96\%$

Design shear strength of M 20 concrete for $p_t = 1.96\%$ as per table 19 of IS 456: 2000 ≃ 0.49 N/mm²

Shear strength of concrete (V_c) = 0.49 x 300 x 500 N = 73.5 kN

Remaining shear force to be taken up by shear stirrups = V_s

= $V - V_c$ = 159.66 – 73.5 kN = 86.16 kN

Vertical shear reinforcement

Assuming 2 legged 8 mm dia. stirrups,

$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\ \text{mm}^2$

Spacing of shear stirrups (s_v) = $\frac{A_{sv} \sigma_{sv} d}{V_s} = \frac{100.53 \times 230 \times 500}{86.16 \times 10^3}\ \text{mm} = 134.2\ \text{mm c/c}$

Minimum shear reinforcement

$$\frac{A_{sv\,min.}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$ } same formula for WSM and LSM

$$s_v \le \frac{A_{sv} \times 0.87 f_y}{0.4b} \le \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

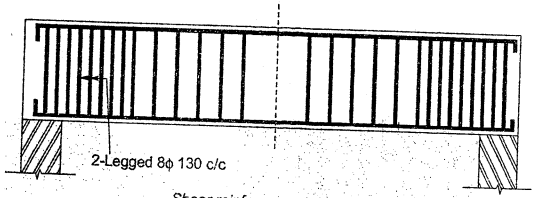
$$\le 302.47 \text{ mm c/c}$$

Maximum spacing of shear reinforcement

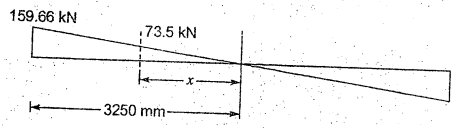
1. $0.75d = 0.75 \times 500 \text{ mm} = 375 \text{ mm c/c}$
2. 300 mm

} whichever is less

∴ At support provide 2 legged 8 mm dia. stirrups @ 130 mm c/c



Shear reinforcement detailing



SFD

For similar triangles

$$\frac{x}{3250} = \frac{V_c}{V} = \frac{73.5}{159.66}$$

$$x = 1496.15 \text{ mm}$$

Solution by LSM

Maximum shear force = 159.66 kN

Factored shear force (V_u) = 1.5 × 159.66 kN = 239.49 kN

Nominal shear stress of concrete (τ_v) $= \frac{V_u}{bd} = \frac{239.49 \times 10^3}{300 \times 500}$ N/mm²

= 1.5966 N/mm² ($< \tau_{c\,max.}$) (OK)

Percentage of tension steel (p_t) = 1.96%

Design shear strength of M 20 concrete for

p_t = 1.96% as per table 19 of IS 456: 2000 (τ_c) = 0.79 N/mm²

∴ Shear strength of concrete, $V_c = \tau_c bd = 0.79 \times 300 \times 500 \text{ N} = 118.5 \text{ kN}$

∴ Shear force to be resisted by shear stirrups

$$V_{us} = V - V_c = 239.49 - 118.5 \text{ kN} = 120.99 \text{ kN}$$

Spacing of shear reinforcement

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{2 \times \frac{\pi}{4} \times 8^2 \times 0.87 \times 415 \times 500}{120.99 \times 10^3} \text{ mm}$$

$$= 150 \text{ mm c/c}$$

Minimum shear reinforcement

$$\frac{A_{sv\,min.}}{bs_v} \ge \frac{0.4}{0.87 f_y}$$ } same formula for WSM and LSM

$$s_v \le \frac{A_{sv} \times 0.87 f_y}{0.4b} \le \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

$$s_v \le 302.47 \text{ mm c/c}$$

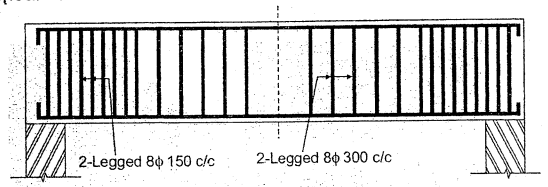
Maximum spacing of shear reinforcement

1. $0.75d = 0.75 \times 500 \text{ mm} = 375 \text{ mm c/c}$
2. 300 mm

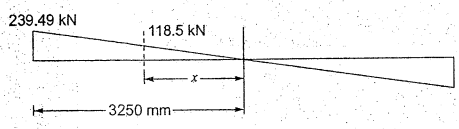
} whichever is less

∴ At support provide 2 legged 8 mm dia. stirrups @ 130 mm c/c

Central portion shear reinforcement



Shear reinforcement detailing



SFD

$$x = \frac{118.5}{239.49} \times 3250 \text{mm} = 1608 \text{ mm}$$

∴ In central portion, provide 2 legged 8 mm dia. stirrups @ 300 mm c/c

Objective Brain Teasers

Q.1 If a concentrated load acts on a beam within a distance of twice the effective depth of beam from the support face then critical section for shear will be located at:
(a) A distance 'd' from the support face
(b) At the support face itself
(c) At a distance '$d/2$' from the support face
(d) At a distance '$2d$' from the support face

Q.2 The design shear strength of concrete as per IS recommendations, depends on:
(a) Grade of concrete
(b) Grade of steel
(c) Percentage of steel provided
(d) All of the above

Q.3 IS 456: 2000 allows for an increase in the shear strength of slabs. Now, thinner the slab is:
(a) Lesser is the shear strength of slab
(b) No change in the shear strength of slab
(c) Greater is the shear strength of slab
(d) Data insufficient

Q.4 The provisions given in IS 456: 2000 for the design shear strength of concrete are applicable for:

(a) One way shear only
(b) Two way shear only
(c) Both (a) and (b)
(d) Neither (a) nor (b)

Q.5 Presence of axial force on a member influences the design shear strength of concrete. Which of the following provision is NOT covered in IS 456: 2000 with reference to the presence of axial force on a concrete member?
(a) Presence of compressive force
(b) Presence of tensile force
(c) Both (a) and (b)
(d) None of (a) or (b)

Q.6 Closed shear stirrups are better than open shear stirrups because:
(a) Closed stirrups resist torsion effectively
(b) Closed stirrups provide confinement to the compression reinforcement
(c) Closed stirrups prevent flexural cracks effectively
(d) Both (a) and (b)

Q.7 In order to control the crack width, the grade of steel in shear stirrups is limited to:
(a) Fe 250 (b) Fe 550
(c) Fe 500 (d) Fe 415

Q.8 Minimum shear reinforcement is provided for the following reasons:
A. A sudden failure of beam is prevented in case the concrete covers bursts and the bond between steel and concrete is lost.
B. Possibility of brittle shear failure is prevented due to non-availability of shear reinforcement.
C. It holds the reinforcing bars in position while pouring concrete.
D. Tie effect of compression steel makes the section more effective.
(a) A, B and C (b) A, C and D
(c) B, C and D (d) A, B, C and D

Q.9 In composite beams, the two dissimilar units are made to act as a single unit by the provision of:
(a) Shear stirrups (b) Lateral ties
(c) Lap splicing (d) Shear connectors

Q.10 If nominal shear stress is less than the half of the design shear strength of concrete, then:
(a) Minimum shear reinforcement is required to be provided
(b) Designed shear reinforcement is required to be provided
(c) Maximum shear reinforcement is required to be provided
(d) No shear reinforcement is required to be provided

Q.11 The ratio of permissible shear stress in LSM and WSM of design is
(a) 5 : 4 (b) 25 : 16
(c) 125 : 64 (d) 5 : 3

Q.12 The contribution of bent up bars towards shear resistance should not be more than
(a) 30% of total shear reinforcement
(b) 40% of total shear reinforcement
(c) 50% of total tension reinforcement
(d) 50% of total shear reinforcement

Q.13 High shear strength of RC beam can be achieved by providing
(a) shear stirrups
(b) TMT bars
(c) flunky aggregates in concrete mix
(d) All of the above

Q.14 As per IS 456: 2000, provision of minimum shear reinforcement in beams is provided in the form of stirrups
(a) to resist principal tension
(b) to resist additional shear force due to live loads
(c) to resist flexural cracks
(d) All of the above

Q.15 Flexural shear failure in RC beam
(i) Occurs under large BM and less SF
(ii) Occurs when shear span to effective depth ratio > 6
(iii) results in cracks inclined at $\pi/2$ radians with the horizontal
of the above statements, the incorrect statement(s) is/are
(a) (i) and (ii) (b) (i) and (iii)
(c) (iii) only (d) (ii) only

Q.16 In a RC section, the shape of shear stress diagram is
(a) 
(b) 
(c) 
(d) 

Q.17 A RC beam of rectangular section with 350 mm effective depth is provided with shear reinforcement. The maximum spacing of vertical shear stirrups is
(a) 262.5 mm (b) 300 mm
(c) 205 mm (d) 210.5 mm

Q.18 In a RC beam section, the nominal shear stress (τ_v) does not exceed the design shear stress of concrete (τ_c). In this case,
(a) No shear reinforcement in provided
(b) A minimum shear reinforcement is provided
(c) Shear reinforcement corresponding to nominal shear stress is provided
(d) Shear reinforcement is provided for $(\tau_c - \tau_v)$

Q.19 The maximum permissible shear stress of concrete ($\tau_{c\,max}$) as per IS 456: 2000 is based on
(a) diagonal compression failure
(b) diagonal tension failure
(c) flexural compression failure
(d) flexural tension failure

Answers

1. (b)	2. (d)	3. (c)	4. (a)	5. (b)
6. (d)	7. (d)	8. (d)	9. (d)	10. (d)
11. (b)	12. (d)	13. (a)	14. (a)	15. (d)
16. (a)	17. (a)	18. (b)	19. (a)	

Hints:

11. (b)

$$\frac{\text{Permissible shear stress in LSM}}{\text{Permissible shear stress in WSM}} = \frac{0.25\sqrt{f_{ck}}}{0.16\sqrt{f_{ck}}} = \frac{25}{16}$$

17. (a)

For vertical stirrups, spacing

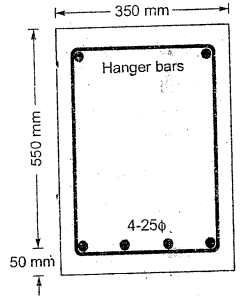
$$\not< \begin{cases} 0.75d \\ 300 \text{ mm} \end{cases}$$

$$\not< \begin{cases} 0.75 \times 350 = 262.5 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

$$= 262.5 \text{ mm}$$

Conventional Practice Questions

Q.1 The beam section as shown in the figure is subjected to a factored SF of 275 kN. Using M20 concrete and Fe415 steel, determine if shear reinforcement is required.



Q.2 Design the shear reinforcement for a beam of size 350 mm × 500 mm effective depth if the beam is reinforced with 4-20 mm dia. bars of Fe415. The beam section is subjected to a factored SF of 50 kN. Use M25 concrete.

Q.3 An intermediate span of continuous beam (of size 200 mm × 450 mm) is having a span length of 4.4 m. The beam carries a uniformly distributed load of 25 kN/m (ultimate) and a point load of 80 kN (ultimate) at a distance of 1.6 m from left support. The beam is subjected to an ultimate hogging moment of 55 kNm at both the ends. Using M20 concrete and Fe 500 steel, design the shear reinforcement for the beam.

CHAPTER 07

Design for Bond in Reinforced Concrete

7.1 Introduction

In the previous chapters, we made an assumption that there exists a perfect bond between the steel and the surrounding concrete in order to have strain compatibility. In this chapter, we will elaborate this assumption along with the concept of development length. We will look into the fact that why anchoring of tensile bars are essential, how tor/deformed bars are better than plain bars, when the question of bond arises? We will understand what does 'bond stress' means along with the concept of 'bond strength'.

7.2 Bond in Reinforced Concrete

'Bond' in reinforced concrete is the adhesion (adhesive force) between the reinforcing steel bar and the surrounding concrete. Due to this bond only it is possible to transfer the axial force from the reinforcing steel to the surrounding concrete thus thereby introducing strain compatibility, composite/combined action of steel and concrete which avoids slippage of reinforcing bars from within the concrete. The basic assumption of flexural theory that plane sections remain plane before and after the bending is valid only if bond between the concrete and the steel is effective.

Remember: In the absence of bond, the reinforcing bar in concrete is just like a string only wherein the stress is constant at all the points on the string.

It is because of the bond only that the axial stress (tensile, compressive) in the reinforcing bar vary along its length from point to point. This is essential to accommodate the variation in bending moment along the length of flexural member.

7.3 Mechanism of Force Transfer Through Bond

Transfer of force due to bond in reinforced concrete is achieved by the following mechanisms:

1. Chemical adhesion due to sticky glue like properties of certain products of hydration of cement in concrete.
2. Frictional resistance due to surface roughness of reinforcement and the grip exerted by the surrounding concrete.

3. Mechanical interlocking due to surface protrusions/ribs on deformed reinforcing bars which is in fact not available for plain reinforcing bars.

7.4 Bond Stress

Bond resistance is achieved by development of tangential (shear) stress along the interface (contact surface) between the reinforcing bars and the concrete. The stress developed at the interface of steel and concrete is called as bond stress.

7.5 Various Types of Bond in Reinforced Concrete

There are two major types of bond according to the types of loading as:

1. **Flexural bond:** It comes into play in flexural members on account of shear or variation in bending moment. This in turn causes a variation in the axial tension along the length of reinforcing bar. Flexural bond is critical at locations where shear ($V = dM/dx$) is significant.
2. **Anchorage/development bond:** It arises over the length of anchorage provided for a bar or at the end of reinforcing bar. This bond resists the pulling out of bar if it is in tension or pushing in of the bar if it is in compression.

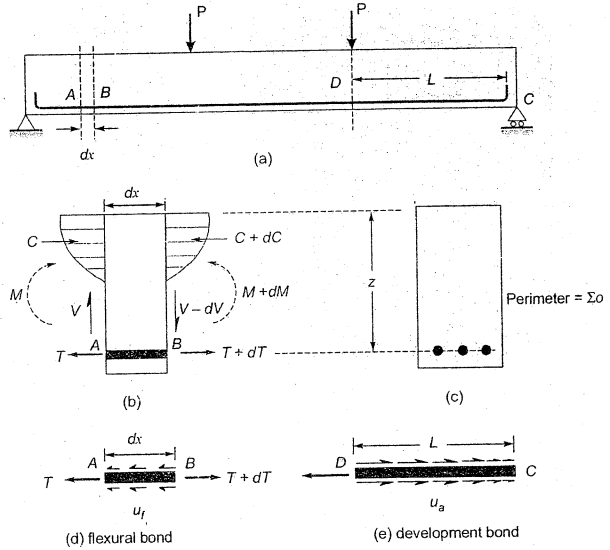


Fig. 7.1 *Different types of bond in reinforced concrete*

7.6 Flexural Bond

The variation in tension along the length of reinforcing bar owing to varying bending moment is due to flexural bond. Consider two sections on a beam distant 'dx' apart as shown in the Fig. 7.1, which are acted upon by a differential moment 'dM'. If 'z' is the lever arm then,

At section A, $M = Cz = Tz$

and at section B, $M + dM = (C + dC)z = (T + dT)z$

From above two equations, $dT = \frac{dM}{z}$ ...(i)

Therefore, force 'dT' which is the unbalanced force in the bar actually gets transferred to the surrounding concrete. This force transfer through this mechanism is called as **flexural bond**. This force gets developed along the interface of reinforcing bar and the surrounding concrete.

Let, Σo = Total perimeter of all the reinforcing bars at the beam section under consideration

dx = Elementary length of the beam under consideration

u_f = Flexural bond stress

Thus, $u_f(\Sigma o)dx = dT$ ...(ii)

From equations (i) and (ii),

$$u_f = \frac{dM/z}{(\Sigma o)dx} = \frac{V}{(\Sigma o)z} \quad \text{...(iii)}$$

where, V = Transverse shear force at the section = $\frac{dM}{dx}$

Thus flexural bond stress is high at locations of high shear force (V) and the term (Σo) in the denominator indicates that flexural bond stress can be reduced by using small diameter of large number of bars.

7.6.1 Factors Affecting The Flexural Bond Stress

The actual flexural bond stress gets affected by the following factors:

1. Appearance of flexural cracking.
2. Local slip of reinforcing bars in concrete.
3. Splitting of concrete and other secondary effects.

Do You Know?

How Flexural Cracking in Reinforced Concrete Affects The Flexural Bond Stress?

From equation (iii) above, it is evident that flexural bond stress (u_f) varies with the transverse shear force (V).

Thus, when, $V = 0$, $u_f = 0$

i.e. no flexural bond stress will be developed at the locations of zero shear force. But in reality, this is NOT true. The tensile force in the reinforcing bar (dT) varies between the flexural crack locations even at the locations of constant bending moments. At the locations of flexural cracks, tension is carried by reinforcing bars only but at locations in between the cracks, concrete do take some tension thereby reducing the tensile stress in reinforcing bar to some extent.

Remember: Flexural cracks are in general NOT present in the compression region of beam.

7.7 Anchorage/Development Bond

This anchorage/development bond gets developed at the extreme cut-off end of the bar subjected to tensile force/ compressive force.

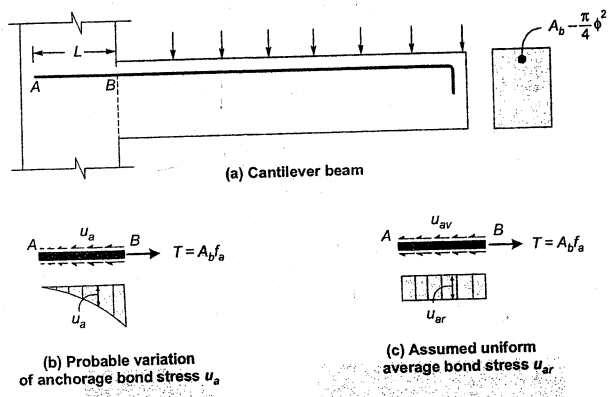


Fig. 7.2 *Anchorage/development bond in reinforced concrete*

In Fig. 7.2 shown above, the stress is zero at the end A of the reinforcing bar and at end B, the bending moment is maximum and hence the tensile stress is maximum at B. This variation of force between the ends A and B is transferred to the surrounding concrete by anchorage bond stress in the beam.

An expression for the average bond stress (u_{avg}) can be obtained by assuming a constant bond stress distribution over the length 'L' of the reinforcing bar.

$$\pi\phi L(u_{avg}) = \frac{\pi}{4}\phi^2 f_s$$

$$u_{avg} = \frac{\phi f_s}{4L}$$

where, f_s = Maximum stress (tensile or compressive) developed at the critical section

7.8 Development Length

As per Cl. 26.2 of IS 456: 2000, development length is:

"The calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or a combination thereof."

Development length prevents the bar from pulling out in tension or pushing it in compression. When the required development length cannot be provided due to certain restrictions or other considerations then in that case, bends, hooks or mechanical anchorages are provided to supplement with an equivalent embedment length.

Development length can be calculated as:

$$\pi\phi L\tau_b = \left(\frac{\pi}{4}\right)\phi^2 f_s \Rightarrow L = \frac{\phi f_s}{4\tau_b}$$

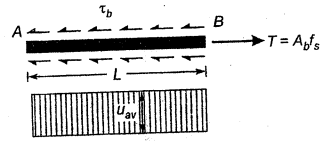


Fig. 7.3 *Assumed uniform average bond stress U_{av}*

where, τ_b = Design bond stress = Permissible value of average anchorage bond stress

The average bond stress is still used in the working stress method of design as given in Cl. B-2.1.2 of IS 456: 2000. But in the

limit state method of design, this average bond stress is referred to as design bond stress,(τ_{bd})as per Cl. 26.2.1.1 of IS 456: 2000.

The value of bond stress is increased by 60% for deformed bars in tension and a further increase of 25% is made for bars in compression.

Table 7.1: *Design Bond Stress for Different Grades of Concrete (in N/mm²)*

CONCRETE GRADE	M15	M20	M25	M30	M35	M40	M45	M50
LSM (Cl. 26.2.1.1 of IS:456)	—	1.2	1.4	1.5	1.7	1.9	1.9	1.9
WSM (Table 21 of IS:456)	0.6	0.8	0.9	1	1.1	1.2	1.3	1.4

Table 7.2: *Development length of fully stressed ($0.87f_y$) single bar in tension (ϕ = Bar diameter)*

Grade of Steel	Concrete Grade		
	M20	M25	M30
Fe 250	46ϕ	39ϕ	37ϕ
Fe 415	47ϕ	41ϕ	38ϕ
Fe 500	57ϕ	49ϕ	46ϕ

For bundled bars, the development length of each bar of bundled bars shall be that of individual bar increased by 10% for two bars in contact, 20% for three bars in contact, 33% for four bars in contact. Such an increase in development length is made because of reduction in the interface area between the steel and surrounding concrete.

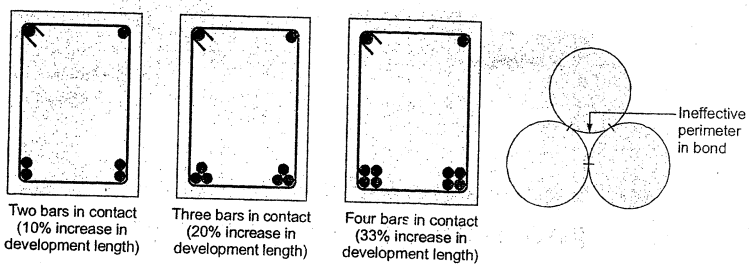


Fig. 7.4 *Bundled Bars in contact*

As per IS 456: 2000 development length is assumed to be satisfied in concrete structure if:

$$L_d \le \frac{M_1}{V} + L_0$$

where, M_1 = Moment of resistance of the section assuming all reinforcing bars to be stressed up to design strength (σ_{st} or $0.87f_y$)

V = Shear force at support

L_0 = Length of reinforcement beyond centre of support

Remember: In the above expression, the value of (M_1/V) can be increased by 30% if ends of reinforcement are confined by a compressive reaction.

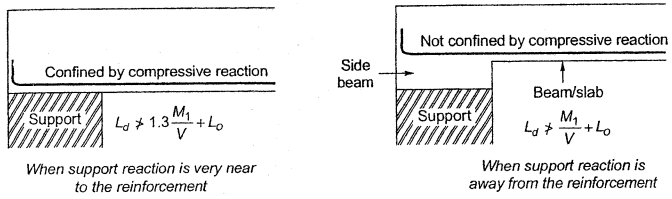


Fig. 7.5 *Confinement of reinforcement ends by compressive support reaction*

The requirement, $L_d \le \frac{M_1}{V} + L_0$

can be satisfied by any of the following methods:

1. Provide more number of bars so that M_1 is increased. This option however adds up the additional reinforcement cost.
2. Increase L_0 but this can be done upto a certain extent since width of support cannot be increased indefinitely.
3. Decrease the diameter of main reinforcing bars.

7.9 Mechanisms of Bond Failure

The bond between the steel and the surrounding concrete may fail in any one or combination of the following:

1. Failure of adhesion between the reinforcing bar and concrete.
2. Splitting of concrete that surrounds the reinforcing bar.
3. Crushing of concrete in front of the bar ribs in case of deformed bars.
4. Shearing of concrete embedded between the ribs along the cylindrical surface surrounding the ribs.

7.10 Factors affecting the Bond Strength

Bond strength is affected by many factors. The following factors attempt to increase the bond strength:

1. Using ribbed/deformed bars instead of plain bars.
2. Using smaller diameter bars.
3. Use of higher concrete grade.
4. Increasing the cover around the bar.
5. Increasing the length of embedment, hooks, bends etc.
6. Using mechanical anchorages.

7.11 Bends, Hooks and Mechanical Anchorages

Bends are provided in order to meet the development length requirements for bars in tension or compression. As per IS 456: 2000, the anchorage value of a bend is taken as 4 times the bar diameter for each 45 degree bend subject to a maximum of 16 times the bar diameter (Cl. 26.2.2.1 (b) of IS 456: 2000).

Bends and hooks introduce bearing stresses in the concrete they bear against. In order that these bearing stresses do not exceed the permissible limits, the turning radius must be large enough.

Mechanical anchorages in the form of nuts, bolts, welded plates etc. can also be provided if they are capable of developing the strength of bar without any damage to concrete.

7.12 Anchoring Bars in Tension

As per Cl. 26.2.2.1 of IS 456: 2000, for anchoring bars in tension, deformed bars may be used without end anchorages provided the development length requirement is satisfied. Hooks should be provided for plain bars in tension. Standard hooks and bends should be as per IS 2502: 1963 or as per Table 67 of SP-16.

7.13 Anchoring Bars in Compression

As per Cl. 26.2.2.2 of IS 456: 2000, for anchoring bars in compression, the anchorage length shall be equal to the development length of the bars in compression. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

7.14 Mechanical Devices for Anchorages

As per Cl. 26.2.2.3 of IS 456: 2000, any mechanical or other device capable of developing the strength of the bar without damage to concrete may be used as anchorage with the approval of engineer-in-charge.

7.15 Anchoring Shear Reinforcement

As per Cl. 26.2.2.4 of IS 456: 2000, for inclined bars, the development length shall be as for bars in tension where this length is measured as:

1. In tension zone, from the end of the sloping or inclined portion of the bar and,
2. In compression zone, from the mid-depth of the beam.

For stirrups and any other secondary reinforcement like transverse ties, complete development length and anchorage shall have to be provided when bar is bent through an angle of at least 90 degree around a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight times diameter, or when the bar is bent through an angle of 135 degree and is continued beyond the end of the curve for a length of at least six times bar diameter or when the bar is bent through an angle of 180 degrees and is continued beyond the end of the curve for a length of at least four times bar diameter.

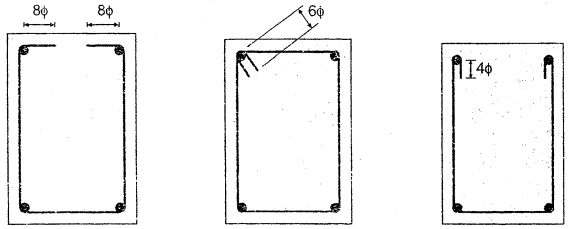


Fig. 7.6 *Anchoring shear reinforcement*

7.16 Reinforcement Splicing

Splicing is required when the bars available are shorter than that required (due to non-availability of longer bars owing to manufacturing or transportation problem), when there is a change in the bar diameter (e.g. in columns) etc. Splicing is done to transfer the axial load from one bar to another with the same line of action. Splicing introduces stress concentrations in the surrounding concrete. To avoid such stress concentrations, IS 456: 2000 recommends that splices in flexural members should not be at sections where the bending moment is more than 50% of the moment of resistance and not more than half the bars shall be spliced at a particular section. Splicing is done by lapping of bars, welding of bars or with mechanical connection.

7.16.1 Lap Splices

It is done by overlapping the bars over a certain length, thereby making possible the transfer of axial force from the terminating bar to the connecting bar by the mechanism of anchorage (development) bond with the surrounding concrete. Lap splices are generally not permitted for very large diameter bars ($\phi > 36$ mm), for which welded splices are recommended. However, where welding is not practicable, Cl. 26.2.5.1(a) of IS 456: 2000 recommends the use of additional spirals around the lapped bars.

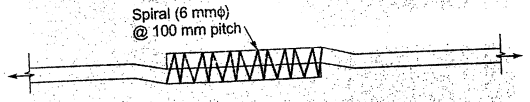


Fig. 7.7 *Lapping of large diameter bars by lap splicing*

It is always desirable to bend the bars slightly (especially for large diameter bars) at the location of splices in order to have a collinear force transfer without any eccentricity. IS 456: 2000 specifies that straight length of the lap should not be less than 15ϕ or 200 mm (whichever is more). The lap length must be atleast equal to development length. In no case the lap length should be less than 30ϕ for flexural and direct tension and 24ϕ for compression.

When two different diameter bars meet at splice location then lap length should be calculated on the basis of smaller bar diameter. Splices in tension members should be enclosed by spirals made up of bars of diameter not less than 6 mm along with pitch not exceeding 100 mm.

7.16.2 Welded Splices and Mechanical Anchors

Welded splices and mechanical connections are suitable for large diameter bars for which lap splicing is not recommended. This leads to reduced consumption of reinforcing steel. Cl. 12.4 of IS 456: 2000 specifies that tension tests should be performed on these welded splices in order to check their adequacy.

Welding of cold worked bars (deformed bars) need special attention as there always exists a possibility of loss of strength due to the heat of welding. Cl. 26.2.5.2 of IS 456: 2000 states that the design strength of welded splice should be limited to 80% of design strength of the bar in tension splice.

Butt welding of bars is generally adopted in welded splices. The bars to be spliced should be of the same diameter. Additional two or three symmetrically positioned small diameter lap bars may also be provided (especially when the bars are subjected to tension) and fillet welded to the main bars. Even in the case of 'lap splices', lap welding (at intervals of 5ϕ) may be provided in order to reduce the lap length.

Cl. 26.2.5.3 of IS: 456-2000 permits the use of end bearing splices for bars in compression.

Example 7.1 A reinforced concrete (M25) simply supported rectangular beam of size 300 mm × 500 mm is reinforced with 3-20 mm diameter bars of Fe 415 steel. The reinforcement bars are bent at 90 degrees at the support. The shear force at the supports is 105 kN under working load conditions. Find the value of anchorage length.

Solution:

Factored shear force (V_u) = 1.5 x 105 kN = 157.5 kN

Let clear cover to reinforcement = 30 mm

Thus effective depth of the beam (d) = $50-30-\frac{20}{2}$ mm = 460 mm

Area of tension steel (A_{st}) = $3\times\left(\frac{\pi}{4}\right)\times 20^2 mm^2 = 942.5\ mm^2$

Calculation of depth of neutral axis:

$$x_u = \frac{0.87 f_y A_{st}}{0.362 f_{ck} b} = \frac{0.87x415x942.5}{0.362x25x300} = 125.34\ mm$$

Limiting depth of neutral axis ($x_{u\,lim}$) = 0.479d = 0.479 × 460 mm = 220.34 mm > x_u (=125.34 mm)

Thus beam section is under-reinforced and $f_{st} = 0.87 f_y$ is correct in the above calculation of depth of neutral axis.

Calculation of moment of resistance:

Moment of resistance of the beam section (M_r) = $0.87 f_y A_{st}(d - 0.416 x_u)$

= 0.87 × 415 × 942.5(460 − 0.416 × 125.34) = 138.79 kNm

For M25 concrete and Fe415, design bond stress (τ_{bd}) = 1.4 x 1.6 N/mm² = 2.24 N/mm²

(Cl. 26.2.1.1 of IS 456: 2000)

Development length:

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87x415\phi}{4x2.24} = 40.296\phi$$

Anchorage length:

For 90 degree bend, anchorage length (L_o) = 8ϕ = 8 × 20 mm = 160 mm

$$40.296\phi \le 1.3\frac{138.79\times10^6}{157.5\times10^3} + 160$$

$$\phi \le 32.4\ mm$$

So there is no need to increase the anchorage length.

If the diameter of bar provided was larger than 32.4 mm then in that case, the anchorage length had to be increased either by extending the bar length beyond the anchorage value provided or by providing the anchorage in the form of a U bend (anchorage value = 16ϕ).

Objective Brain Teasers

Q.1 Which of the following option is the best way to have proper bond between the steel and surrounding concrete?
(a) Use small number of large diameter bars
(b) Use large number of small diameter bars
(c) Use of shear stirrups
(d) All of the above

Q.2 When mild steel bars are used in place of deformed bars then bond strength:
(a) Increases
(b) Decreases
(c) Is independent of type of steel
(d) Becomes zero

Q.3 Match List-I and List –II using the codes given below:

List–I
(i) Modulus of rupture of concrete
(ii) Development length
(iii) Modulus of elasticity of concrete
(iv) Nominal shear stress

List–II
(A) $5000\sqrt{f_{ck}}$
(B) $\phi f_s / 4\tau_{bd}$
(C) V_u / bd
(D) $0.7\sqrt{f_{ck}}$

(a) (i)-(D), (ii)-(B), (iii)-(A), (iv)-(C)
(b) (i)-(B), (ii)-(D), (iii)-(C), (iv)-(A)
(c) (i)-(A), (ii)-(C), (iii)-(B), (iv)-(D)
(d) (i)-(C), (ii)-(A), (iii)-(D), (iv)-(B)

Q.4 In limit state design, if τ_{bd} is the design bond stress for plain bars in tension, then the bond stress for deformed bars in compression will be:
(a) $1.6\tau_{bd}$ (b) $1.25\tau_{bd}$
(c) τ_{bd} (d) $2\tau_{bd}$

Q.5 The type of bond which gets developed due to variation of bending moment along the beam is:
(a) Development bond
(b) Anchorage bond
(c) Flexural bond
(d) All of the above

Q.6 The minimum lap length required for reinforcing bars in tension is:
(a) 30 times the bar diameter
(b) 24 times the bar diameter
(c) 16 times the bar diameter
(d) None of the above

Q.7 Lap splicing is generally not permitted if the bar diameter exceeds:
(a) 25 mm (b) 32 mm
(c) 36 mm (d) 40 mm

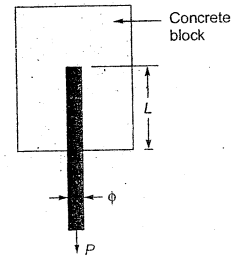
Q.8 The minimum diameter of spirals recommended for splicing in tension members is:
(a) 5 mm (b) 6 mm
(c) 8 mm (d) 10 mm

Q.9 The pitch of the spirals for splicing in tension members should not exceed:
(a) 20 mm (b) 15 mm
(c) 25 mm (d) 100 mm

Q.10 As per IS: 456-2000 recommendations, the design strength of welded splice should be limited to _____ % of design strength of bar in tension splice.
(a) 50 (b) 60
(c) 80 (d) 90

Q.11 The bond strength for M20 concrete as per IS 456: 2000 is 1.2 N/mm². For reinforcing steel of grade Fe 360, the development length in terms of bar diameter (ϕ) is
(a) 47ϕ (b) 45ϕ
(c) 50ϕ (d) 36ϕ

Q.12 A bar of diameter ϕ is embedded in a large concrete block as shown. A force P is being applied to the bar. Let δ_b is the bond strength between the steel and concrete and σ_{st} is the tensile strength of steel bar. Assuming the material of block does not fail, the maximum value of P is



(a) Max.$\left(\frac{\pi}{4}\phi^2\sigma_b, \pi\phi L\sigma_{st}\right)$
(b) Max.$\left(\frac{\pi}{4}\phi^2\sigma_{st}, \pi\phi L\sigma_b\right)$
(c) Min.$\left(\frac{\pi}{4}\phi^2\sigma_{st}, \pi\phi L\sigma_b\right)$
(d) Min.$\left(\frac{\pi}{4}\phi^2\sigma_b, \pi\phi L\sigma_{st}\right)$

Answers

1. (b) 2. (b) 3. (a) 4. (d) 5. (c)
6. (a) 7. (c) 8. (b) 9. (d) 10. (c)
11. (a) 12. (c)

Hints:

4. (d)
(1.6 for deformed bars) × (1.25 for compression)
$\tau_{bd} = 2\tau_{bd}$

11. (a)
$$L_d = \frac{f_{st}\phi}{4\tau_{bd}} = \frac{360\phi}{4\times1.6\times1.2} = 46.875\phi \approx 47\phi$$

Conventional Practice Questions

Q.1 A hook of 20 mm dia. plain bar is embedded in concrete for a length of 150 mm. A load of 15 kN is applied on the hook. If the ultimate bond stress of concrete is 1.2 N/mm², check whether the load is safe to be carried by the hook.

Ans. [Unsafe, the load will fall down]

CHAPTER 08

Design for Torsion in Reinforced Concrete

8.1 Introduction

It is another important *limit state of collapse* just like shear and flexure. Torsion invariably occurs in most of the type of structures. In this chapter, we will define the two different types of torsion that exist in actual structures, the approach followed for the design of members subjected to combined bending/flexure, shear and torsion along with the concept of equivalent moment and equivalent shear.

8.2 Design for Torsion

Loads acting normal to the plane of bending in case of beams and slabs, gives rise to bending moments and shear force. However, when the loads act away from the plane of bending at an eccentricity then this will induce torsion in members. In reinforced concrete, the state of pure torsion rarely exists unlike the case of shafts where we encounter pure torsion. It generally occurs in combination with transverse shear and flexure. Behavior of reinforced concrete members under the combined influence of flexure, shear and torsion is quite complex owing to the fact that concrete is a non-homogeneous material. Presence of cracks in concrete members and composite nature of material i.e. concrete and steel adds to the complexity of torsion analysis. Research in this area is still going on. However, various design codes in the world describe very simplified procedure for torsion design of reinforced concrete members.

Torsional Moment v/s Flexural Moment

FLEXURAL MOMENT	TORSIONAL MOMENT
Flexural/bending moments are distributed among the connected members according to their distribution factors which are proportional to flexural stiffness $\frac{EI}{L}$.	Torsional moments are distributed among the connected members according to their distribution factors which are proportional to torsional stiffness $\frac{GJ}{L}$.

8.3 Mechanism of Torsion in Reinforced Concrete Structures

There are many mechanisms by which torsion can be induced in reinforced concrete members during load transfer stage (during the process of load transfer) in the structural system. Depending upon the situations which induce torsion in reinforced concrete, torsion can be classified as:

1. Equilibrium torsion (or primary torsion)
2. Compatibility torsion (or secondary torsion)

1. **Equilibrium Torsion:** This torsion gets induced in a reinforced concrete member due to eccentric loading w.r.t. shear centre of the cross section. Here, the equilibrium conditions are sufficient in itself to determine the twisting/torsional moments especially in **statically determinate structures**. This torsional component **must be considered in design (as per Cl. 41.4 of IS: 456-2000)** as it is a major component in statically determinate structures. Also, the ends of the member should be suitably designed to resist this type of torsion.

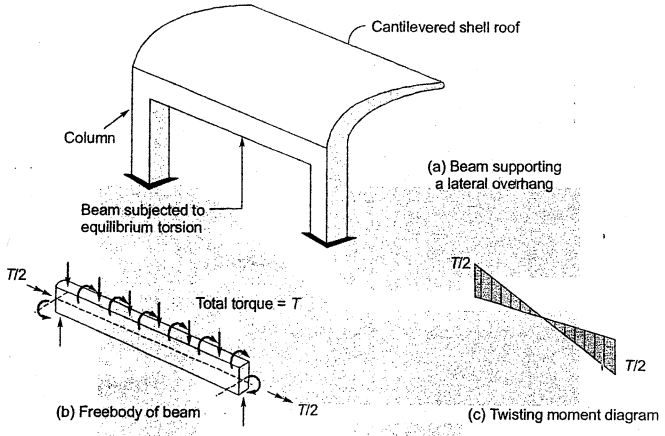


Fig. 8.1 *Concept of Equilibrium Torsion*

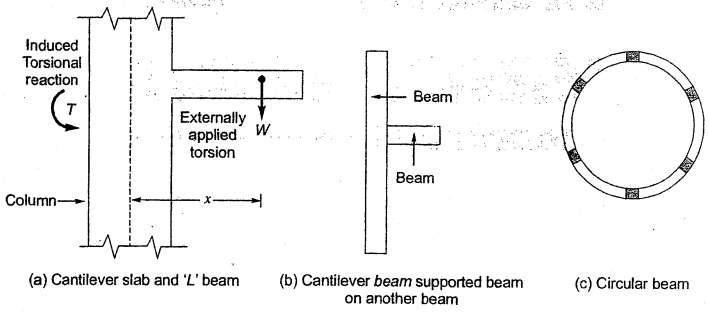


Fig. 8.2 *Simplified explanation of equilibrium torsion*

2. **Compatibility Torsion:** This type of torsion arises due to need of the member to undergo a certain angle of twist to maintain the compatibility conditions. Here, the twisting moments developed are dependent on torsional stiffness of the member and these twisting moments are **statically indeterminate**, minor amounts of which **can be ignored in design due to multiple load paths available in statically indeterminate structures.**

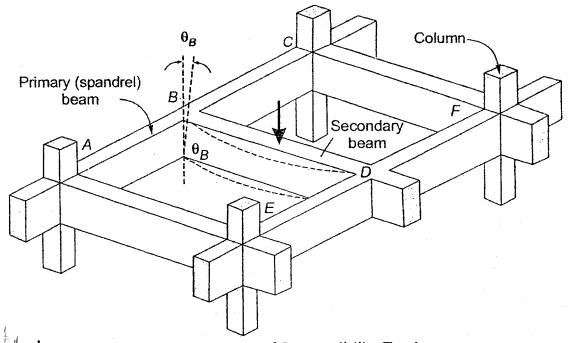


Fig. 8.3 *Concept of Compatibility Torsion*

IS 456: 2000 (Cl. 41.1) states that where the torsion can be eliminated by releasing the redundant restrains, there, no specific design for torsion is necessary provided torsional stiffness is not taken into the analysis of internal stresses.

Remember

Just like *Rupture Moment* for flexural moment, twisting moment (or torsional moment) is associated with ***Cracking Torque*** which implies after the first time loading of plain concrete member, the cracks that develop prior to torsional cracking. A minimum torsional reinforcement is always provided in reinforced concrete members in order to control cracks and impart ductility to the member. Minimum stirrups requirement as per **Cl. 26.5.1.6 of IS 456: 2000** also reinforces the fact that some degree of torsional cracking can be controlled in concrete members due to compatibility torsion.

8.4 Plain Concrete Subjected to Torsion

From the principles of **Solid Mechanics**, it is known that torsion induces shear stresses and causes warping in non-circular sections. **Fig. 8.5** shows the distribution of torsional shear stresses over a rectangular cross section which follows linear elastic behavior.

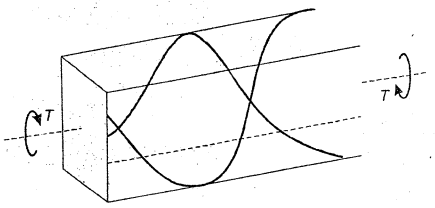


Fig. 8.4 *Plain concrete acted upon by torsional moment*

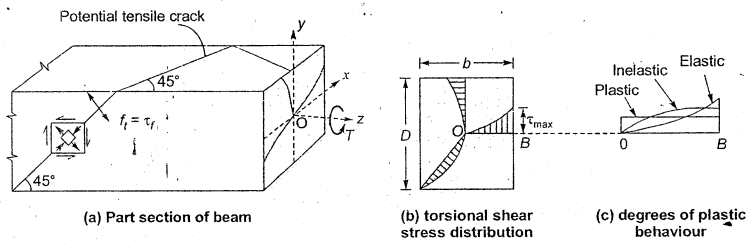


Fig. 8.5 *Distribution of Torsional Shear Stress in Elastic Rectangular Beam Section*

Maximum torsional shear stress occurs at the middle portion of the larger face of the section and is given by:

$$\tau_{tor,\,max} = \frac{T}{\alpha D b^2}$$

where T is the applied torsional moment, b and D are the cross section dimensions and α is a constant which depends on the ratio D/b.

Pure shear induces diagonal tensile and diagonal compressive stresses.

As shown in Fig. 8.5, principle tensile and compressive stress paths spiral around the beam in orthogonal directions at 45 degrees to the beam axis. When this diagonal tensile stress reaches the tensile strength of concrete then cracks start appearing on the section. Due to brittle nature of concrete, this crack penetrates rapidly in the inward direction from the exposed surface which nullifies the torsional resistance of the member.

Thus in plain concrete members, the diagonal torsional cracking leads to the failure of the entire section almost immediately.

The ultimate torsional resistance of the plain concrete can be assessed by measuring **cracking torque** (T_{cr}). The expression for the cracking torque (T_{cr}) can be computed from any of the following proposed theories viz.:

1. Elastic theory
2. Plastic theory
3. Skew bending theory
4. Theory of equivalent tube analogy

The cracking torque (T_{cr}) computed from any of the theory has to be verified and correlated experimentally with the actual tensile strength of concrete. IS 456: 2000 has adopted the **design shear strength** of concrete (τ_c) (table 19 of IS 456: 2000) as the measure of tensile strength of concrete.

8.5 Torsionally Reinforced Concrete Subjected to Torsion

As discussed in the previous section, failure of plain concrete in torsion occurs due to the diagonal tensile stresses and thus to prevent this failure, steel should be provided in the form of spirals around the member in the direction of principle tensile stresses.

Thus **to prevent a beam against torsional failure, torsional reinforcement should be provided in spirals along the direction of principal tensile stress which is in fact not a practical solution**. So, torsional reinforcement is provided in the form of longitudinal reinforcement (as longitudinal bars) and transverse reinforcement (as stirrups).

The twisting behaviour of torsionally reinforced concrete beam is similar to that of plain concrete beam until the formation of first crack (which corresponds to **torsional cracking moment** T_{cr}). After the occurrence of first crack, there is a large increase in twist at constant torque due to abrupt loss of torsional stiffness. After this the torsional behavior of reinforced concrete member depends on the amount of torsional reinforcement present.

Very small amount of torsional reinforcement may not be able to prevent the torsional cracks and thus no increase in strength is possible beyond T_{cr}. As the reinforcement is increased, torsional strength increases but this cannot be done indefinitely because ultimate failure occurs by crushing of concrete. Increasing the torsional reinforcement increases the ductile behavior but this is felt at very large angle of twists.

Consequences of Torsional Moment

1. Torsional moments are associated with shear stresses in beams since torsion causes diagonal tension thereby leading to spiral cracks.
2. Due to spiral cracks, reinforcement should be provided in the form of spirals along the direction of principal tensile stresses but this is often not possible. Thus the requirement of spiral reinforcement is converted to an equivalent longitudinal and transverse reinforcement. **Thus effect of torsion is split into equivalent shear and equivalent moment.**

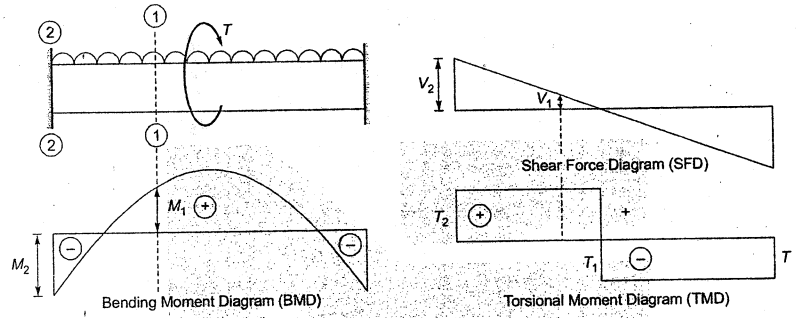


Fig. 8.6 *Variation of BM, SF and TM diagram in a fixed beam*

8.6 Analysis for Torsion

There are many methods available to understand the behavior of reinforced concrete members under torsion like **Skew Bending Theory, Space-Truss Analogy** etc.

8.7 Torsional Reinforcement

When torsional shear stress (τ_t) exceeds the design shear strength of concrete (τ_c) **(table 19 of IS 456: 2000)**, then torsional reinforcement is required to be provided in concrete members. As stated earlier, normally torsional shear acts in association with flexural shear (V_u) and in that case **equivalent shear** (V_e) has to be considered as per **Cl. 41.3.1 of IS 456: 2000.**

The expression for equivalent shear (V_e) is given as:

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right)$$

Remember: The flexural shear and torsional shear are additive only on one side of the beam and they act in opposite directions on the other side of the beam.

The equivalent nominal shear stress (τ_{ve}) is then given by:

$$\tau_{ve} = \frac{V_u + 1.6\left(\frac{T_u}{b}\right)}{bd}$$

If this equivalent nominal shear stress (τ_{ve}) exceeds the maximum shear strength of concrete ($\tau_{c\,max}$), then section has to be redesigned.

The strength of a member subjected to both torsion and flexure is described in terms of interaction of T_u/T_{uR} with M_u/M_{uR}. Where, T_{uR} and M_{uR} are respectively the strength of the member subjected to pure torsion and pure flexure.

Cl. 41.4.2 of IS 456: 2000 gives recommendations for the design of concrete members subjected to both flexure and torsion. This recommendation is based on simplified **Skew Bending Theory**.

Accordingly, the torsional moment can be converted to an equivalent flexural moment as:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right)$$

Here the M_t so calculated is combined with the flexural moment (M_u) to give equivalent bending moments M_{e1} and M_{e2} as:

$$M_{e1} = M_t + M_u$$
$$M_{e2} = M_t - M_u$$

Reinforcement is designed to resist the equivalent bending moment M_{e1} and the corresponding required steel is provided in the flexural tension zone.

Now when $M_t > M_u$ i.e. $M_{e2} > 0$, then a reinforcement for this equivalent moment (M_{e2}) has also to be designed and is provided in flexural compression zone.

When $M_u = 0$ i.e. in case of **pure torsion**, then $M_{e1} = M_{e2} = M_t$ and equal longitudinal reinforcement has to be provided in the both flexural tension zone and flexural compression zone.

8.8 IS 456: 2000 Provisions for the Design of Reinforcement in Members Subjected to Torsion

1. **Cl. 41.4.3 of IS 456: 2000** provides recommendations for the design of **2 legged, closed transverse stirrup reinforcement** the area of which is given by:

$$A_{sv} = \frac{T_u S_v}{0.87 f_y b_1 d_1} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)} = \frac{S_v}{0.87 f_y d_1}\left(\frac{\tau_u}{b_1} + \frac{V_u}{2.5}\right)$$

2. In addition to the above, **Cl. 41.4.3 of IS 456: 2000** specifies limit on minimum area of transverse reinforcement also as:

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b . S_v}{0.87 f_y} \quad \text{or} \quad \frac{A_{sv}}{b S_v} \geq \frac{(\tau_{ve} - \tau_c)}{0.87 f_y}$$

3. As per **Cl. 41.1 of IS 456: 2000,** in structures where torsion is required to maintain the equilibrium, members shall be designed for torsion. However where torsion can be eliminated by releasing the redundant restraints, no specific design for torsion is necessary provided torsional stiffness is neglected in the calculation of internal forces. There, adequate control of any torsional cracking is being taken care of by the required nominal shear reinforcement.
4. **Cl. 41.2** of **IS 456: 2000** states that sections located at less than distance '*d*' from the face of the support may be designed for the same torsion as computed at a distance '*d*'.

5. **Cl. 26.5.1.7a of IS 456: 2000** specifies the distribution of torsional reinforcement in terms of maximum spacing of stirrups (s_v), in order to have sufficient post crack torsional resistance and to control crack widths.

$$s_v \leq \begin{cases} x_1 \\ \dfrac{x_1 + y_1}{4} \\ 300\,mm \end{cases}$$

where, $x_1 = b_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

$y_1 = d_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

} shown in Fig.8.7

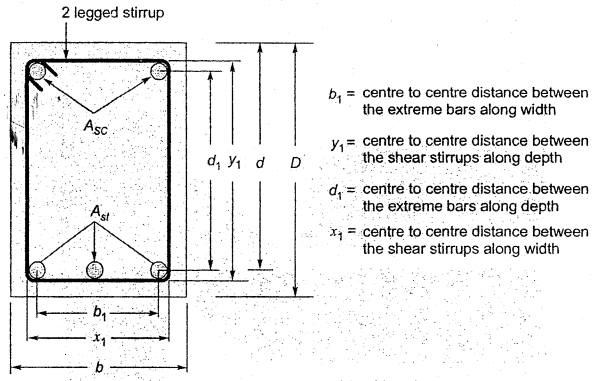


Fig. 8.7 *Stirrups in beams*

6. **Cl. 26.5.1.7b of IS 456: 2000** states that *longitudinal reinforcement shall be placed as close as possible to the corners of the cross section and in all cases, there shall be atleast one longitudinal bar at each corner of ties.*
7. **If depth of the member subjected to torsion exceeds 450 mm,** then additional longitudinal bars are required to be provided at the faces with total area not less than 0.1% of web area. These side face bars are to be distributed equally on each face at a spacing not exceeding 300 mm or web thickness, whichever is small.

8.9 Design for Torsion as per Working Stress Method

1. **Shear and Torsion: Cl. 41.3.1** states that equivalent shear taking into account the effect of torsion, is given by:

$$V_e = V + 1.6\left(\frac{T}{B}\right)$$

where, T = Torsional moment
V = Shear force
V_e = Equivalent shear force
B = Beam width

Thus equivalent nominal shear stress is given by:

$$\tau_{ve} = \frac{V_e}{Bd} \leq \tau_{c_{max}}$$

2. **Longitudinal Reinforcement:** Longitudinal reinforcement must be designed to resist an equivalent bending moment M_{e1} given by:

$M_{e1} = M + M_t$ where, M = Flexural / Bending moment at the cross section under working conditions

$$M_t = \left(\frac{T}{1.7}\right)\left(1+\frac{D}{B}\right)$$

Case 1: When $M_t < M$

In this case, only tension reinforcement is designed without the requirement of any compression reinforcement.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \frac{M_{e_1}}{\sigma_{st} \cdot jd} = \frac{M_{e_1}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$

Case 2: When $M_t > M$

In this case, longitudinal reinforcement shall be provided on flexural compression face, such that the beam can also withstand an equivalent moment M_{e2} (= $M_t - M$) where moment M_{e2} being taken as acting in the opposite sense to the moment M_x.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \frac{M_{e1}}{\sigma_{st} \cdot jd} = \frac{M_{e1}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$

Area of compression steel (A_{sc}) is also designed for a moment M_{e2} (= $M_t - M$) taken in opposite sense to that of M_x.

(Design tension reinforcement for M_{e2} and provide as negative (–ve) reinforcement).

$$A_{st2} = \frac{M_{e2}}{\sigma_{st} \cdot jd} = \frac{M_{e2}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$$

Thus, provide this much reinforcement at top.

As per Cl. 28.5.1.2 of IS 456: 2000, area of compression reinforcement should be less than or equal to 4% of gross sectional area of beam.

8.9.1 Shear/Transverse Reinforcement

IS 456: 2000 states that two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross section A_{sv} given by:

$$A_{sv} = \frac{TS_v}{b_1 d_1 \sigma_{SV}} + \frac{VS_v}{2.5 d_1 \sigma_{SV}} = \frac{S_v}{d_1 \sigma_{SV}}\left[\frac{T}{b_1} + \frac{V}{2.5}\right]$$

But total reinforcement shall not be less than **minimum shear reinforcement which is given by:**

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{\sigma_{SV}}, \quad \frac{A_{sv}}{bs_v} \geq \frac{(\tau_{ve} - \tau_c)}{\sigma_{SV}}$$

Spacing of shear reinforcement is given by:

$$S_V = \frac{A_{sv}\sigma_{SV}d_1}{\left[\frac{T}{b_1} + \frac{V}{2.5}\right]} = \frac{A_{sv}\sigma_{SV}d_1}{V_{seq}} \qquad \text{where,} \quad V_{Seq} = \frac{T}{b_1} + \frac{V}{2.5}$$

where, S_V = spacing of shear reinforcement

b_1 = centre to centre distance between corner bars in the direction of width

= b – clear cover – 2 × stirrups dia – 2 (longitudinal bar dia/2)

d_1 = centre to centre distance between corner bars

= D – clear cover – 2 × stirrups dia – 2 (longitudinal bar dia/2)

τ_{ve} = equivalent shear stress

τ_c = design shear strength of concrete as per Table 19 of IS: 456-2000

Maximum Spacing

$$S_v \leq \begin{cases} x_1 \\ \frac{x_1 + y_1}{4} \\ 300\ mm \end{cases}$$

where, $x_1 = b_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

$y_1 = d_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

Side face reinforcement

Side face reinforcement shall be provided as per Cl. 26.5.1.3 and 26.5.17(6) of IS 456: 2000.

1. Beam depth > 450 mm (if beam subjected to torsion)
2. Beam depth > 750 mm (if beam not subjected to torsion)

Provide @ 0.1% of web area and distribute it equally on both side faces.

8.10 Design for Torsion as per Limit State Method

Factored bending moment = 1.5 $M = M_u$

Factored shear force = 1.5 $V = V_u$

Factored torsion moment = 1.5 $T = T_u$

Shear and torsion: Cl. 41.3.1 states that equivalent shear taking into account the effect of torsion, is given by:

$$V_e = V_u + 1.6\left(\frac{T_u}{B}\right)$$

where, T_u = Factored torsional moment, V_u = Factored shear

V_e = Equivalent shear, B = Beam width

Thus equivalent nominal shear stress is given by:

$$\tau_{ve} = \frac{V_e}{Bd} \leq \tau_{c\,max}$$

Longitudinal Reinforcement: Cl. 41.4.2 states that longitudinal reinforcement must be designed to resist an equivalent bending moment M_{e1} given by:

$$M_{e1} = M_u + M_t$$

where, M_u = Factored bending moment at the cross section

Case 1: When $M_t < M_u$

In this case, only tension reinforcement is designed without the requirement of any compression reinforcement.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \frac{M_{e1}}{0.87f_y \cdot jd}$

Case 2: When $M_t > M_u$ (Cl. 41.4.2.1 of IS 456: 2000)

In this case, longitudinal reinforcement shall be provided on flexural compression face, such that the beam can also withstand an equivalent moment M_{e2} (= $M_t - M_u$) where, moment M_{e2} being taken as acting in the opposite sense to the moment M_u.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (Ast) is given by: $A_{st} = \frac{M_{e1}}{0.87f_y \cdot jd}$

Area of compression steel (A_{sc}) is also provided for M_{e2} (=$M_t - M_u$).

$$A_{st2} = \frac{M_{e2}}{0.87f_y \cdot jd}$$

Provide this much reinforcement at top.

As per **Cl. 26.5.1.2 of IS 456: 2000**, area of compression reinforcement area should not exceed 4% of gross sectional area of beam.

Shear/Transverse reinforcement: Cl. 41.4.3 of IS 456: 2000 states that two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross section A_{sv} given by:

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87f_y)} = \frac{S_v}{d_1(0.87f_y)}\left[\frac{T_u}{b_1} + \frac{V_u}{2.5}\right]$$

$$\Rightarrow \quad S_v = \frac{A_{sv}(0.87f_y)d_1}{\frac{T_u}{b_1} + \frac{V_u}{2.5}}$$

But total reinforcement shall not be less than **minimum shear reinforcement** which is given by:

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{0.87f_y}, \qquad \frac{A_{sv}}{bs_v} \geq \frac{(\tau_{ve} - \tau_c)}{0.87f_y}$$

Spacing of shear reinforcement is given by:

$$S_v = \frac{A_{sv}(0.87f_y)d_1}{\left[\frac{T_u}{b_1} + \frac{V_u}{2.5}\right]} = \frac{A_{sv}(0.87f_y)d_1}{V_{seq}}$$

where $V_{seq} = \frac{T_u}{b_1} + \frac{V_u}{2.5}$ = Equivalent shear force

S_v = spacing of shear reinforcement

b_1 = centre to centre distance between corner bars in the direction of width

= b – clear cover – 2 x stirrups dia – 2 (longitudinal bar dia/2)

d_1 = centre to centre distance between corner bars

= D – clear cover – 2 x stirrups dia – 2 (longitudinal bar dia/2)

τ_{ve} = equivalent shear stress

τ_c = design shear strength of concrete as per **Table 19 of IS: 456-2000**

Maximum spacing

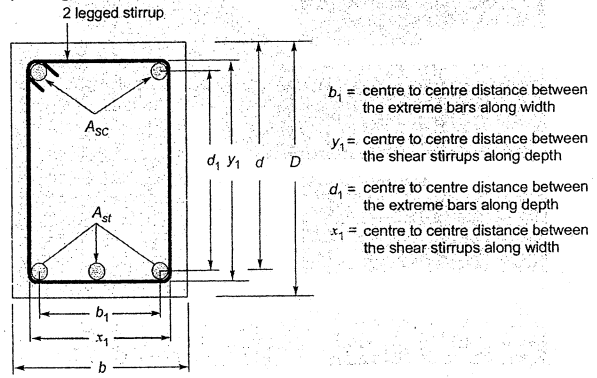


Fig. 8.8 *Stirrups in beams*

$$s_v \leq \begin{cases} x_1 \\ \frac{x_1 + y_1}{4} \\ 300\ mm \end{cases}$$

where, x_1 = b_1 + 2 (longitudinal bar dia/2) + 2 (stirrups dia/2)

y_1 = d_1 + 2 (longitudinal bar dia/2) + 2 (stirrups dia/2)

Example 8.1 Design a rectangular beam of size 350 x 750 mm which is acted upon by a factored twisting moment of 150 kNm in combination with an ultimate negative moment of 210 kNm and an ultimate shear force of 110 kN. Use M 30 grade of concrete and Fe 415 steel.

Solution:

Given,

b	= 350 mm	D	= 750 mm
f_{ck}	= 30 N/mm^2	f_y	= 415 N/mm^2
T_u	= 150 kNm	M_u	= 210 kNm
V_u	= 110 kNm		

Let effective cover to the reinforcing bars = 50 mm

Thus effective depth, d = 750 mm – 50 mm = 700 mm

Equivalent bending moment due to torsion:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right) = 150/1.7 \times (1 + 750/350) = 277.31 \text{ kNm}$$

Bending moment for design:

$$M_e = M_t \pm M_u$$
$$= 277.31 \pm 210 = 487.31 \text{ kNm} \quad \text{(flexural tension at bottom)}$$
$$= 67.31 \text{ kNm} \quad \text{(flexural compression at top)}$$

Design of bottom reinforcement:

$$R_1 = \frac{M_{e1}}{bd^2} = 487.31 \times 10^6/(350) \times (700)^2 = 2.84146 \text{ N/mm}^2$$

and $$\frac{M_{u,lim}}{bd^2} = 0.138 \times 30 = 4.14 \text{ N/mm}^2 > 2.84145 \text{ N/mm}^2$$

Thus $$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_1}{f_{ck}}}\right] = 8.98798 \times 10^{-3}$$

$$p_t = 0.89\%$$

$$A_{st,reqd} = 2202.06 \text{ mm}^2$$

Provide 5-25 mm diameter bars at the bottom.

Thus $A_{st,\,provided}$ = 2454.37 mm²

Design of top reinforcement:

$$R_2 = M_{e2}/bd^2 = 67.31 \times 10^6/(350) \times (700)^2 = 0.392478 \text{ N/mm}^2$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_2}{f_{ck}}}\right] = 1.10398 \times 10^{-3}$$

$$p_t = 0.11\ \%$$

$$A_{st,reqd} = \frac{0.110398}{100} \times 350 \times 700 = 270.48 \text{ mm}^2$$

Provide 2-16 mm diameter bars at the top.

Thus $A_{st,\,provided}$ = 402.12 mm²

Side face reinforcement:

Since depth of the beam (D) is greater than 450 mm, side face reinforcement @ 0.1 % of beam cross sectional area, for torsion is required to be provided.

Thus torsional reinforcement on each face = 0.05 % = $\left(\frac{0.05}{100}\right) \times 350 \times 750 = 131.25 \text{ mm}^2$

Provide 2-10 mm diameter bars on each face.

Side face reinforcement provided on each face = 157.08 mm²

Vertical spacing between the longitudinal bars should not exceed 300 mm.

Design of shear/transverse reinforcement:

$$\text{Equivalent nominal shear stress} = \tau_{ve} = \frac{V_u + 1.6\frac{T_u}{b}}{bd}$$

$$= \frac{110\times10^3 + 1.6\left(\frac{150\times10^6}{350}\right)}{350\times700} = 3.24 \text{ N/mm}^2$$

$\tau_{c,\,max}$ (for M30 concrete) = 3.5 N/mm² > 3.2478 N/mm² (OK)

Now for $p_t = \frac{2454.37\times100}{350\times700}$ = 1 % and M30 concrete, τ_c = 0.66 N/mm²

It can be seen that torsional shear stress (= 3.24 N/mm²) is much higher than design shear strength of concrete (= 0.66 N/mm²).

Using 2 legged 10 mm diameter bars as shear stirrups, A_{sv} = 2 × 78.54 mm² = 157 mm²

Since effective cover assumed is 50 mm.

So d_1 = 750 mm – 2 × 50 mm = 650 mm

b_1 = 350 mm – 2 × 50 mm = 250 mm

$$s_v = \frac{0.87f_yA_{sv}}{\frac{T_u}{b_1d_1} + \frac{V_u}{2.5d_1}} = \frac{0.87(415)157}{\frac{150\times10^6}{250\times650} + \frac{110\times10^3}{2.5\times650}}$$

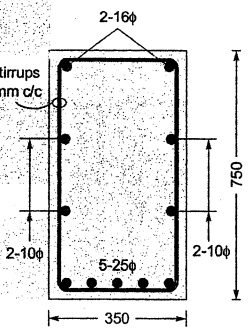
= 57.21 mm

Maximum shear spacing requirement:

$$s_v \le \begin{cases} x_1 = 250+25+10 = 285 \text{ mm} \\ \frac{x_1+y_1}{4} = \frac{285+(650+12.5+8+10)}{4} \\ 300 \text{ mm} \end{cases}$$

= 241.375 mm

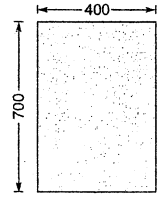
Provide 2 legged 10 mm diameter bars as shear stirrups @ 50 mm c/c.



Check for clear cover:

Clear cover = 50 – 10 – 14 mm = 26 mm > 20 mm (O.K.)

Example 8.2 Determine the amount of reinforcement required for a beam of size 400 mm x 700 mm and subjected to a working moment of 150 kNm, twisting moment of 60 kNm and working shear force of 80 kN. Use M 20 concrete and Fe 415 steel.



Solution:

Let effective cover = 50 mm

∴ Effective depth (d) = 700 mm – 50 mm = 650 mm

Step 1: Check for adequacy of beam depth

$$\text{Equivalent shear force } (V_e) = V_u + 1.6\frac{T_u}{b}$$
$$= (1.5\times80) + 1.6\left(\frac{1.5\times60}{0.4}\right)$$
$$= 120 + 360 = 480 \text{ kN}$$

$\therefore$ Equivalent shear stress $(\tau_{ve}) = \dfrac{V_e}{bd} = \dfrac{480\times10^3}{400\times650} = 1.846\ \text{N/mm}^2$

For M-20 concrete, $\tau_{max} = 2.8\ \text{N/mm}^2$ (Table 20 of IS 456: 2000)

$> \tau_{ve}\ (= 1.846\ \text{N/mm}^2)$ (OK)

Thus depth provided is OK.

Step 2: Check for requirement of shear reinforcement

Let percentage of tension reinforcement $(p_t) = 0.5\%$

For M 20 concrete and $p_t = 0.5\%$, design shear stress of concrete

$\tau_c = 0.48\ \text{N/mm}^2$ (Table 19 of IS 456: 2000)

$\because \quad \tau_c < \tau_{ve} < \tau_{max}$

$\therefore$ Shear reinforcement is required.

Step 3: Tension reinforcement requirement

Equivalent bending moment $(M_e) = M_u + M_t$

$$= (1.5\times150) + \frac{T_u}{1.7}\left(1+\frac{D}{b}\right)$$

$$= 225 + \frac{1.5\times60}{1.7}\left(1+\frac{700}{400}\right) = 225 + 145.6 = 370.6\ \text{kNm}$$

$$\therefore \quad R = \frac{M_e}{bd^2} = \frac{370.6\times10^6}{400\times650^2} = 2.1929$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{20}{2(415)}\left[1-\sqrt{1-4.598\times\frac{2.1929}{20}}\right] = 0.0071285$$

$\therefore \quad p_t = 0.71\% > 0.5\%$ (assumed)

$\Rightarrow \quad \tau_c = 0.5472\ \text{N/mm}^2$ for M 20 concrete

and $p_t = 0.71\%$ (Table 19 of IS: 456 - 2000)

From here also, $\tau_c > \tau_{ve} < \tau_{max}$

$\Rightarrow$ Shear reinforcement is required.

Also $A_{st} = 0.0071\times400\times650 = 1846\ \text{mm}^2$

$\therefore$ No. of 25 mm ϕ bars required $= \dfrac{1846}{\frac{\pi}{4}\times25^2} = 3.76 = 4$ bars (say)

$\therefore \quad p_{t\,(\text{provided})} = \dfrac{4\times\frac{\pi}{4}\times25^2}{400\times650}\times100 = 0.76\%$

$\Rightarrow \quad \tau_c = 0.5624\ \text{N/mm}^2$ (Table 19 of IS: 456 - 2000)

Minimum reinforcement percentage $= \left(\dfrac{0.85}{f_y}\right)\times1000 = \left(\dfrac{0.85}{415}\right)\times100 = 0.20\% < 0.76\%$ (OK)

Maximum reinforcement percentage $(P_{t\,(max)}) = 4\% > 1.76\%$ (OK)

Step 4: Compression reinforcement requirement

From Step-3 above, $M_t = 145.6\ \text{kNm} < M_u\ (= 225\ \text{kNm})$

$\therefore$ No compression reinforcement is required.

However provide 2-12ϕ bars as hanger bars for holding the transverse reinforcement.

Step 5: Side face reinforcement requirement

$\because \quad D > 450\ \text{mm}$

$\therefore$ Side face reinforcement is required @ 0.1% of bD

$$= \frac{0.1}{100}\times400\times700 = 280\ \text{mm}^2 \text{ equally distributed on the side face}$$

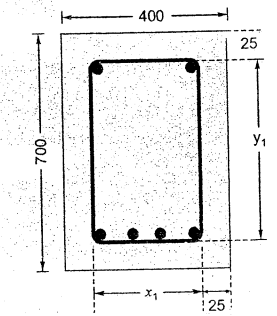
$\therefore$ Side face reinforcement on one face $= \dfrac{280}{2}\text{mm}^2 = 140\,\text{mm}^2$

Providing 2-10ϕ bars, area of side face reinforcement provided

$$= 2\times\frac{\pi}{4}\times10^2 = 157\,\text{mm}^2 > 140\ \text{mm}^2 \quad \text{(OK)}$$

Step 6: Transverse reinforcement

Using 2 legged 10ϕ bars as transverse reinforcement



$d_1 = 700 - 50 - 50 = 600\ \text{mm}$

$b_1 = 400 - (25 + 10 + 12.5) - (25 + 10 + 12.5)$

$= 305\ \text{mm}$

$$\therefore \quad \frac{0.87 f_y A_{sv}}{s_v} = \frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}$$

$$= \frac{1.5\times60\times10^6}{305\times600} + \frac{1.5\times80\times10^3}{2.5\times600}$$

$= 491.8 + 80 = 571.8\ \text{N/mm} \quad ...(i)$

Also, $A_{sv} \geq (\tau_{ve} - \tau_c)\dfrac{b.s_v}{0.87 f_y} \geq (1.846 - 0.5624)\dfrac{400\,s_v}{0.87(415)}$

$$\Rightarrow \quad \frac{0.87 f_y A_{sv}}{s_v} \geq (1.846 - 0.5624)\,400 = 513.44\ \text{N/mm} \quad ...(ii)$$

Comparing equations (i) and (ii), it is clear that equation (i) is governing equation.

$$\therefore \quad \frac{0.87 f_y A_{sv}}{s_v} = 571.8\ \text{N/mm}$$

With 2 legged 10ϕ bars (as assumed),

$$A_{sv} = 2\times\frac{\pi}{4}\times10^2 = 157\ \text{mm}^2$$

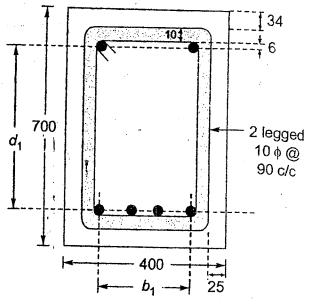
$$\therefore \quad s_v = \frac{0.87 f_y A_{st}}{571.8} = \frac{0.87(415)157}{571.8} = 99.13\,\text{mm } c/c$$

Step 7: Check for adequacy of s_v

$x_1 = b_1 + (12.5 + 12.5) + (5 + 5)$
$= 305 + 25 + 10 = 340$ mm

$y_1 = d_1 + (6 + 12.5) + (5 + 5)$
$= 600 + 18.5 + 10 = 628.5$ mm

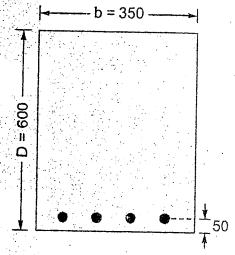
$$s_v \le \begin{bmatrix} x_1 \\ \frac{(x_1 + y_1)}{4} \\ 300\,\text{mm} \end{bmatrix} = \begin{bmatrix} 340\,\text{mm} \\ \frac{340 + 628.5}{4} \\ 300\,\text{mm} \end{bmatrix} = 242.125\,\text{mm}$$



$\therefore$ $s_v \le 242.125$ mm

$\therefore$ Provide 2 legged 10ϕ stirrups @ 90 c/c.

Example 8.3 A rectangular beam as shown below is subjected to a moment of 174 kNm, torsion of 35 kNm and shear force of 88 kN at service load. Determine the reinforcement required for the beam. use M 20 and Fe 415.



Solution:

Given
Moment (M) = 174 kN
Torsion (T) = 35 kNm
SF (V) = 88 kN

At service load conditions

Thus
Factored moment (M_u) = 1.5 × 174 = 361 kNm
Factored torsion (T_u) = 1.5 × 35 = 52.5 kNm
Factored shear force (V_u) = 1.5 × 88 = 132 kN

Check for adequacy of beam depth

$$\text{Equivalent shear force } (V_e) = V_u + 1.6\left(\frac{T_u}{b}\right) = 132 + 1.6\left(\frac{52.5}{\frac{350}{1000}}\right) = 372\,\text{kN}$$

$$\text{Equivalent shear stress } (\tau_v) = \frac{V_e}{bd} = \frac{372 \times 1000}{350 \times (600 - 50)} = 1.932\,\text{N/mm}^2$$

As per table 20 of IS 456: 2000, for M 20,

$\tau_{c\,max}$ = 2.8 N/mm2
> τ_v (= 1.932 N/mm²)

Thus there is no need to redesign the section.
Thus depth of beam is OK.

Requirement of shear reinforcement

Let percentage of tension reinforcement
(p_t) = 0.5 %

$\therefore$ Design shear stress of M 20 concrete for
p_t = 0.5 %
as per Table 19 of IS 456: 2000, τ_c = 0.48 N/mm² < τ_v (1.932 N/mm²)
Thus shear reinforcement is required.

Requirement of flexural reinforcement

Equivalent bending moment $(M_e) = M_u + M_t = M_u + \left(\frac{T_u}{1.7}\right)\left(1 + \frac{D}{b}\right)$

$= 361 + \left(\frac{52.5}{1.7}\right)\left(1 + \frac{0.6}{0.35}\right) = 261 + 83.82 = 344.82$ kNm

$\therefore$ $R = \frac{M_e}{bd^2} = \frac{344.82 \times 10^6}{350(550)^2} = 3.25686$ N/mm²

$\therefore$ $\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{20}{2(415)}\left[1 - \sqrt{1 - \frac{4.598(3.25686)}{20}}\right] = 0.012018$

$\therefore$ p_t = 1.2 %
$\Rightarrow$ A_{st} = 2313.5 mm²
But we assumed p_t = 0.5 % (initially)

Design shear strength of M 20 concrete for
p_t = 1.2 %
as per table 19 of IS 456: 2000, τ_c = 0.65 N/mm² < τ_v (=1.932 N/mm²)

Compression reinforcement

Here M_t = 83.82 kNm
< M_u (= 261 kNm)

Thus no compression reinforcement is required

Side face reinforcement

Depth of the beam (D) = 600 mm > 450 mm
So side face reinforcement is required
Total area of side face reinforcement ≥ 0.1 % of web area
≥ $\frac{0.1}{100}(350)(600)$
≥ 210 mm²

Provide two 10 ϕ bar on each face of the beam,

Total area = $4 \times \frac{\pi}{4} \times 10^2 = 314.16\,\text{mm}^2 > 210\,\text{mm}^2$

Example 8.4 Design a rectangular reinforced concrete beam of width 400 mm subjected to the following at any particular section:
(i) Bending moment = 300 kNm (ii) Shear force = 200 kN (iii) Torsional moment = 100 kNm
Use M 30 concrete and Fe 415 steel.
Design the beam using WSM.

Solution:

By WSM
Width of the beam, B = 400 mm
Assume overall depth of the beam, D = 800 mm
and effective cover = 50 mm
Effective depth of the beam = 800 – 50 mm = 750 mm

Equivalent shear

$$V_e = V + \frac{1.6T}{B} = 200 + \frac{1.6 \times 100}{0.4}\,\text{kN} = 600\,\text{kN}$$

Nominal shear stress, $\tau_{ve} = \frac{V_e}{Bd} = \frac{600\times10^3}{400\times750}\text{N/mm}^2$

$= 2\text{ N/mm}^2 < \tau_{c\,max}$ (= 2.2 N/mm² for M 30 concrete)

Equivalent moment

$M_e = M + M_T$

$= 300 + \frac{T}{1.7}\left(1+\frac{D}{B}\right) = 300 + 176.47\text{ kNm}$

$= 300 + \frac{100}{1.7}\left(1+\frac{0.8}{0.4}\right)\text{kNm} = 476.47\text{ kNm}$

Now $M_T < M$

∴ Reinforcement on compression side is not required

Depth required

$k = \frac{mc}{t+mc} = \frac{9\times10}{230+9\times10} = 0.28$

$j = 1-\frac{k}{3} = 1-\frac{0.28}{3} = 0.906$

$Q = \frac{1}{2}cjk = \frac{1}{2}\times10\times0.906\times0.28 = 1.27$

$d = \sqrt{\frac{M_e}{BQ}} = \sqrt{\frac{476.47\times10^6}{400\times1.27}} = 968.47\text{ mm}$

∴ $D = 968.47 + 50\text{ mm}$

$= 1018.47\text{ mm} \simeq 1020\text{ mm} > 800\text{ mm}$ (Not OK)

Take $D = 1050\text{ mm}$

∴ $d = 1050 - 50\text{ mm} = 1000\text{ mm}$

$M_e = M + M_T = M + \frac{T}{1.7}\left(1+\frac{D}{B}\right)$

$= 300 + \frac{100}{1.7}\times\left(1+\frac{1050}{400}\right)$

$= 300 + 213.23\text{ kNm} = 513.23\text{ kNm}$

∴ $d = \sqrt{\frac{M_e}{QB}} = \sqrt{\frac{513.23\times10^6}{1.27\times400}}\text{ mm} = 1005\text{ mm}$

Provide $d = 1010\text{ mm}$

So that $D = 1010 + 50\text{ mm} = 1060\text{ mm}$

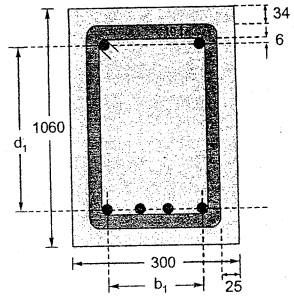
$M_e = 300 + \frac{100}{1.7}\left(1+\frac{1.060}{0.4}\right) = 514.7\text{ kNm}$

∴ $d = \sqrt{\frac{M_e}{QB}} = \sqrt{\frac{514.7\times10^6}{1.27\times400}}\text{ mm} = 1006\text{ mm}$

Area of steel required

$A_{st} = \frac{M_e}{\sigma_{st}jd} = \frac{514.7\times10^6}{230\times0.906\times1010}\text{mm}^2 = 2431.1\text{ mm}^2$

∴ No. of 25 mm dia. bars required = $\frac{2431.1}{\frac{\pi}{4}\times25^2} = 4.95 = 5$ (say)



Provide 5-25 mm dia. bars

$b_1 = B - 100 = 400 - 100 = 300\text{ mm}$

$d_1 = D - 100 = 1060 - 100 = 960\text{ mm}$

$x_1 = 300 + (25 + 5 + 5) = 300 + 35 = 335\text{ mm}$

$y_1 = 960 + (25 + 5 + 5) = 960 + 35 = 995\text{ mm}$

Design of shear reinforcement

Using 2-legged 8 mm dia. stirrups,

Spacing, $s_v = \frac{A_{sv}\sigma_{sv}d_1}{\left(\frac{V}{2.5}+\frac{T}{b_1}\right)} = \frac{\frac{\pi}{4}\times2\times8^2\times230\times960}{\left(\frac{200\times10^3}{2.5}+\frac{100\times10^6}{300}\right)} = 53.7\text{ mm}$

With 10 mm dia. bar

$s_v = \frac{10^2}{8^2}\times53.7 = 83.9\text{ mm}$

With 2-legged 12 mm dia. stirrups

$s_v = \frac{12^2}{8^2}\times53.7 = 120.8\text{mm}$

Minimum shear reinforcement

$A_{sv} \not< \frac{(\tau_{ve}-\tau_c)Bs_v}{\sigma_{sv}}$

$\frac{A_{sv}}{Bs_v} \not< \frac{\tau_{ve}-\tau_c}{\sigma_{sv}}$

$\tau_{ve} = \frac{600\times10^3}{400\times1010} = 1.48\text{ N/mm}^2$

Percentage of steel, $p_t = \frac{5\times\frac{\pi}{4}\times25^2\times100}{400\times1010} = 0.60\%$

∴ $\tau_c = 0.334\text{ N/mm}^2$

∴ $s_v = \frac{A_{sv}\sigma_{sv}}{(\tau_{ve}-\tau_c)B} = \frac{2\times\frac{\pi}{4}\times12^2\times230}{(1.48-0.334)\times400} = 113.5\text{ mm}$

Maximum spacing

$s_v \le \begin{cases} x_1 = 335\text{ mm} \\ \frac{x_1+y_1}{2} = \frac{335+995}{4} = 332.5\text{ mm} \\ = 300\text{ mm} \end{cases}$ (whichever is less.)

Provide 2-legged 12 mm dia. stirrup @ 110 mm c/c

Side face reinforcement

∵ $D > 450\text{ mm}$

∴ Side face reinforcement is required @ 0.1% = $\frac{0.1}{100}\times400\times1060 = 424\text{ mm}^2$

No. of 10 mm dia. bars required = $\frac{424}{\frac{\pi}{4}\times10^2} = 5.398 \approx 6$

Provide 3-10 mm dia. bars on each face

Spacing $= \frac{d_1}{4} = \frac{960}{4} = 240 \text{ mm} < 300 \text{ mm}$ (OK)

Objective Brain Teasers

Q.1 As per IS 456: 2000, which of the following type of torsion must be considered in the design of reinforced concrete members?
(a) Equilibrium torsion
(b) Compatibility torsion
(c) Both (a) and (b)
(d) No torsion need to be considered for design

Q.2 Minor amounts of which of the following torsion can be ignored in design due to the availability of multiple load paths?
(a) Equilibrium torsion
(b) Compatibility torsion
(c) Both (a) and (b)
(d) None of (a) or (b)

Q.3 Side face reinforcement shall be provided in beams if:
(i) Beam depth exceeds 750 mm without torsion
(ii) Beam depth exceeds 450 mm without torsion
(iii) Beam depth exceeds 450 mm with torsion
(iv) Beam depth exceeds 750 mm with torsion
Among the above statements, true one(s) is/are:
(a) (i) and (iii) (b) (i) and (ii)
(c) (ii) and (iv) (d) (iii) and (iv)

Q.4 The torsion moment in a reinforced concrete beam needs to be converted to an equivalent moment and equivalent shear because:
(a) It is economical to provide reinforcement for moment and shear than for torsion directly.
(b) It is recommended by the principles of Structural Analysis.
(c) It is difficult to provide inclined stirrups to check torsional cracks.
(d) All of the above

Q.5 In a reinforced concrete rectangular beam, the flexural shear and the torsional shear are:
(a) Additive only on one side of the beam
(b) Additive on both the sides of beam
(c) Additive on one or both the sides of beam depending on the type of loading
(d) Cannot be said with certainty

Q.6 Compression reinforcement in a beam subjected to torsion shall be provided when:
(a) Equivalent flexural moment due to torsion exceeds the flexural moment on beam.
(b) Equivalent flexural moment due to torsion is less than the flexural moment on beam.
(c) It is provided in all cases irrespective of the value of equivalent flexural moment due to torsion.
(d) None of the above

Q.7 The horizontal distance between two parallel reinforcing bars in a beam shall not be less than:
(a) Dia of larger bar
(b) Maximum aggregate size + 5 mm
(c) Twice the dia. of bar
(d) Maximum aggregate size

Q.8 Additional longitudinal bars shall be provided in beams subjected to torsion if depth of beam exceeds
(a) 750 mm (b) 450 mm
(c) 500 mm (d) None of these

Q.9 In a beam subjected to torsion, the equivalent torsion moment is less than the flexural moment. In this case
(a) steel is provided only on tension side
(b) steel is provided only on compression side
(c) steel is provided in both tension and compression side
(d) data insufficient

Q.10 Torsion in a reinforced concrete member gives rise to
(a) diagonal cracks (b) vertical cracks
(c) inclined cracks (d) spiral cracks

Q.11 State true or false
'In indeterminate structures, torsion can be eliminated by releasing the redundant reactions.'

Q.12 A RC beam is subjected to a bending moment of 200 kNm, shear force of 20 kN and a torque of 9 kNm. The size of the beam is 300 mm width × 425 mm overall depth. The effective cover is 25 mm. The equivalent shear force is
(a) 68 kN (b) 50 kN
(c) 79 kN (d) 55 kN

Q.13 The equivalent moment in the above question is
(a) 200 kNm (b) 250 kNm
(c) 213 kNm (d) 400 kNm

Q.14 Torsion resisting capacity of a given RC section
(a) increases with increase in stirrup and longitudinal steel
(b) does not depend on stirrup and longitudinal steel
(c) decreases with decrease in stirrup spacing
(d) decreases with increase in longitudinal bars

Answers

1. (a)	2. (b)	3. (a)	4. (c)	5. (a)
6. (a)	7. (a, b)	8. (b)	9. (a)	10. (d)
11. True	12. (a)	13. (c)	14. (a)	

Hints:

12. (a)

$$V_e = V + \frac{1.6T}{b} = 20 + \frac{1.6 \times 9}{0.3} = 68 \text{ kN}$$

$$M_e = M + \frac{T}{1.7}\left(1 + \frac{D}{b}\right)$$

$$= 200 + \frac{9}{1.7}\left(1 + \frac{425}{300}\right) = 213 \text{ kNm}$$

Conventional Practice Questions

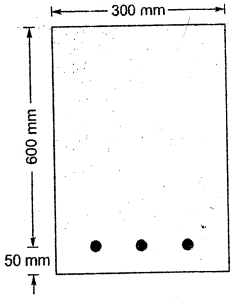
Q.1 A beam of size 300 mm × 450 mm is subjected to a torsional moment of 20 kNm along with a flexural moment of 100 kNm. The shear force acting on the beam is 105 kN at ultimate state of collapse. Design the beam using M25 concrete and Fe415 steel.

Q.2 Find the shear stress in a 300 mm × 500 mm sized beam which is subjected to a shear force of 25 kN and a torsional moment of 15 kNm at service loads. Assume 50 mm effective cover and tension reinforcement as 0.75%. Use M20 concrete and Fe415 steel.

Q.3 A rectangular beam of size 300 mm × 500 mm effective depth is reinforced with 0.5% tension steel of Fe415 grade. It is subjected to a shear force of 25 kN and a torsional moment of 10 kNm. Check if the beam needs torsional reinforcement. Use M20 concrete.

Q.4 A RC rectangular beam of size 350 mm × 450 mm effective depth is subjected to a flexural moment of 25 kNm, a shear force of 55 kN and a torsional moment of 40 kNm. Using M20 concrete and Fe415 steel design the reinforcement for the beam.

Q.5 A reinforced concrete beam is as shown. The section is subjected to a SF of 47 kN under service load conditions. Assuming the percentage tensile reinforcement as 0.5%, find the factored torsional moment that the section can resist when



(a) no torsional reinforcement is provided
(b) maximum steel for torsion is provided
Use M30 concrete and Fe500 steel.

CHAPTER 09

Analysis and Design of Flanged Beams by LSM

9.1 Introduction

In the previous chapters, we dealt with the analysis and design of RCC beams of rectangular section. The analysis procedure described earlier is applicable to beams of any other cross sectional shape also. In practice, the reinforced concrete floors/slabs are casted monolithically with beams. Such type of rectangular beams having slab at the top casted monolithically behaves differently from the simple reinforced concrete rectangular beam with no slab or with disconnected slabs. Such type of flanged systems where the beam and slab are monolithic, can be of either '*T*' or '*L*' type. In this chapter, we will discuss the analysis and design of such types of beams which are encountered more frequently in practice than simple rectangular beams.

9.2 Flanged Beams

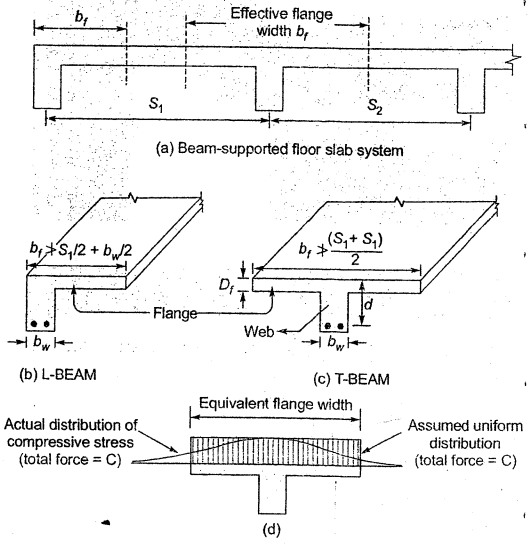


***Fig.9.1** Explanation of T and L beams*

In general, the floor slabs are casted monolithically with beams and hence the behavior of such beams is different from simple rectangular beams. In such type of beams, the slab acts in conjunction with beam. The slab portion is called as **flange** of '*T*' or '*L*' beam. The beam portion which is below the slab (flange) is called as **web** of '*T*' or '*L*' beam.

Actual width of the flange is the spacing of the beam which is same as the center to center distance between the adjacent spans of the slab as shown in **Fig. 9.1**. However in the flanged systems, a part of the width which is less than or equal to the actual width of the slab is considered to be effective. Thus the width of the slab is the effective width of the flange.

When the flange is too wide, then compressive stress distribution is not uniform over the entire width of web. But theory of flexure as discussed in previous chapters, assume a uniform stress distribution across the width of the section. Thus it becomes imperative to reduce the non-uniform stress distribution in the flange to a uniform stress distribution.

9.3 Effective Width of Flange

The **effective width of flange** is in fact a hypothetical concept and is defined as **the width of the flange that resists the uniform compressive stress that is equal to maximum stress in the original wide flange in such a way that resultant longitudinal compressive force is same in both the cases.** This is shown in Fig. 9.1(d).

The effective width of flange depends upon the following:

1. Span of the beam.
2. Width of the web (b_w).
3. Thickness of flange (D_f).
4. Type of loading i.e. whether the load is concentrated or uniformly distributed.
5. Type of beam supports.

As per **Cl. 23.1.1 of IS 456: 2000**, the following requirements must be satisfied for ensuring combined action of slab and rib (beam):

1. The slab shall be casted integrally with the web/rectangular beam or they shall be effectively bonded in any other manner.
2. At the mid-span of the slab, the transverse reinforcement must be atleast 60% of the main longitudinal reinforcement of the slab which is parallel to the transverse beam.

9.3.1 IS 456: 2000 Recommendations for Effective Width of Flange

Cl. 23.1.2 of IS 456: 2000 specifies the following values of effective flange width of *T*-beam and *L*-beams:

1. For *T*-beams: Minimum of the following is adopted as effective width of the flange:

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

or

$$b_f = b_w + \frac{l_1}{2} + \frac{l_2}{2}$$

2. For *L*-beams: Minimum of the following is adopted as effective width of the flange:

$$b_f = \frac{l_0}{12} + b_w + 3D_f$$

or

$$b_f = b_w + \frac{l_1}{2}$$

3. For isolated *T*-beam: $b_f = \dfrac{l_o}{\frac{l_o}{b}+4} + b_w$

4. For isolated *L*-beam: $b_f = \dfrac{0.5l_o}{\frac{l_o}{b}+4} + b_w$

Where, l_o = Distance between points of zero moment in the beam, which is the effective span of simply supported beam and 0.7 times the effective span for continuous beams and frames.

D_f = Thickness of the flange

b_w = Width of the web

b_f = Effective width of the flange

b = Actual width of the flange

Remember: For all the above cases, in no case the effective width of flange shall exceed the actual width of the flange.

9.4 Compressive Stress Distribution in the Flange

Maximum compressive stress in the flange occurs at the location above the rib of the flanged beam as shown in Fig. 9.2. Due to this complex variation of stress distribution, the concept of effective width of the flange was introduced.

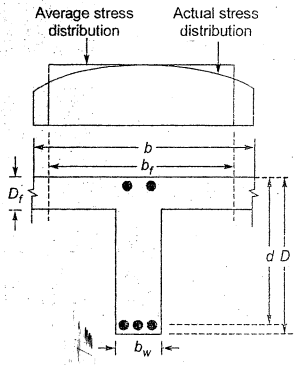


Fig. 9.2 Variation of Compressive Stress in Flanged Beams

9.5 Analysis of Flanged Beams Sections (by Limit State Method)

In a flanged beam, the neutral axis may either be in the flange or web of the section which depends upon the dimensions of effective flange width (b_f), web width (b_w), and thickness of flange (D_f) of the flanged beam.

If neutral axis lies in the flange of the T beam (i.e. $x_u \le D_f$) then flanged beam can be taken as a rectangular beam of width b_f and effective depth d because the concrete on tension side is not taken into account. Thus equations derived in the previous chapters on rectangular beams are still applicable with the only difference that width of the beam (b) will get replaced by effective width of the flange (b_f).

But if the neutral axis lies in the web of the flanged beam (i.e. $x_u > D_f$) then compression is taken by flange and part of the web of the beam. In this case, the total compressive force (C_u) comprises compressive force due to the web portion ($b_w x_u$) and compressive force due to the flange portion [$(b_f - b_w) D_f$]. The compressive force and moment of resistance in the web portion is given by:

$$C_{uw} = 0.36 f_{ck} b_w x_u$$

$$M_{uw} = C_{uw}(d - 0.42x_u) = 0.36 f_{ck} b_w x_u (d - 0.42x_u)$$

But estimation of compressive force in the flange is difficult because it depends on the shape of the compressive stress distribution curve i.e. whether only the rectangular portion of compressive stress distribution or the rectangular and part of parabolic portion of the compressive stress distribution lies in the flange portion.

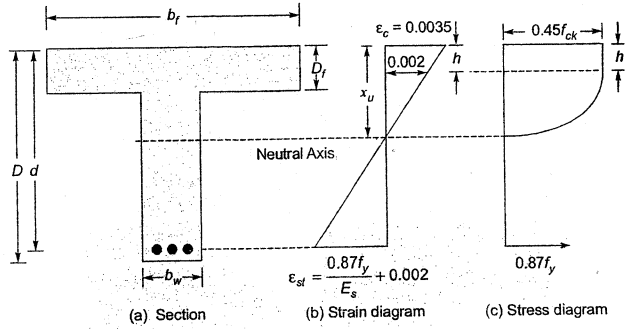


Fig.9.3 A typical T-beam section.

However it is to be noted that the compressive stress remains constant between the concrete compressive strains of 0.002 and 0.0035 which is equal to 0.45 f_{ck}.

Let h = Depth of concrete stress block where the compressive stress remains constant i.e. depth between compressive strain of 0.002 and 0.0035

If depth 'h' where the compressive strain is 0.002 lies in the web of the flange beam then whole of the flange will be under a constant compressive stress of 0.45 f_{ck}.

From Fig. 9.3(b)

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u} \quad \text{or} \quad \frac{h}{x_u} = \frac{3}{7}$$

In the limiting case when $x_u = x_{u\,lim}$,

$$h = \frac{3}{7} x_{u\,lim}$$

$= 0.227d$ for *Fe* 250 since $x_{u\,lim} = 0.53d$

$= 0.205d$ for *Fe* 415 since $x_{u\,lim} = 0.479d$

$= 0.197d$ for *Fe* 500 since $x_{u\,lim} = 0.46d$

From the above results, it can be inferred that,

$$\frac{h}{d} \simeq 0.2$$

Thus for all the three grades of steel i.e. Fe 250, Fe 415 and Fe 500, $h \approx 0.2d$. Let the value of 'h' reaches the depth of flange i.e., at the bottom of flange so that $h = D_f$ and thus the strain will be 0.002 if $\dfrac{D_f}{d} = 0.2$.

NOTE: Thus, the flange thickness can be considered as small if D_f/d is less than 0.2 and in that case the entire flange will be under a constant compressive stress of $0.45 f_{ck}$.

If $\frac{D_f}{d} > 0.2$, then only a part of flange of the beam will be under a constant compressive stress of $0.45f_{ck}$, i.e. the strain is more than 0.002.

Thus, in balanced flanged beams (when $x_u = x_{u\,lim}$), the ratio of $\frac{D_f}{d}$ is important to determine i.e. the rectangular stress block is for the full depth of flange (when $\frac{D_f}{d} \le 0.2$) or for a part of the flange (where $\frac{D_f}{d} > 0.2$).

Similarly, for the case of under reinforced flanged beams (when $x_u < x_{u\,lim}$), replace $\frac{D_f}{d}$ by $\frac{D_f}{x_u}$.

If $\frac{D_f}{x_u} \le 0.43\left(=\frac{3}{7}\right)$, then the constant stress block (= $0.45\,f_{ck}$) will be for the full depth of the flange.

If $\frac{D_f}{x_u} > 0.43\left(=\frac{3}{7}\right)$, then only a part of flange depth will be having a constant stress of $0.45f_{ck}$.

Thus, there are four cases of flanged beams which are explained below:

Case 1: When $x_u < D_f$ i.e. when neutral axis is in the flange

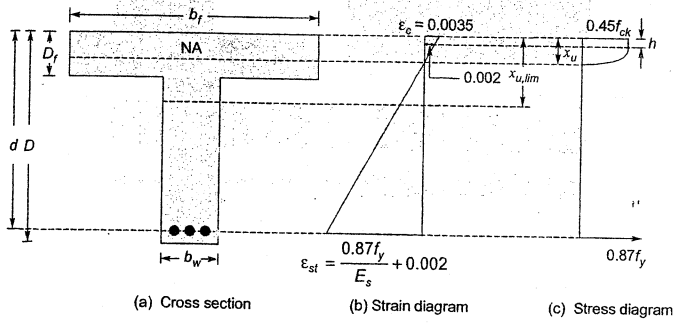


***Fig.9.4** T-beam, when $x_u < D_f$*

Concrete below the neutral axis remains under tension and hence ignored. Thus the beam is considered as a rectangular beam of width b_f and effective depth d.

Actual depth of neutral axis is given by,

$$C = T$$

$$0.36f_{ck}b_f x_u = 0.87f_y A_{st}$$

$$x_u = \frac{0.87f_y A_{st}}{0.36f_{ck}b_f}$$

Moment of resistance is given by,

$$M_u = C \times \text{Lever Arm} = T \times \text{Lever Arm}$$

$$= 0.36f_{ck}b_f x_u(d - 0.42x_u) = 0.87f_y A_{st}(d - 0.42x_u)$$

Case 2: When $x_{u\,lim} > D_f$ i.e. when neutral axis is in the web and the section is balanced (i.e., $x_u = x_{u\,lim}$)

Sub case (i) When $\frac{D_f}{d} \le 0.2$.

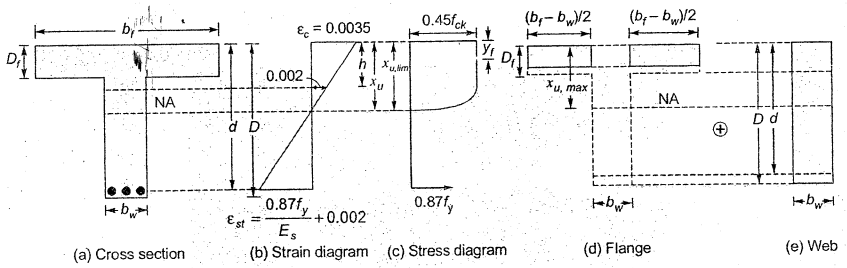


***Fig.9.5** T-beam when $D_f/d \le 0.2$ and balanced $x_{u\,lim} \ge D_f$*

In this case, the depth of constant stress (= $0.45f_{ck}$) block is greater than D_f. The section is divided into two parts viz.

1. Rectangular beam of width b_w and effective depth d
2. Rectangular flange of width $(b_f - b_w)$ and depth D_f

Total compressive force (C) = Compressive force of rectangular beam of width b_w and depth d + Compressive force of rectangular flange of width $(b_f - b_w)$ and depth D_f

Compressive force, $C = 0.36f_{ck}\,b_w x_{u\,lim} + 0.45f_{ck}(b_f - b_w)D_f$

Do you know?: In the above expression it is assumed that constant compressive stress is $0.45f_{ck}$ and NOT $0.446f_{ck}$ as per Cl. G-2 of IS 456: 2000.

Tensile force, $T = 0.87f_y A_{st}$

Lever arm for rectangular web part is $(d - 0.42\,x_{u\,lim})$ and for the flange part is $(d - 0.5D_f)$.

Total moment of resistance = Moment due to rectangular web beam + Moment due to rectangular flange

$$M_u = 0.36f_{ck}b_w x_{u\,lim}(d - 0.42x_{u\,lim}) + 0.45f_{ck}(b_f - b_w)\,D_f\left(d - \frac{D_f}{2}\right)$$

or $$M_u = 0.36\left(\frac{x_{u\lim}}{d}\right)\left[1-0.42\left(\frac{x_{u\lim}}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

The above equation is given in Cl. G-2.2 of IS 456: 2000.

Sub case (ii) When $\frac{D_f}{d} > 0.2$

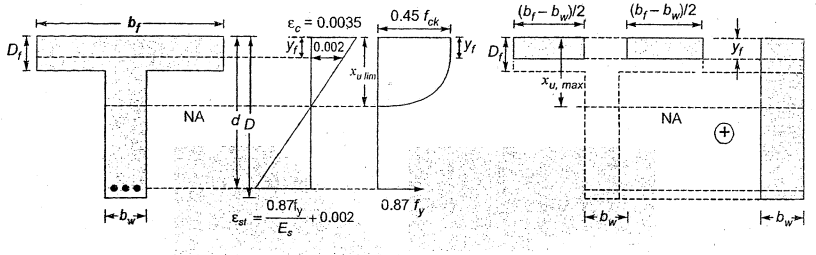


***Fig.9.6** T-beam, Case (ii-b), $D_f/d > 0.2$ and balanced $x_{u,lim} > D_f$*

In this case, the depth of rectangular portion of stress block lies in the flange of the beam only. The depth of constant stress (= $0.45 f_{ck}$) is taken as 'y_f' where 'y_f' is given by:

$$\boxed{y_f = (0.15 x_{u\lim} + 0.65 D_f) \le D_f}$$

The derivation of above expression for y_f is given in the next section.

When $\frac{D_f}{d} \le 0.2$, the equations derived in sub-case(i) are applicable here also just by replacing D_f by y_f.

Thus the equations are:

$$C = 0.36\, f_{ck} b_w x_{u\,\lim} + 0.45\, f_{ck}(b_f - b_w) y_f$$
$$T = 0.87 f_y A_{st}$$

Lever arm for rectangular web part is $(d - 0.42\, x_{u\,\lim})$ and for the flange part is $(d - 0.5 y_f)$. Thus the moment equations are:

$$M_u = 0.36 f_{ck} b_w x_{u\lim}(d - 0.42 x_{u\lim}) + 0.45 f_{ck}(b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

or $$M_u = 0.36\left(\frac{x_{u\lim}}{d}\right)\left[1-0.42\left(\frac{x_{u\lim}}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

Case 3: When $x_u > D_f$ i.e. when neutral axis is in the web and the section is under reinforced

Sub case (i) When $\frac{D_f}{x_u} \le 0.43$

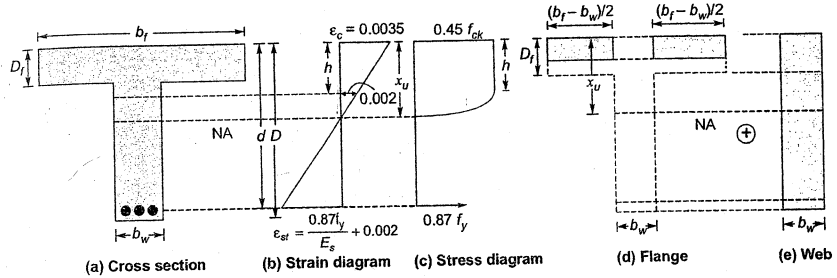


***Fig.9.7** T-beam, Case (iii-a), when $D_f/x_u \le 0.43$ and under-reinforced $x_u > D_f$*

Now since the flange depth D_f does not exceed $0.43x_u$ and h (depth of compressive fibre of strain 0.002) is $0.43x_u$, the whole flange is under constant compressive stress of $0.45 f_{ck}$.

Thus, $$C = 0.36\, f_{ck} b_w x_u + 0.45\, f_{ck}(b_f - b_w) D_f$$
$$T = 0.87 f_y A_{st}$$
$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1-0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\left(d - \frac{D_f}{2}\right) D_f$$

Sub case (ii): When $\frac{D_f}{x_u} > 0.43$

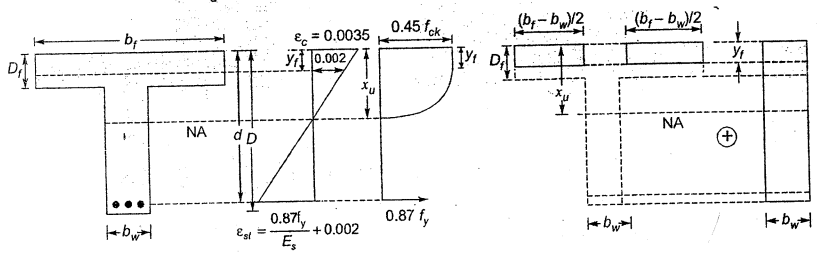


Fig.9.8 *T-beam, when $D_f/d \le 0.2$ and $x_u > D_f$*

In this case, $D_f > 0.43x_u$ and h (depth of compressive fibre of strain 0.002) is at a depth of $0.43x_u$, part of the flange will be under the constant compressive stress of $0.45 f_{ck}$ and this depth is denoted as y_f.

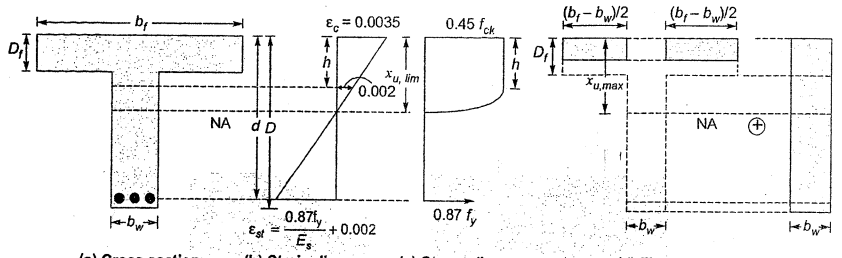
Thus
$$y_f = (0.15 x_u + 0.65 D_f) \le D_f$$
$$C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) y_f$$
$$T = 0.87\, f_y A_{st}$$
$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1-0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

Case 4: When $x_u > D_f$ i.e. when neutral axis is in the web and the section is over reinforced

In case of over reinforced sections, the actual depth of neutral axis (x_u) is greater than the limiting depth of neutral axis ($x_{u\,lim}$).

However x_u is restricted to $x_{u\,lim}$.

Sub case (i): When $\frac{D_f}{d} \le 0.2$



(a) Cross section (b) Strain diagram (c) Stress diagram (d) Flange (e) Web

Fig.9.9 *T-beam, when $D_f/d \le 0.2$ and $x_u > D_f$*

$$C = 0.36\,f_{ck} b_w x_{u\,lim} + 0.45\,f_{ck}(b_f - b_w)D_f$$

$$T = 0.87 f_y A_{st}$$

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right)$$

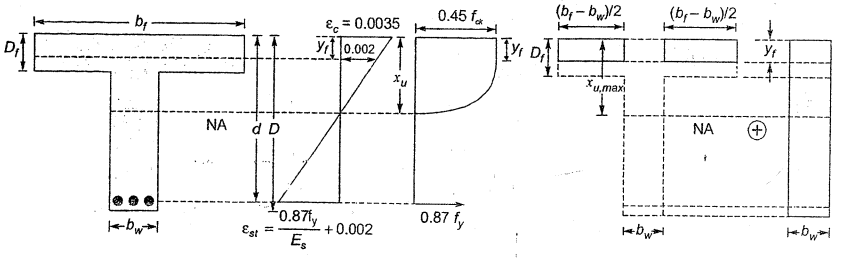
Sub case (ii): When $D_f/d > 0.2$

$$y_f = (0.15x_u + 0.65D_f) \le D_f$$

$$C = 0.36\,f_{ck} b_w x_{u\,lim} + 0.45\,f_{ck}(b_f - b_w)y_f$$

$$T = 0.87 f_y A_{st}$$

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)y_f\left(d - \frac{y_f}{2}\right)$$



(a) Cross section (b) Strain diagram (c) Stress diagram (d) Flange (e) Web

Fig.9.10 *T-beam, $D_f/d > 0.2$ and $x_u > D_f$*

From the above expressions, it is quite evident that for over reinforced beams, the additional moment of resistance beyond balanced section is not possible because it will restrict the failure/yielding of steel. Thus the section is either redesigned or we go for doubly reinforced beams.

9.6 Derivation of the Expression to Determine 'y_f'

Fig. 9.11 shows two stress blocks viz. the IS 456: 2000 stress block and the Whitney's stress block.

Now,

y_f = Depth of constant portion of stress block when $D_f/d > 0.2$.

Since y_f is a function of x_u and D_f and thus y_f can be assumed as:

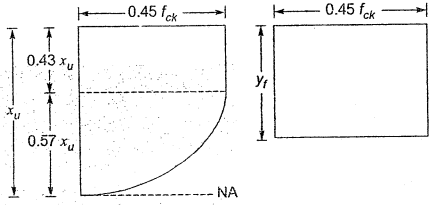
$y_f = Ax_u + BD_f$

Here A and B are constants to be determined from the following two conditions viz.:

1. When $D_f = 0.43x_u$, $y_f = 0.43x_u$
2. When $D_f = x_u$, $y_f = 0.8x_u$

Thus solving equation of y_f for the conditions 1. and 2. gives,

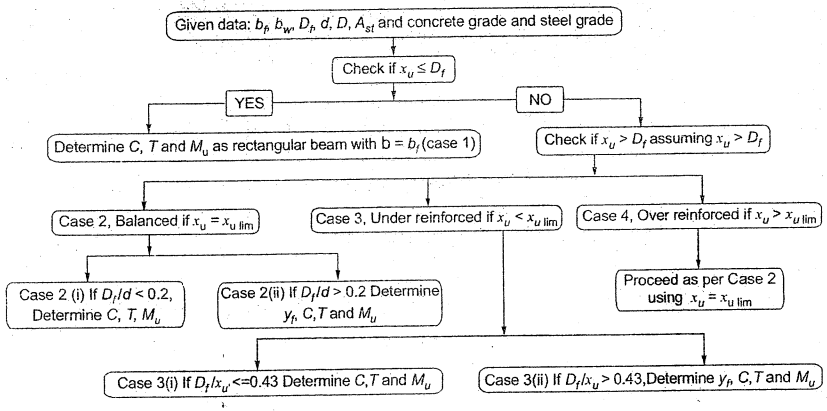
$$y_f = 0.15x_u + 0.65D_f$$



(a) IS 456 stress block (b) Whitney's stress block

Fig.9.11 *Stress blocks*

9.7 Types of Analysis Problems



9.8 Integral Action of Slab and Beam

The combined (or the integral) action between the slab and beam can be ensured by providing stirrups. Sufficient transverse reinforcement must be provided near the top of the flange which is present in the form of

negative moment reinforcement in continuous slabs which spans across and form the flange of *T* beams as shown in the Fig. 9.12.

In the other case (i.e. when bars run parallel to the beam in flange as shown in Fig. 9.13), Cl. 23.1.1b of IS 456: 2000 specifies to provide transverse reinforcement in the flange of the ***T or L beam.*** The area of such steel should not be less than 60% of the area of main mid-span steel reinforcement and should extend on either side of the beam for a distance not less than (span/4).

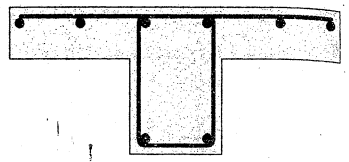


Fig.9.12 *Integral action of slab and beam*

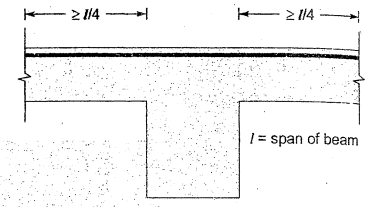


Fig.9.13 *Flanged beam with top bars running parallel to flange*

9.9 Design of Flanged Beam Sections (by Limit State Method)

The combined action of slab and beam occurs in *cast in-situ* constructions. If the slab and beam are not casted monolithically, flanged action on the beam cannot be considered unless special shear connectors are provided at the interface of slab and beam.

In **continuous flanged beams**, the proportioning of the section is governed by the negative moment at the support which is invariably larger than the mid span positive moment. For this case (i.e. negative moment at support in the flanged beam), the flange portion is under tension and thus the concrete in the flange portion is ignored. In other words, the flanged beam section at supports is designed as a rectangular beam section for the factored negative moment. At the mid span of the beam, the beam is actually a flanged beam with flange portion under compression. Now as the web width (b_w) and effective beam depth (d) has already been determined (hence fixed) from the factored negative moment at support, only the reinforcing steel required is to be determined. The effective width of the flange (b_f) is determined as per provisions of **IS 456: 2000.**

In case of **simply supported flanged beams**, the web dimension has to be determined / designed. The web width is usually fixed as 250 mm, 300 mm or 350 mm etc. with overall depth preliminary taken as 1/13th to 1/16th of span. The area of tension steel is then determined as:

$$A_{st\,required} = \frac{M_u}{0.87 f_y z}$$

Where, the lever arm (z) is approximately taken larger of $0.9d$ or $\left(d - \frac{D_f}{2}\right)$.

Determination of reinforcing steel in flanged beams depends on the location of neutral axis (x_u), which in any case should not exceed $x_{u\,lim}$. If factored moment (M_u) exceeds the limiting moment of resistance of the flanged beam section ($M_{u\,lim}$) then either the beam depth should be increased or the section has to be designed as doubly reinforced.

Case 1: Neutral Axis Lies Within the Flange ($x_u \le D_f$)

In practical situations, the compressive force contributed by the flange of *T* or *L* beams is usually very large and thus the depth of neutral axis (x_u) is less than the flange depth (D_f). Here, the section behaves as a rectangular section of width b_f and effective depth d.

Now the question arises,

"How to check whether $x_u \le D_f$ without actually calculating the neutral axis depth x_u?"

The answer to the above question is that first calculate moment of resistance of the section at $x_u = D_f$ i.e. $(M_{uR})_{x_u=D_f}$ as:

$$(M_{uR})_{x_u=D_f} = 0.362 f_{ck} b_f D_f (d - 0.42 D_f)$$

Compare the above calculated moment $(M_{uR})_{x_u=D_f}$ with the factored design moment (M_u). If $M_u \le (M_{uR})_{x_u=D_f}$ then it implies that $x_u \le D_f$. But all this is valid only if $x_{u\,lim} > D_f$. If $x_{u\,lim} < D_f$ then $M_{u\,lim}$ is calculated from the above expression by replacing D_f by $x_{u\,lim}$.

Case 2: Neutral Axis Lies within the Web ($x_u > D_f$)

In this case, $M_u > (M_{uR})_{x_u=D_f}$ and thus $x_u > D_f$. This leads to the accurate determination of the depth of neutral axis (x_u) which is quite tedious. Now as discussed above, the compressive force in concrete consists of compressive force due to the web (C_{uw}) and due to the flange (C_{uf}) as:

$$C_{uw} = 0.362 f_{ck} b_w x_u$$

$$C_{uf} = 0.45 f_{ck} (b_f - b_w) y_f$$

Here the equivalent flange thickness (y_f) is less than or more than D_f depends on whether x_u is greater than or less than, $\frac{7D_f}{3}$.

If $x_{u\,lim} > \frac{7D_f}{3}$, then the ultimate moment of resistance $(M_{uR})_{x_u=7D_f/3}$ (i.e. at $x_u = \frac{7D_f}{3}$) and $y_f = D_f$ is computed first. If $M_u \ge (M_{uR})_{x_u=7D_f/3}$ (i.e. at $x_u = \frac{7D_f}{3}$) then it implies that $x_u > \frac{7D_f}{3}$ and $y_f = D_f$.

Else $D_f < x_u < \frac{7D_f}{3}$ for $(M_u)_{x_u=D_f} < M_u < (M_{uR})_{x_u=7D_f/3}$ and

$$y_f = 0.15 x_u + 0.65 D_f$$

x_u is then calculated from the following equation,

$$M_u = C_{uw}(d - 0.42 x_u) + C_{uf}\left(d - \frac{y_f}{2}\right)$$

Once x_u has been determined, C_{uw} and C_{uf} can be determined from the above equations. Area of reinforcing steel required is computed as:

$$A_{st\,required} = \frac{C_{uw} + C_{uf}}{0.87 f_y}$$

Example 9.1 Determine the moment of resistance of a flanged beam as shown below. The area of steel in the tension zone is 2945 mm^2 (6-25 ϕ). Use M 20 concrete and Fe 415 steel.

Solution:

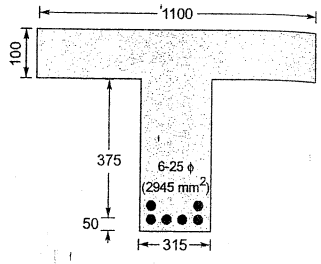
Calculating the depth of natural axis

From statical equilibrium, $C = T$

$\Rightarrow \quad 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$\Rightarrow \quad x_u = \dfrac{0.87 \times 415 \times 2945}{0.36 \times 20 \times 1100} = 133.51 \text{ mm} > D_f (= 100 \text{ mm})$

$\dfrac{D_f}{d} = \dfrac{100}{475} = 0.211 > 0.2$



$\therefore \quad y_f = 0.15 x_u + 0.65 D_f$

$= 0.15 x_u + 0.65 (100)$

$= 0.15 x_u + 65$

and from $C = T$

$0.36 f_{ck} b_w x_u + 0.445 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$

$\Rightarrow 0.36 (20)(315) x_u + 0.445 (20)(1100 - 315)(0.15 x_u + 65)$

$= 0.87 (415)(2945)$

$\Rightarrow 2280.6 x_u + 1050.33 x_u + 455143 = 1063292.25$

$\Rightarrow \quad x_u = 182.58 \text{ mm}$

$x_{u\,lim} = 0.479 \text{ d} = 0.479(475) = 227.525 \text{ mm} > x_u$

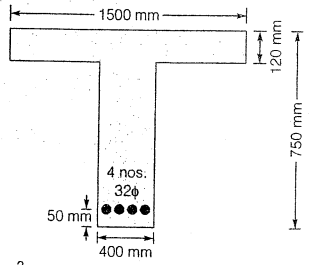
Thus beam is under-reinforced

Moment of resistance $(MR) = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 2945 (475 - 0.42 \times 182.58) \text{ Nmm}$

$= 423.53 \text{ kNm}$

Example 9.2 Calculate the moment of resistance of a continuous isolated T-beam of clear span 9 m and cross-section as shown in the figure. Use M 25 concrete and Fe 500 steel. Use WSM.



Solution:

Given: Flange width $(b) = 1500 \text{ mm}$

Effective depth $(d) = 750 - 50$

$= 700 \text{ mm}$

Flange depth $(d_f) = 120 \text{ mm}$

Area of tension steel $(A_{st}) = 4 \times \dfrac{\pi}{4} \times 32^2 \text{ mm}^2$

$= 3216.9 \text{ mm}^2$

Design coefficients

$m = 11, c = 8.5, t = 275 \text{ N/mm}^2$

$k = \dfrac{mc}{mc + t} = \dfrac{11 \times 8.5}{11 \times 8.5 + 275} = 0.253$

Critical depth of neutral axis

$x_c = kd = 0.253 \times 750 \text{ mm} = 189.75 \text{ mm}$

Effective width of flange

$b_f = \dfrac{l_0}{\dfrac{l_0}{b} + 4} + b_w$

$\because \quad l_0 = 0.7l = 0.7 \times 9 \text{ m} = 6.3 \text{ m} = 6300 \text{ mm}$

$b_f = \dfrac{6300}{\dfrac{6300}{1500} + 4} + 400 = 1168.3 \text{ mm} < 1500 \text{ mm}$

Actual depth of neutral axis

(a) If neutral axis lies in the flange of the beam $(x_a \le d_f)$

$b_f \dfrac{x_a^2}{2} = m A_{st} (d - x_a)$

$1168.3 \times \dfrac{x_a^2}{2} = 11 \times 3216.9 (750 - x_a)$

$389.43 x_a^2 = 35385.9 (750 - x_a)$

$x_a = 185 \text{ mm} > 120 \text{ mm}$

$\therefore$ Neutral axis does not lies in the flange of the beam

(b) If neutral axis lies in the web of the beam $(x_a > d_f)$

$b_f d_f \left(x_a - \dfrac{d_f}{2} \right) + b_w \dfrac{(x_a - d_f)^2}{2} = m A_{st} (d - x_a)$

$1168.3 \times 120 \left(x_a - \dfrac{120}{2} \right) + \dfrac{400}{2} (x_a - 120)^2 = 11 \times 3216.9 (750 - x_a)$

$140196 (x_a - 60) + 200 (x_a^2 + 120^2 - 240 x_a) = 35385.9 (750 - x_a)$

$x_a = 193 \text{ mm}$

$x_a = 193 \text{ mm} > 120 \text{ mm} \quad \therefore$ NA lies in the web of the beam

Now, $x_a > x_c$

$\therefore$ Section is over reinforced $c_a = \sigma_{cbc} = 8.5 \text{ N/mm}^2$

Stress at the junction of flange and web

$c_1 = \dfrac{x_a - d_f}{x_a} \times \sigma_{cbc} = \dfrac{193 - 120}{193} \times 8.5 = 3.215 \text{ N/mm}^2$

Moment of resistance

$M_R = b_f d_f \left(\dfrac{c_a + c_1}{2} \right) \left(d - \dfrac{c_a + 2c_1}{c_a + c_1} \times \dfrac{d_f}{3} \right)$

$+ b_w (x_a - d_f) \dfrac{c_1}{2} \left(d - d_f - \dfrac{x_a - d_f}{3} \right)$

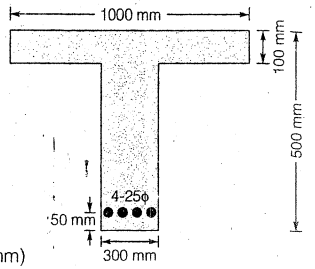
$= 1168.3 \times 120 \left(\dfrac{8.5 + 3.215}{2} \right) \left(700 - \dfrac{8.5 + 2 \times 3.215}{8.5 + 3.215} \times \dfrac{120}{3} \right)$

$+ 400 (193 - 120) \dfrac{3.215}{2} \left(700 - 120 - \dfrac{193 - 120}{3} \right)$

$= 559.05 \text{ kNm}$

Example 9.3 Find the moment of resistance of the T-beam of cross-section as shown in the figure below. Use M 20 concrete and Fe 415 steel.

Solution:



Step-1: Determine the depth of N.A.

Let NA lies in the flange i.e., $x_u \le D_f$

$$C = T$$

$$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87(415)4\times\frac{\pi}{4}(25)^2}{0.36\times 20\times 1000}$$

$$= 97.92 \text{ mm} < D_f (= 100 \text{ mm})$$

Thus assumption of $x_u \le D_f$ is correct.

Also $x_{u\,lim} = 0.48d$ (for Fe 415) $= 0.48\,(450) = 216 \text{ mm} > x_u$

So section is under reinforced

Step-2: Determine moment of resistance of section

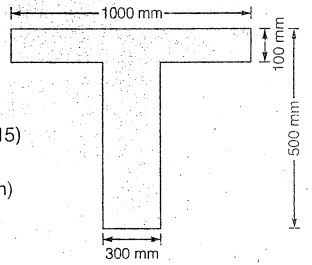
∵ Section is under reinforced

∴ Steel will yield first

∴ Moment of resistance $= 0.87 f_y A_{st}(d - 0.42x_u)$

$= 0.87 \times 415 \times 1963\,(450 - 0.42 \times 97.92) = 289.78$ kNm

Example 9.4 A T-beam of cross-section as shown in the figure below. Using M20 concrete and Fe 415 steel, determine $A_{st\,lim}$ and $M_{u\,lim}$.



Solution:

Step-1: Determine (D_f/d) ratio

Let Effective cover = 50 mm

In the limiting case, $x_u = x_{u\,lim} = 0.48d$ (for Fe 415)

$= 0.48\,(450)$

$= 216 \text{ mm} > D_f (= 100 \text{ mm})$

$$\therefore \quad \frac{D_f}{d} = \frac{100}{450} = 0.222 > 0.2$$

Let N.A. lies in the web.

Step-2: Calculate y_f, C and T.

$$y_f = 0.15\, x_{u\,lim} + 0.65\, D_f \not> D_f$$

$$= 0.15\,(0.48 \times 450) + 0.65\,(100)$$

$$= 97.4 \text{ mm} < D_f (= 100 \text{ mm}) \quad \text{(OK)}$$

$$\therefore \quad C = 0.362\, f_{ck}\, b_w x_{u\,lim} + 0.45 f_{ck}\,(b_f - b_w) y_f$$

$$= 0.362\,(20)\,(300)\,(216) + 0.45\,(20)(1000 - 300)97.4$$

$$= 1082.772 \text{ kN}$$

In the limiting case, $f_{st} = 0.87 f_y$

$$\therefore \quad T = 0.87 f_y A_{st}$$

from $C = T$

$$A_{st} = \frac{1082.772\times 10^3}{0.87 f_y} = \frac{1082.772\times 10^3}{0.87(415)} = 2998.95 \text{ mm}^2$$

$$\therefore \quad A_{st\,lim} = 2998.95 \text{ mm}^2 \approx 2999 \text{ mm}^2$$

Step-3: Calculate $M_{u\,lim}$

$$M_{u\,lim} = 0.362\, f_{ck}\, b_w x_{u\,lim}(d - 0.42x_{u\,lim}) + 0.45 f_{ck}\,(b_f - b_w) y_f (d - y_f/2)$$

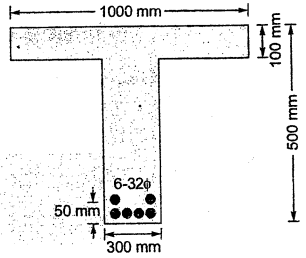
$$= 0.362 \times 20 \times 300 \times 216\,(450 - 0.42 \times 216) + 0.45 \times 20(1000 - 300)$$

$$97.4\,(450 - 97.4/2)$$

$$= 414.81 \text{ kNm}$$

Example 9.5 In the previous question, determine the moment of resistance of the T-beam when it is reinforced with 6-32ϕ bars on tension side. Use M20 concrete and Fe415 steel.

Solution:



Area of tension steel $A_{st} = 6\times\frac{\pi}{4}\times 32^2 = 4825.5 \text{ mm}^2$

Step-1: Determine depth of N.A. (x_u)

Let N.A. lies in the flange i.e., $x_u \le D_f$

$$\therefore \quad x_u = \frac{0.87 f_y A_{st}}{0.362 f_{ck} b_f} = \frac{0.87(415)4825.5}{0.362(20)1000}$$

$$= 240.64 \text{ mm}$$

Limiting depth of N.A. $x_{u\,lim} = 0.48d = 0.48(450) = 216$ mm

$$\therefore \quad x_u > x_{u\,lim}$$

So section is over reinforced

∴ Limit x_u to $x_{u\,lim}$ i.e., $x_u = x_{u\,lim} = 216$ mm

Also $\frac{D_f}{d} = \frac{100}{450} = 0.222 > 0.2$

∴ Depth of constant stress block $(= 0.45 f_{ck}) < D_f$

$$\therefore \quad y_f = 0.15\, x_u + 0.65\, D_f \not> D_f$$

Here $x_u = x_{u\,lim} = 216$ mm

$$\therefore \quad y_f = 0.15\,(216) + 0.65\,(100) = 97.4 \text{ mm}$$

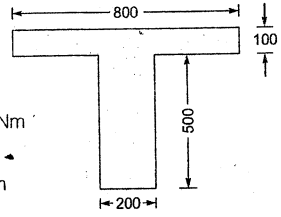
Step-2: Calculate moment of resistance of beam

$$M_u = 0.362\, f_{ck}\, b_w x_{u\,lim}(d - 0.42x_{u\,lim}) + 0.45 f_{ck}\,(b_f - b_w) y_f (d - y_f/2)$$

$$= 0.362\,(20)\,(300)\,(216)\,(450 - 0.42 \times 216) + 0.45(20)(1000 - 300)97.4$$

$$(450 - 97.4/2) = 414.81 \text{ kNm}$$

Example 9.6 A T-beam section as shown below is supposed to resist a moment of 300 kNm at working loads, Using M20 concrete and Fe415 steel, design the reinforcement required in the beam.



Solution:

Factored BM $(M_u) = 1.5 \times 300$ kNm $= 450$ kNm

Let effective cover to reinforcement is 50 mm.

So, effective depth of the section $(d) = 600 - 50$ mm $= 550$ mm

Depth of the flange (D_f) = 100 mm

So, $\frac{D_f}{d} = \frac{100}{550} = 0.1818 \approx 0.2$

Limiting depth of neutral axis ($x_{u\,lim}$) = $0.479d = 0.479 \times 550$ mm = 263.45 mm

Let neutral axis lies in the web of the T-beam section.

Ultimate moment of resistance of the T-beam section:

$$M_{uR} = 0.138 f_{ck} b_w d^2 + 0.362 f_{ck} (b_f - b_w) D_f (d - 0.5 D_f)$$

$$= 0.138(20)(200)(550)^2 + 0.362(20)(800-200)(100)(550 - 0.5 \times 100)$$

$$= 166.98 \times 10^6 + 217.2 \times 10^6 \text{ Nmm}$$

$$= 384.18 \text{ kNm} < 450 \text{ kNm}$$

Thus a doubly reinforced beam section is required.

Area of tension steel corresponding to limiting moment of resistance is $p_{t\,lim}$ which is given by:

$$p_{t\,lim} = 41.61 \left(\frac{f_{ck}}{f_y}\right) \cdot \frac{x_{ulim}}{d}$$

$$p_{t\,lim} = 41.61 \left(\frac{20}{415}\right)(0.479) = 0.9605\,\%$$

$$A_{st\,lim} = \frac{0.9605}{100}(200)(550) = 1056.55 \text{ mm}^2$$

Balance moment to be resisted by compression steel (ΔM_u) = 450 – 384.18 kNm = 65.82 kNm

Let effective cover to compression steel (d') = 50 mm

So $\frac{d'}{d} = \frac{50}{550} = 0.0909 \approx 0.1$

For $\frac{d'}{d} = 0.1$ and Fe415, $f_{sc} = 351.9$ N/mm²

A_{sc} Required in the compression side:

$$\Delta M_u = f_{sc} A_{sc} (d - d')$$

$$65.82 \times 10^6 = (351.9) A_{sc} (550 - 50)$$

$$A_{sc} = 374.08 \text{ mm}^2$$

Provide 2-16 mm diameter bars in the compression side.

Thus, $A_{sc\,provided} = 402.13 \text{ mm}^2$

A_{st} Required in the tension side:

$$f_{sc} A_{sc} = 0.87 f_y \Delta A_{st}$$

$$\Delta A_{st} = \frac{(351.9)(402.13)}{(0.87 \times 415)} = 391.94 \text{ mm}^2$$

Thus total area of tension steel = $A_{st\,lim} + \Delta A_{st} = 1056.55 + 391.94 \text{ mm}^2 = 1448.49 \text{ mm}^2$

Provide 5-20 mm diameter bars in the tension side.

Thus, $A_{st\,provided} = 1570.8 \text{ mm}^2 > 1448.49 \text{ mm}^2$

Maximum percentage of steel in a beam section ≤ 4%

$= 0.04 \times 200 \times 600 \text{ mm}^2$

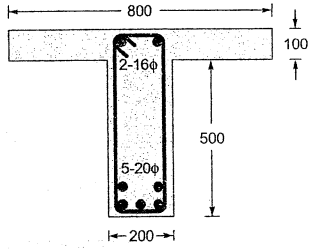
$= 4800 \text{ mm}^2$

Thus, $A_{sc\,provided} = 402.13 \text{ mm}^2 < 4800 \text{ mm}^2$ (OK)

and $A_{st\,provided} = 1570.8 \text{ mm}^2 < 4800 \text{ mm}^2$ (OK)

Minimum tension steel $A_{st\,min} \geq 0.85 \times 200 \times \frac{550}{415} = 225.3 \text{ mm}^2$

$< A_{st\,provided} (= 1570.8 \text{ mm}^2)$ (OK)



Reinforcement Detailing

Objective Brain Teasers

Q.1 A slab casted monolithically on a cantilever beam under the influence of gravity loads will act as a
(a) Flanged beam
(b) Curved beam
(c) Deep beam
(d) Rectangular beam

Q.2 A shed is casted with a central row of columns with a central beam on the columns. Cantilevered slabs are casted on either side of the central beam to cover the shed area. The mid span transverse section will be designed as a
(a) Flanged beam
(b) Continuous beam
(c) Spandrel beam
(d) None of the above

Q.3 When $\frac{D_f}{d} < 0.2$ is the limiting case then the flange of T beam will have:
(a) A constant compressive stress of $0.45 f_{ck}$
(b) A varying compressive stress
(c) A constant shear stress
(d) All of the above

Q.4 In order to have a constant compressive stress in the flange (D_f) of a T beam, D_f/x_u should be
(a) Greater than 3/7 (b) Greater than 4/7
(c) Lesser than 4/7 (d) Lesser than 3/7

Q.5 The expression for equivalent depth of flange 'y_f' for a constant compressive stress is based on
(a) Marcus stress block
(b) Grashoff's stress block
(c) Whitney's stress block
(d) Rankine stress block

Q.6 State true or false
An inverted T-beam subjected to gravity loads act as T beam.

Q.7 The location of neutral axis in a RC T-beam
(a) depends on flange depth and total depth
(b) Lies in the flange always
(c) Lies in the web always
(d) Lies at the junction of flange and web

Answers

1. (d) 2. (a) 3. (a) 4. (d) 5. (c)
6. False 7. (a)

Conventional Practice Questions

Q.1 Find the steel reinforcement required for a simply supported flanged beam of flange width 1000 mm, flange depth 100 mm, web width 300 mm and overall depth of 700 mm. Take effective cover to reinforcement as 50 mm. The span of beam is 12 m and it carries an imposed load of 10 kN/m^2. Use M20 and Fe415.

Q.2 Find the moment of resistance of a flanged beam with b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, effective cover = 50 mm and overall depth of 500 mm. (a) using M20 and Fe250, (b) using M20 and Fe415.

Q.3 Find the area of steel required for a simply supported flanged beam of flange width 1500 mm, flange depth 120 mm, web width 350 mm and overall depth of 700 mm. Take effective cover to reinforcement as 60 mm. The span of beam is 10 m and it carries a live load of 5 kN/m^2. Use M25 and Fe500.

Q.4 Design a flanged beam with b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, subjected to a live load of 4 kN/m^2. Use M30 and Fe500.

CHAPTER 10

Limit State of Serviceability-Deflection and Cracking

10.1 Introduction

As discussed in the previous chapters, there are two limit states in the limit state method of design viz. the *limit state of collapse* and the *limit state of serviceability.* After designing a structure for the limit state of collapse (flexure, shear, torsion etc.), it needs to be checked for deflections, vibrations, cracking etc. i.e. collectively known as *limit state of serviceability.* These *limit states of serviceability* play a crucial role in the design of structures and they ensure that the structure performs its intended function very well with safety with respect to limit state of collapse.

Deflection and cracking are two important limit states of serviceability. For a check on deflection, we need to consider the both short term and long term deflection. We will cover the factors which are primarily responsible to influence these limit states of serviceability. As we can see during normal day to day experiences, normally a structure gets fail in serviceability and NOT in collapse.

10.2 The Limit States of Serviceability-Deflection and Cracking

As per limit state design philosophy, there are two types of limit state that are to be considered in design viz. ultimate limit states and serviceability limit states. The former i.e. the ultimate limit state deals with safety in terms of strength, overturning, sliding, buckling, fatigue fracture, etc., and the latter i.e. serviceability limit state deals with serviceability in terms of deflection, cracking, vibration, etc. The major consideration in structural design by the LSM is to ensure both safety and serviceability, so that the structure performs its intended function satisfactorily.

Use of limit state method of design with higher grades of concrete and steel has lead to smaller sections, which are otherwise large in the traditional working stress method of design. Thus those structures must be checked for deflection and cracking. Cl. 42 and 43 of IS 456: 2000 stipulates not to check every normal structure for deflection and crack widths provided codal provisions for limiting *l/d* ratios and maximum permissible reinforcement spacing recommendations are followed. However many a times (like where the dead and live loads are very heavy), it is specifically required to calculate the crack widths and deflections in the structure.

Remember: Structures often fail in terms of serviceability and rarely fail in terms of safety.

Cl. 35.1.1 of IS 456: 2000 states that the designer should consider all the relevant limit states to ensure a suitable degree of safety and serviceability. Cl. 35.3.1 of IS 456: 2000 specifies limit state of serviceability comprising **deflection** and Cl. 35.3.2 of IS 456: 2000 specifies limit state of serviceability comprising **cracking**. Cl. 8 of IS 456: 2000 refers to **durability** consideration of concrete. Stability of the structure against **overturning** and **sliding (as** per Cl. 20 of IS 456: 2000) and **fire resistance** (as per Cl. 21 of IS 456: 2000) are other indispensable aspects to be taken into account while designing reinforced concrete structures.

Use of limit state design with improved grades of concrete and steel have resulted in concrete sections that are thinner in size and higher stress levels at service loads. Consequently, the structures undergo larger deflections, crack widths, vibrations etc. Thus the requirement of serviceability limit state check has attained a greater importance now.

10.3 Limit State of Serviceability: Deflection

The limits on deflection of flexural members are prescribed in order to have:

1. Aesthetic and psychological comfort to occupants.
2. A limit on crack width in the structures.
3. Proper functioning of attached and unattached floor and other structural elements.
4. No ponding in roof slabs.

Short Term Deflection: It refers to immediate deflection after casting of concrete and application of partial or full service loads. The following factors affect the short term deflection of structures:

1. Live loads magnitude and its distribution on the structure.
2. Span and type of end supports.
3. Cross sectional areas of the elements of the structure.
4. Amount of reinforcement and stress developed in the reinforcement.
5. Characteristic strengths of concrete and steel used in the structure.
6. Amount and extent of cracks developed.

Long Term Deflection: It occurs over a long period of time mainly due to creep and shrinkage. **Long term deflection is almost 2-3 times the short term deflection**. The following factors affect the long term deflection of concrete:

1. Humidity and temperature range during the time of curing.
2. Age of concrete at the time of loading.
3. Type and the size of aggregates, water-cement ratio, member sizes, amount of compressive steel which ultimately affects the creep and shrinkage of concrete.

10.3.1 Control of Deflection

Cl. 23.2 of IS 456: 2000 specifies limiting deflection under two groups as given below:

1. The maximum final deflection should not normally exceed **span/250** due to all loads including the long term effects of creep, shrinkage and temperature and measured from as cast level of the support of floors, roofs and other horizontal elements of the structure.

2. The maximum deflection should not normally exceed the lesser of **span/350 or 20 mm** including the effects of temperature, creep and shrinkage occurring after the erections of partitions and application of finishes.

Both of the above requirements must be satisfied by a structure.

10.3.2 Selection of Preliminary Dimensions of the Structural Elements

The above two requirements for deflection control are checked after designing the structure. But the structural design has to be revised if it fails to satisfy either condition (1) or (2) or both as stated in section 10.3.1. In order to avoid all this repetitive type of work, IS 456: 2000 recommends to assume some initial dimensions of the structural elements which will generally satisfy the deflection limits. Cl. 23.2.1 of IS 456: 2000 specifies different ratios of effective span to effective depth and Cl. 23.3 of IS 456: 2000 limits slenderness of simply supported, continuous and cantilever beams.

For deflection requirement : Basic values of effective span to effective depth ratios for span upto 10 m and different support conditions are specified in IS 456: 2000, which is given as below:

Table 10.1: *Limiting span to depth ratios*

S. No.	Item	Cantilever	Simply Supported	Continuous
1.	Basic values of effective span to effective depth ratios for spans up to 10 m	7	20	26
2.	Modification factor for span > 10 m	Not applicable as deflection calculations are to be done	Multiply basic values of row 1 by (10/span in m)	
3.	Modification factor depending on area and stress in steel	Multiply values of Row 1 or Row 2 (as the case may be) with modification factors given in Figure 4 of IS 456: 2000		
4.	Modification factor depending on area of compression steel	Further multiply earlier respective value with modification factors given in Figure 5 of IS 456: 2000		
5.	Modification factors for flanged beams	(i) Modify values of Row 1 or Row 2 (as the case may be) as per values given in Fig. 6 of IS 456: 2000 (ii) Further modify as per Row 3 and/or Row 4 where reinforcement percentage to be used on area of section equal to $b_f d$ where b_f is the width of rib.		

For lateral stability : Cl. 23.3 of IS 456: 2000 specifies the following requirements

1. For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the minimum of $60b$ or $250b^2/d$, where b is the width of compression face of the beam midway between the lateral restraints and d is the effective depth of the beam.
2. For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the minimum of $25b$ or $100b^2/d$.

All limits prescribed in IS 456: 2000 (Cl: 23.2) with reference to deflections are based on service loads and thus partial factor of safety to characteristic loads should be unity (one). Cl. 36.4.2.2 of IS 456: 2000 states that the modulus of elasticity and other relevant parameters to be considered in deflection calculations should be based on *characteristic strengths of steel and concrete.*



Why Precise Prediction of Deflection is Difficult?

Precise prediction of deflections in reinforced concrete members is difficult because:

1. There are uncertainties in predicting the flexural rigidity (EI) of the member which depends on degree of tensile cracking, amount of flexural reinforcement, modulus of elasticity of concrete, modulus of rupture of concrete.

2. There are other environmental and time dependent effects which influence shrinkage and creep.
3. Concrete members shows inelastic behavior also.
4. There are large variations in observed deflections even under laboratory conditions.

Due to the above said reasons, approximations and simplifications are necessary for calculation of deflections. The deflection calculation is considered in two parts viz. immediate or short term deflection and long term deflection. Calculation of short term deflection involves the consideration of full dead and live loads. However for calculating the long term deflection due to creep and shrinkage, the permanent load i.e. dead load and that part of live load which is sustained to the structure are considered. Even after making suitable approximations and simplifications, the deflection calculations are quite rigorous and complicated.

10.3.3 Calculation of Short-term Deflection

Short term deflection under service loads are calculated assuming linear elastic behavior of concrete and steel. Usual expressions for maximum deflections in beams subjected to different types of loading are covered in the subject of *Structural Analysis*. In the deflection calculations, an important parameter is the flexural rigidity (EI) where 'I' is the second moment of area of the member section and 'E' is the short term modulus of elasticity of concrete which is expressed as:

$$E = 5000\sqrt{f_{ck}} \qquad \text{(As per IS 456: 2000)}$$

$$E = 5700\sqrt{f_{ck}} \qquad \text{(As per IS 456: 1978)}$$

The second moment of area of the section (I) depends on amount of reinforcement and amount of flexural cracking which in turn depends on the applied bending moment and the modulus of rupture of concrete.

Cl. C-2 of Annexure C of IS 456: 2000 specifies the steps of calculating the short term deflections. The said expression is based on earlier version of the British Code. The code recommends the usual methods for elastic deflections using short term modulus of elasticity of concrete (E_c) and effective moment of inertia (I_{eff}) which is given as:

$$I_{eff} = \frac{I_{cr}}{1.2 - \frac{M_{cr}}{M}\left(\frac{z}{d}\right)\left(1-\frac{x}{d}\right)\left(\frac{b_w}{b}\right)}$$

But $I_{cr} \le I_{eff} \le I_{gr}$

where,
- I_{cr} = Moment of inertia of cracked section
- M_{cr} = Cracking moment ($=f_{cr}I_{gr}/y_t$) where f_{cr} is the modulus of rupture of concrete
- I_{gr} = Moment of inertia of gross section about the centroidal axis ignoring the reinforcement
- y_t = Distance of extreme fibre in tension from the centroidal axis of gross section ignoring the reinforcement
- M = Maximum moment under service loads
- z = Lever arm
- x = Depth of neutral axis
- d = Effective depth
- b_w = Width of web
- b = Width of compression face

For continuous beams, the values of I_{gr}, I_r and M_r has to be modified by the following equation:

$$X_e = k_1\left[\frac{X_1 + X_2}{2}\right] + (1-k_1)X_0$$

where, X_e = Modified value of X, X_1, X_2 = Values of X at supports
X_o = Value of X at mid span, k_1 = Coefficient given in **Table 25 of IS 456: 2000**
X = Values of I_{gr}, I_r or M_r as desired

Table 10.2 : *Values of Coefficient k_1*

k_2	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_1	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Here k_2 is given by:

$$k_2 = \frac{M_1 + M_2}{M_{F1} + M_{F2}}$$

where, M_{F1}, M_{F2} = Fixed end moments
M_1, M_2 = Support moments

Additional Short-term Deflection due to Live Loads alone

As described at the start of this chapter that the deflection calculations require calculation of deflections after the structure has been constructed (including the long term effects of creep and shrinkage). This needs the calculation of deflections due to live loads alone.

Now because of the fact that with the variation in flexural rigidity (EI) with applied moment, the load deflection behavior of reinforced concrete member is non-linear. This implies that principle of superposition cannot be applied in deflection calculations.

Thus deflection due to live loads alone is computed from the difference of deflection due to both dead and live loads and the deflection due to dead loads alone.

10.3.4 Long-term Deflections

The deflection of reinforced concrete members increase with time because of the following reasons:

1. Variation in temperature which results in differential shrinkage in concrete members.
2. Creep due to sustained loading.

The combined long term deflection due to creep, shrinkage and temperature may be about 2-3 times the short term deflection. However, the excessive short term deflections can be nullified by providing a camber (just like in slab casting), but this cannot be applied for long term deflections in the structure.

NOTE

If observed deflections in a structure just after its completion are within the prescribed limits, it cannot be said with 100% surety that the same structure will meet the deflection requirements in long run also. In fact, long term deflections gradually build up over a period of time to such an extent that the total deflection exceeds the prescribed deflection limits.
Other factors which increase the long term deflection are:

1. Formation of new cracks.
2. Widening/enlarging of existing cracks.
3. Repeated load cycles acting on the structure.

10.3.5 Calculation of Deflection Due to Shrinkage

As usually observed, in unrestrained concrete members, drying shrinkage of concrete shortens the member but the embedded reinforcing steel does not shorten to same extent. This introduces compressive stress in reinforcing steel and tensile stress in the surrounding concrete.

When reinforcement is placed symmetrically in a concrete section, except in indeterminate structures, no curvature results due to shrinkage. But unsymmetrically placed reinforcement in concrete section (which is usually the case) results differential strains across the section.

Cl. C-3 of Annexure-C of IS 456: 2000 describes the method of calculating the deflection due to shrinkage (α_{cs}) as:

$$\alpha_{cs} = k_3 \Psi_{cs} l^2$$

where, k_3 = Constant which depends on support conditions

= 0.5 for cantilevers

= 0.125 for simply supported members

= 0.086 for members continuous at one end

= 0.063 for fully continuous members

Ψ_{cs} = shrinkage curvature = $k_4(\varepsilon_{cs}/D)$ where ε_{cs} is the ultimate shrinkage strain of concrete

$$k_4 = 0.72\left(\frac{P_t - P_c}{\sqrt{P_t}}\right) \le 1.0 \quad \text{for } 0.25 \le (P_t - P_c) < 1.0$$

$$= 0.65\left(\frac{P_t - P_c}{\sqrt{P_t}}\right) \le 1.0 \quad \text{for } (P_t - P_c) \ge 1.0$$

where, $p_t = 100\frac{A_{st}}{bd}$ and $p_c = 100\frac{A_{sc}}{bd}$

D is the total depth of section and l is the span length

10.3.6 Calculation of Deflection Due to Creep

It is a usual fact that under sustained loading, the compressive strain in concrete keeps increasing in a non-linear fashion. This phenomenon is referred as *creep*. The creep coefficient is defined as:

$$C_t = \frac{\text{Creep strain } (\epsilon_{cp})}{\text{Initial elastic strain } (\alpha_i)}$$

This creep coefficient provides a method to calculate creep in concrete.

The maximum value of creep coefficient (C_t) is called as the **ultimate creep coefficient (denoted as θ in IS 456: 2000).** This ultimate creep coefficient (θ) is used for assessing the maximum deflection in a concrete member due to creep.

Cl. 6.2.5.1 of IS 456: 2000 states that in the absence of any data on factors influencing creep, the value of ultimate creep coefficient (θ) can be taken as 2.2, 1.6 and 1.1 respectively for loading ages of 7 days, 28 days and 1 year (or 365 days).

The distribution of creep strain across the depth of concrete section is non uniform and it in fact varies linearly similar to the flexural strain produced by applied bending moment.

NOTE

Creep strain is primarily related to increase in compressive strain of concrete but there is a marginal increase in tensile strain in steel. Due to creep in concrete, there is small increase in the depth of neutral axis thereby reducing the lever arm. In order to maintain the static equilibrium with the applied loading (or applied moment), there is a little increase in the steel stress which in turn increases the strain in steel.

Cl. C-4 of Annexure C of IS 456: 2000 describes the following method of calculating the deflection due to creep. The creep deflection due to permanent loads is given by:

$$\alpha_{cc(\text{perm})} = \alpha_{i,cc(\text{perm})} - \alpha_{i(\text{perm})}$$

where, $\alpha_{icc(\text{perm})}$ = Initial deflection plus creep deflections due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity E_{ce}, which is given by:

$E_{ce} = \frac{E_c}{(1+\theta)}$, where θ is the creep coefficient

$\alpha_{i(\text{perm})}$ = Short term deflection due to permanent load using E_c

10.3.7 Deflection Due to the Effect of Temperature

From the *Structural Analysis*, it is found that variation in temperature causes stresses in indeterminate structures. The calculation of bending moments due to variation in temperatures can be done by any of the principles described in *Structural Analysis*. This requires a suitable value of coefficient of thermal expansion. In addition to overall stresses in a structure, local stresses in members may also get induced due to unsymmetrical reinforcement. The calculation of such deflections is identical to calculations done for assessing the deflection due to differential shrinkage.

In common design practice, deflections due to temperature is usually NOT computed because such deflections are not significant and are also reversible in nature owing to increase or decrease in temperature. But tensile stresses and thus resulting tensile cracks induced by the restraints against temperature changes can be quite significant which require special attention by proper detailing of reinforcement.

10.3.8 Check on Total Deflection

Once we have calculated the deflections due to various components viz. short term deflection and long term deflection, the overall total deflection need to be checked with the limit prescribed in **Cl. 23.2 of IS 456: 2000.**

$$\Delta_{DL+LL} + \Delta_{long\text{-}term} \le \frac{l}{250}$$

$$\Delta_{LL} + \Delta_{long\text{-}term} \le \frac{l}{350} \text{ or } 20 \text{ mm} \quad \text{(whichever is less)}$$

where, $\Delta_{long\text{-}term} = \Delta_{shrinkage} + \Delta_{creep} + \Delta_{temperature}$

and l = Effective span of the member

If these limits exceed, then the member needs to be redesigned by increasing the depth (and thereby stiffness) of the member. If there are limitations on the depth of the member to be provided, then grade of concrete has to be improved thereby increasing the modulus of elasticity of concrete. Provision of compression reinforcement also reduces the deflection in members.

10.4 Limit State of Serviceability: Cracking

Cracking in concrete occurs whenever the tensile stress of concrete exceeds the permissible limit. The tensile strength of concrete is usually very low and thus it is generally not possible to have almost zero cracks in concrete. But proper design of concrete member can control the cracks in terms of crack widths and spacing of cracks.

Disadvantages of Cracking in Concrete

The occurrence of cracks in concrete brings with it the following drawbacks:

1. Cracks in concrete spoil aesthetic of structure.
2. Cracks reduce the durability of concrete.
3. Development of cracks in concrete leads to corrosion of reinforcement.
4. In liquid retaining structures, appearance of cracks can in fact adversely affect the functional requirement of the structure at all.

Thus it is always tried to limit the crack width in concrete structures.

Cracking in concrete occurs due to the following reasons:

1. Flexural tensile stresses due to the applied loadings.
2. Shear and torsion induces diagonal tension in concrete members.
3. Restraints against volume changes due to variation in temperatures.
4. Very high compressive stresses induce lateral tensile strains owing to Poisson's effect.
5. Direct tensile stress under the applied loads (like hoop tension in circular tanks).

As stated above, cracking in concrete occurs in tension resulting in splitting of concrete at the surface which penetrates inwards. The consequent appearance of cracks in concrete, their spacing and widths depend on magnitude of tensile stresses, reinforcement detailing, grade and properties of concrete and the thickness/depth of the concrete section/member. It is always desirable to have closely spaced cracks with smaller crack widths rather than widely spaced cracks with large crack widths. The later type of cracks are associated with relatively low percentages of steel, widely spaced reinforcing bars of high strength steel (Fe 415 and Fe 500).

Factors Affecting Crack Widths

The following factors affect the wodth of cracks in reinforced concrete members:

1. Tensile stresses in the reinforcing bars.
2. Bond strength and tensile strength of concrete.
3. Depth of the concrete member and the location of neutral axis.
4. Diameter and spacing of reinforcing bars.
5. Concrete cover.

10.4.1 Limits on Crack Widths

Cl. 35.3.2 of IS 456: 2000 limits the width of cracks to a maximum limit of 0.3 mm for mild exposure conditions. This limit is based purely on aesthetic considerations. Fortunately, this limit on crack width is also found to be adequate for durability purpose also.

A lower limit of crack width of 0.2 mm is prescribed for structures exposed to continuous moisture conditions or which are in direct contact with soil or the ground water ('moderate' exposure condition). This limit on crack width is further reduced to 0.1 mm for severe, very severe and extreme exposure conditions.

Earlier versions of IS 456: 2000 didn't provide any procedure to calculate the crack width. But however IS 456: 2000 (Annexure F) provides a procedure to calculate crack widths. Cl. 43.1 of IS 456: 2000 states that explicit calculations for crack widths are required where reinforcement spacing exceeds the prescribed limits irrespective of the value of cover to reinforcement provided.

The MAJOR DRAWBACK in IS 456: 2000 is that the earlier limit (IS 456: 1978) on clear cover of 75 mm has been eliminated. Increased cover to reinforcement implies increased cracks in concrete especially in flexural concrete members.

'Annexure F' of IS 456: 2000 describes the expression for calculating the crack widths as,

$$w_{cr} = \frac{3a_{cr}\,\varepsilon_m}{1+2(a_{cr}-C_{min})/(D-x)}$$

where, D = Overall depth of the member
C_{min} = Minimum cover to the main longitudinal bar
x = Depth of neutral axis
a_{cr} = Distance from the point considered to the surface of the nearest longitudinal bar
ε_m = Average strain at the level where cracking is being considered (steel level)

10.5 Other Limit States of Serviceability

Some other limit states of serviceability are:

1. **Tension Stiffening Effect:** The increase in the stiffness of concrete over and above the cracked section stiffness due to the ability of concrete between the cracks to take tension is called as tension stiffening effect.
2. **Creep in steel:** Steel itself is NOT at all subjected to creep but due to creep in concrete, there is a slight increase in the actual depth of neutral axis thereby reducing the lever arm of the member section. In order to maintain the static equilibrium with the applied moments, there is slight increase in the steel stress and consequently the steel strain.
3. **Cracking in concrete:** Practically it is very difficult to avoid cracks in concrete and thus it is always beneficial to have a large number of well distributed hair line cracks in concrete rather than a few wide cracks. This is the main objective of Limit State of Serviceability of Cracking for design of a member by Limit State Method.

Example 10.1 There is a cantilever beam of span 6000 mm of cross-section 200 × 500 mm. Check the beam for deflection and lateral stability.

Solution:

Check for deflection

For cantilever, minimum effective depth required

$$= \frac{\text{span}}{7} = \frac{6000}{7} = 857 \text{ mm}$$

But effective depth available (d) = 450 mm (Assuming 50 mm effective cover)

∴ Beam fails in deflection

Check for lateral stability

For lateral stability, span $\leq$ { (1) $25b = 25 \times 200$ mm $= 5000$ mm ; (2) $\frac{100b^2}{d} = \frac{100 \times 200^2}{450} = 8888.9$ mm } whichever is small

$= 5000$ mm

But, actual span $= 6000$ mm > 5000 mm

$\therefore$ Beam fails in lateral stability

$\therefore$ The above beam fails in both deflection and lateral stability.

Objective Brain Teasers

Q.1 In case of rectangular RC beams, shrinkage deflection can be reduced by
(a) providing less compression steel than tension steel
(b) providing more compression steel than tension steel
(c) providing equal tension and compression steel
(d) tension steel greater than compression steel by $33\frac{1}{3}\%$

Q.2 In composite construction, the effect of creep and shrinkage
(a) can be ignored at the ultimate limit state since large inelastic strains are developed
(b) can be ignored at limit state of serviceability
(c) can be totally eliminated by removing the props after 28 days
(d) insufficient data

Q.3 Unequal top and bottom reinforcement in a beam section leads to
(a) large deflection (b) shrinkage deflection
(c) creep deflection (d) long term deflection

Answers

1. (c) 2. (a) 3. (b)

CHAPTER 11

Two Way Slab Design

11.1 Introduction

In this chapter, we will identify different types of slabs with different end conditions. We shall state the limits of span ratio (or aspect ratio) i.e. l_y/l_x to decide whether the slab is one way or two way, load sharing pattern of slab by supporting beams and effective span of slabs. The various methods available for the design of two way slabs will be briefly discussed with special emphasis on *IS Code method* of slab design.

Further we will see that how the depth of slab helps in controlling the slab deflection which is an important limit state of serviceability. Also we will look into the variation of design shear strength of concrete in slabs with slab depth and percentage of reinforcement. We will determine the bending moments in slab (both positive and negative bending moments), state the maximum bar diameter which can be used in slabs and spacing along with placement of reinforcing bars. The aspect of torsion in slabs and corresponding torsion reinforcement will also be covered.

At last, we will look into the details of reinforcement in slabs as per IS 456: 2000.

11.2 One Way v/s Two Way Slabs

In case of one way slabs, the slab panel gets deformed primarily in one direction and the initially plain surface takes the form of a cylindrical surface in which curvatures and bending moments develop in one direction only. Main reinforcement is provided along this main direction of curvature and distribution reinforcement is provided in the normal/perpendicular direction.

On the other hand, in two way slabs, the slab panel gets deformed in two mutually perpendicular directions with significant curvatures taking the form which resembles a saucer. Thus, main reinforcement is provided in two mutually perpendicular directions. In the ideal case, the reinforcement in two-way slabs must be provided along the principal direction of moments at every point in the slab but this is generally not feasible and reinforcement in two way slab is provided in transverse and longitudinal directions only. Such slabs are called as **orthotropically reinforced slabs**. Orthotropically reinforced slab is said to be **isotropically reinforced slab** when bending moment per unit width of the slab is same in the two mutually perpendicular directions at the point under consideration.

Do You Know?

- Orthogonally placed set of longitudinal bars in slab is capable of developing the required flexural strength in any direction.
- In two way slabs, there exists multiple load paths and thus two way slabs have the capacity of considerable stress redistribution. In view of this fact, the reinforcement in two way slabs can be designed in many different ways.

In square slabs, the two way action is more or less equal in both the directions in the plane of the slab while in slabs where the length of the slab exceeds twice the width, two way action reduces predominantly to one way slab action only.

Different types of two way slabs are:

1. Slab supported on flexible beams
2. Slab supported on masonry walls/rigid beams
3. Slab supported directly on columns (like flat slabs etc.)

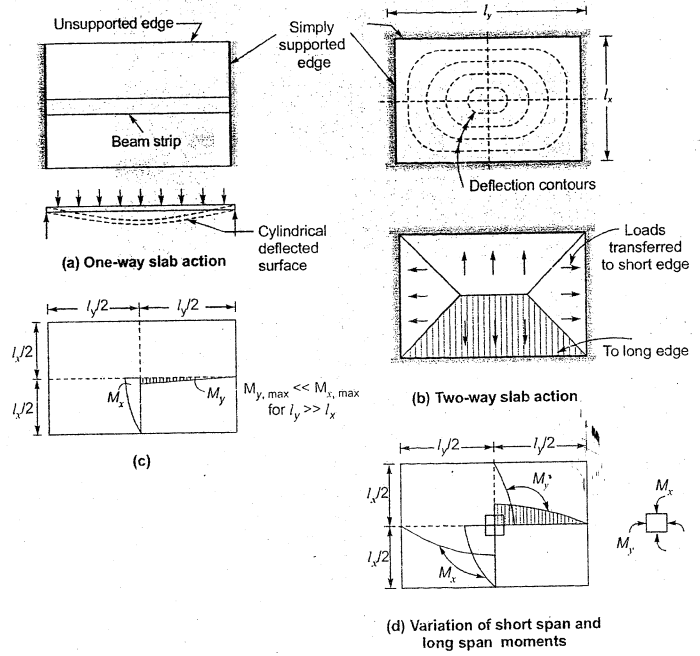


Fig. 11.1 *One way and two way types of slab*

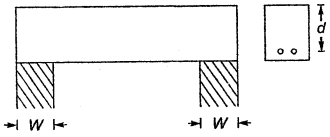
Some Important Provision Pertaining to Beam/Slab

1. Effective span (Cl. 22.2)

A. Simply supported beam/slab

Effective span

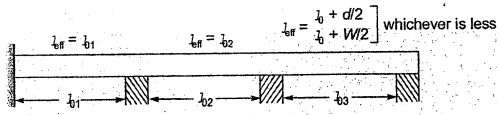
$$l_{eff} = \left.\begin{matrix} l_0 + \phi \\ l_0 + W \end{matrix}\right\}$$ whichevers is less



B. For continuous beam/slab

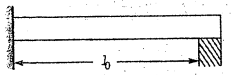
(i) If width of support (W) $< \frac{l_0}{12}$, then same as that of S.S. beam/slab case.

(ii) If width of support (W) $> \frac{l_0}{12}$

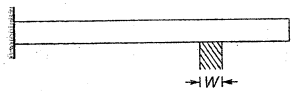


C. Cantilever beam

(i) With fixed end $l_{eff} = l_0 + \frac{d}{2}$



(ii) With continuous end $l_{eff} = l_0 + \frac{W}{2}$



Moment and shear coefficients for continuous beams:

Table 11.1 (IS : 456 : 2000) *Bending moment coefficient (Clause 22.5.1)*

Type of Load	Span Moments		Support Moments	
	Near Middle of End span	At Middle of Interior Span	At Support Next to the End Support	At Other Interior Supports
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

Note: For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span.

Table: 11.2 (IS 456 : 2000) *Shear for coefficients (Clause 22.5.1 and 22.5.2)*

Type of Load	At End Support	At Support Next to the End Support		At All Other Interior Supports
	Near Middle of End span	Outer side	Inner Side	
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

Note: For obtaining the shear force, the coefficient shall be multiplied by the total design load.

11.3 Dealing with Torsion in Two Way Slabs

Apart from two principal moments in slabs, torsional/twisting moments also get developed in slabs. These twisting moments are significant along the slab diagonals. For a simply supported, uniformly loaded square slab, the principal moment along the diagonal at regions close to corner of slab is negative (hogging) and hence downward reactions develop there. If such downward reactions are not amicably taken care of, then the corners of the slab will get lift up.

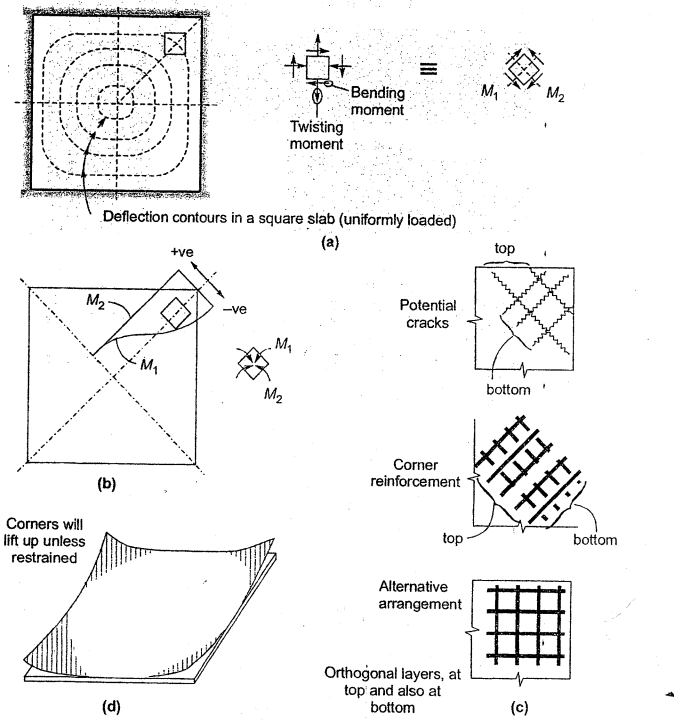


Fig. 11.2 *Torsion in two way slabs*

But in general, corners of the slab are prevented from getting lifted up by monolithic construction with beams. Such slabs are called as **torsionally restrained slabs**. For such slabs, separate reinforcement has to be provided at top along the diagonal at top and at bottom perpendicular to the diagonal or orthogonally in the transverse and longitudinal directions.

11.4 Wall-Supported and Column/Beam Supported Slabs

The uniformly distributed load '*w*' on a typical two-way slab is transmitted partly (w_x) along the short span to the long edge supports, and partly (w_y) along the long span to the short edge supports. In wall-supported panels, these load portions (w_x, w_y) of the load are transmitted by the respective wall supports directly to their foundations (or other supports) vertically below. On the other hand, when the edge supports comprise beams spanning between columns, the portion of the load transmitted by the slab in any one direction is in turn transmitted by the beam in the perpendicular direction to the two supporting columns.

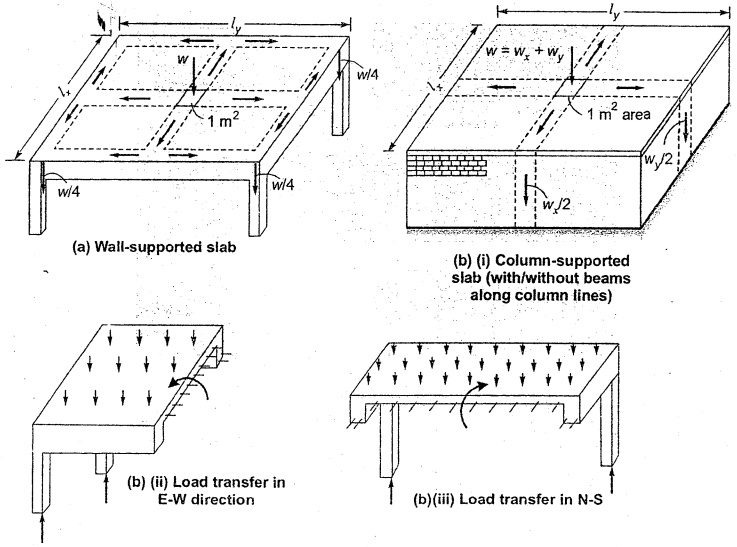


Fig. 11.3 *Wall Supported and Beam/Column Supported Slabs*

11.5 Design of Wall-supported (or Rigid Beam Supported) Two-Way Slabs

The design of wall-supported two-way slabs is very similar to one-way slabs. The thickness of the slab primarily depends on deflection control criteria and the reinforcements in the two mutually perpendicular directions are designed to resist the calculated maximum bending moments in the respective directions at the critical sections. Additional reinforcement may be required at the corners of two-way slabs depending on the support condition of the slab i.e. whether the slab corner is held down or is resting freely on the supports. The slab thickness should be sufficiently large so as to take care of shear forces in the slab but usually shear is not a big problem in two-way slabs subjected to uniformly distributed loads.

11.6 Thickness of Slabs

The initial proportioning of thickness of slab is based on span/depth ratios but effective span in short direction should be considered for this purpose. The percentage tension reinforcement in short direction for two way slab is usually smaller than that of one way slab for the same effective span. For preliminary design, the value of modification factor 'k_t' is generally taken as 1.5. The depth of the slab provided should finally be checked/verified for the actual percentage of tension reinforcement provided.

Remember



Special Case: For two-way slabs with spans up to 3.5 m and live loads not exceeding 3.0 kN/m², **Cl. 24.1, Note 2 of IS 456: 2000**, recommends the slab thickness (overall depth D) to be calculated directly as follows, without the need for further check with respect to deflection control criteria:

1. For mild steel (Fe 250 grade), $D > \frac{l_x}{35}$ for simply supported slabs
   $D > \frac{l_x}{40}$ for continuous slabs
2. For Fe 415 grade steel, $D > \frac{l_x}{28}$ for simply supported slabs
   $D > \frac{l_x}{32}$ for continuous slabs

11.7 Analysis of Two Way Slabs

In two way slabs, numerous load paths are possible for transferring the load. Availability of large number of load paths make these slabs capable of undergoing considerable **stress redistribution**. So reinforcement in such slabs can be designed in many possible ways and not just restricting to only one particular way of providing the reinforcement.

However two way slabs are highly statically indeterminate. Due to this, rigorous solutions are generally not feasible. Thus, standard classical solutions based on conventional **plate theory** are modified due to non-homogeneity and nonlinear behaviour of concrete. Such solutions have been proposed by **Westergaard** and many others in the form of **moment coefficients**.

Another approximate method is the **Rankine-Grashoff method**. This method simplifies the highly indeterminate problem to a determinate one. This method along with later versions given by **Marcus** is used widely in the world.

Modern computer methods include **finite difference method** and the **finite element method** which are in fact capable of solving any type and shape of slab.

The method which is particularly suitable for limit state and is relatively simple as **the inelastic method based on *yield line analysis***.

Cl. 24.4 of IS 456: 2000 recommends for designing two way slabs by any acceptable theory. IS 456 : 2000 describes the procedure for the following types of two way slabs:

1. Simply supported slabs whose corners are not restrained from lifting up (Cl. D-2 of IS 456: 2000).
2. Torsionally restrained slabs whose corners are restrained from lifting up and whose edges may be continuous or discontinuous (Cl. D-1 of IS 456: 2000).

11.7.1 Uniformly Loaded & Otherwise Simply Supported Rectangular Slab (Torsionally Unrestrained Slab)

Cl. D-2 of IS 456: 2000 describes moment coefficients in order to assess the maximum moments in the short and long span directions considering unit width of the slab. These moment coefficients are based on **Rankine Grashoff theory.**

As per this theory, the slab is divided into a series of orthogonal unit strips and load is be divided into the short span and long span strip such that deflection δ at the central portion/middle of strip is same for both the orthogonal strips. If influence of adjoining strips and torsion due to interconnecting strips are ignored then each of the two orthogonal strips can be treated as simply supported strips subjected to uniformly distributed load 'w_x' (on short span) and 'w_y' (on long span). Thus mid-point deflection δ is given by:

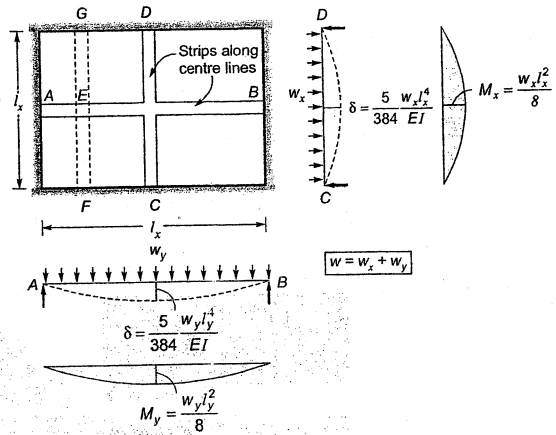


Fig. 11.4 *Explanation of Rankine Grashoff Theory*

$$\delta = \left(\frac{5}{384}\right)\frac{w_x l_x^4}{EI} = \left(\frac{5}{384}\right)\frac{w_y l_y^4}{EI}$$

Assuming that second moment of area (I) is same for both the strips.
From the above relation,

$$w_x = w_y\left(\frac{l_y}{l_x}\right)^4$$

Also, $w = w_x + w_y$

From the above equations,

$$w_x = \frac{wr^4}{\left(1+r^4\right)}, \quad w_y = \frac{w}{\left(1+r^4\right)}$$

where, $r = \frac{l_y}{l_x}$ = span ratio

Also maximum short span and maximum long span moments are,

$$M_x = \frac{w_x l_x^2}{8}, \quad M_y = \frac{w_y l_y^2}{8}$$

From above equations,

$$M_x = \alpha_x w l_x^2, \quad M_y = \alpha_y w l_x^2$$

where moment coefficients α_x and α_y are given by:

$$\alpha_x = \frac{r^4}{8\left(1+r^4\right)}, \quad \alpha_y = \frac{r^2}{8\left(1+r^4\right)}$$

These moment coefficients are given in **Table 27 of IS 456: 2000** which is as given below:

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	2.5	3.0
α_x	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118	0.122	0.124
α_y	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029	0.020	0.014

Reinforcement Detailing for Simply Supported Slabs

In slabs, the reinforcement is provided along the two orthogonal directions according to moments $M_x = \alpha_x w l_x^2$ and $M_y = \alpha_y w l_x^2$. This steel requirement progressively reduces as one proceeds away from the middle portion of the slab. But in practice, steel bars are provided equally spaced with spacing NOT to exceed $3d$ or 300 mm (whichever is less).

Also the bending moments in slab varies from a maximum value at mid span to zero at simply supported supports. Thus the reinforcement bars can be suitably curtailed as per the provisions of IS 456: 2000.

Cl. D-2.1.1 of IS 456: 2000 provides simple procedure for curtailment of bars according to which, upto 50 % bars may be terminated within a distance of $0.1l$ from the support while remaining bars extend fully into the supports.

For a purely simply supported slab, theoretically there are zero moments at the supports. But in practice, slab and beams are provided with some amount of fixity which induces some torsional moments. Thus it is beneficial to safe guard the slab against any such fixity. This is achieved by providing a separate top steel or bending up the alternate bottom bars to extend into the supports with area equal to 50% of bottom steel area.

NOTE: In recent practice, the bending up of alternate bars has been dispensed with and instead we go for separate layers of top and bottom steel.

11.7.2 Uniformly Loaded Restrained Rectangular Slab

Restrained Slab: Cl. D-1 of IS 456: 2000 defines restrained slabs are the slab whose corners are prevented from getting lifted up and are provided with suitable reinforcement to resist torsion and are called as restrained slab. All the four edges of the slab are assumed to be rigidly tied with the beams or walls underneath and the edges may be either continuous (fixed) or discontinuous. Thus nine different configurations of the slab are possible which are as shown below:

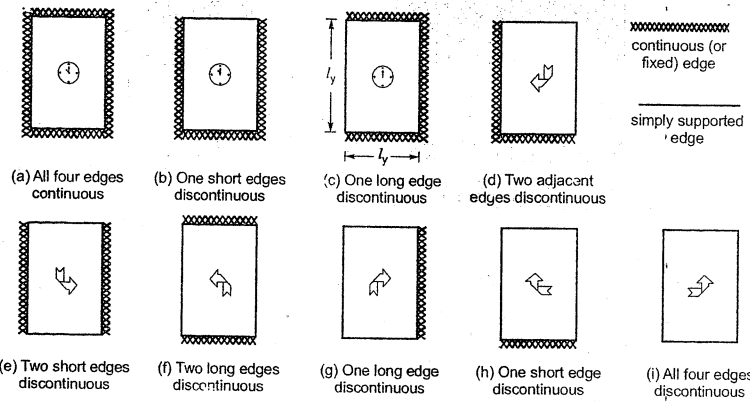


Fig. 11.5 *Nine different types of restrained two way slabs*

Corner restraints in the slab reduce the bending moment and deflection in the middle of slab just like fixed beams which reduce the mid-span moment and deflections as compared to simply supported beams.

The expression for the design moment coefficient for both continuous (fixed) and discontinuous edges of the slab are given by many authors based on elastic analysis and also based on approximate solutions out of which some of them are:

1. Marcus method 2. Pegeaud's method 3. IS Code method

1. **Marcus Method:** Modifications to approximate solutions (like Rankine Grashoff theory) were given by **Marcus** (called as **Marcus Correction**) wherein the moment coefficients are decreased in order to account for torsional restraints at the corners and also the torsional restraint provided by the longitudinal and transverse unit strips. The Marcus coefficient which is multiplied to the Rankine formula is given by:

$$C = \left(1 - \left(\frac{5r^2}{6(1+r^4)}\right)\right)$$

Thus $$M_x = \frac{C_1 w_x l_x^2}{8} = \left(1 - \left(\frac{5r^2}{6(1+r^4)}\right)\right)\left(\frac{r^4}{(1+r^4)}\right)\frac{wl_x^2}{8}$$

and $$M_y = \frac{C_1 w_y l_y^2}{8} = \left(1 - \left(\frac{5r^2}{6(1+r^4)}\right)\right)\left(\frac{r^2}{(1+r^4)}\right)\frac{wl_y^2}{8}$$

For example: For simply supported square slab ($l_y/l_x = 1$), Marcus correction reduces the design moment by about 42%.

2. **Pegeaud's Method:** In this method, the design moments are given by:

$$M_x = \frac{r_x' w l_x^2}{8}, \quad M_y = \frac{r_y' w l_y^2}{8}$$

where, r_x' and r_y' are the two moment coefficients the values of which can be read from tables.

3. **IS Code Method:** As per IS Code method of slab design, the **moment coefficients given in Cl. D-1 of IS 456: 2000 are based on inelastic analysis (yield line theory) and NOT on elastic theory.**

The analysis is based on following assumptions:

(i) The bottom steel in the two orthogonal directions is uniformly distributed over the middle strip which extends over 3/4th or 75% of span.

(ii) The edge strip lies on either side of the middle strip which collectively extends 1/4th or 25% of either span i.e. edge strips are of width $l_y/8$ or $l_x/8$.

(iii) Top steel is provided in the edge strip adjoining a continuous edge and perpendicular to the edge such that corresponding flexural strength (ultimate negative moment capacity) is 4/3 times (or 33% higher than) the corresponding ultimate positive moment capacity due to bottom steel provided in the middle strip in the said direction. Table 26 at Annexure 'D' of IS 456: 2000 gives the moment coefficients for both positive and negative moments in two way slabs for continuous and discontinuous edges (as the case may be).

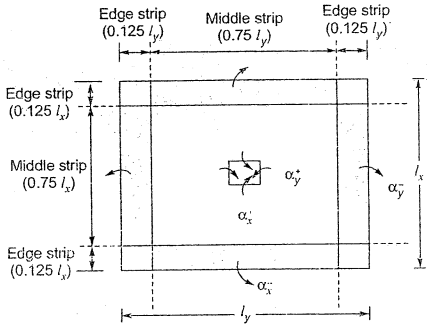


Fig. 11.6 *Explanation of 'Middle strip' and 'edge strips' in restrained two way slab*

Positive Moment Coefficients α_x^+ and α_y^+

The value of positive moment coefficient along long edge α_y^+ varies with the span or aspect ratio l_y/l_x. However, IS 456: 2000 recommends a constant value for the moment coefficient α_y^+ for all values of l_y/l_x.

The value of α_y^+ is obtained as:

$$\alpha_y^+ = \frac{\left(24 + 2n_d + 1.5n_d^2\right)}{1000}$$ Where, n_d = number of discontinuous edges.

For n_d values of 0,1,2,3 and 4, the values of α_y^+ obtained from the above expression are respectively 0.024, 0.0275, 0.034, 0.0435 and 0.056.

The value of α_x^+ can be obtained from α_y^+ for different values of l_y/l_x from the following expression:

$$\alpha_x^+ = \frac{2}{9}\left[\frac{3 - \sqrt{18\alpha_y^+(C_{S1} + C_{S2})/r}}{(C_{l1} + C_{l2})}\right]$$ where, $C = 1$ for discontinuous edge

$= \sqrt{\left(\frac{7}{3}\right)}$ for discontinuous edge

Here, the subscripts '*s*' and '*l*' denote respectively the short and long edge of the slab and the additional subscripts '1' and '2' denote the two edges in either direction i.e., short edge and long edge.

Negative moment coefficients α_x^- and α_y^-

As stated in assumption (iii) above, for 'yield line analysis', that the ultimate moment of resistance at continuous edge is 4/3 times of mid span moment, then,

$$\alpha^- = \left(\frac{4}{3}\right)\alpha^+$$ for continuous edge

$= 0$ for discontinuous edge

Reinforcement Detailing in Two Way Slabs

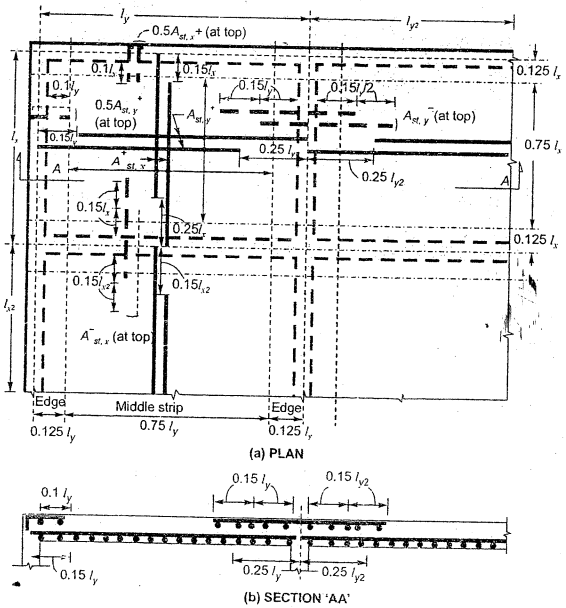


Fig. 11.7 *Typical flexural reinforcement detailing in two way slabs*

The following points must be noted for reinforcement detailing in two way slabs:

1. The bottom steel for the design moments $M_x^+ = \alpha_x^+ w l_x^2$ and $M_y^+ = \alpha_y^+ w l_x^2$ should be uniformly distributed in the middle strips in both the mutually orthogonal directions. Cl. D-1.4 of IS 456: 2000 states that bars should extend to within $0.25l$ of a continuous edge and $0.15l$ for discontinuous edge.
2. Top steel for the design moments $M_x^- = \alpha_x^- w l_x^2$ and $M_y^- = \alpha_y^- w l_x^2$ at continuous supports should be uniformly distributed in the edge strips in the short and long span directions respectively. Cl. D-1.5 of IS 456: 2000 recommends that at least 50% bars should extend to $0.3l$ from the support face of continuous support and $0.15l$ from the support face of discontinuous support.
3. There exists always a possibility of partial fixity at support at discontinuous edge which gives rise to occasional torsional moments in slab at discontinuous edges. Cl. D-1.6 of IS 456: 2000 recommends to use top steel with area 50% of bottom steel area at mid span in the same direction should be provided extending over a length of $0.1l$.
4. In the edge strips, distribution bars parallel to the edge should be provided at the top and bottom and tied up with the main reinforcing bars.

Torsional Reinforcement Detailing in Slab

Torsional reinforcement is provided in the form of a grid or mesh both at the top and bottom of slab. Cl. D-1.8 of IS 456: 2000 recommends that the torsional reinforcement grid/mesh should extend beyond the edge for a distance not less than 20% of shorter span i.e. $l_x/5$. The total area of torsional steel provided in each of the four layers should not be less than:

1. $0.75A_{stx}^+$ if both the meeting edges are discontinuous.
2. $0.375A_{stx}^+$ if one of the two meeting edges, one is continuous and other discontinuous.

Here A_{stx}^+ denotes area of flexural steel required for maximum mid span moment in the short direction.

As per Cl. D-1.10 of IS 456: 2000, no torsional reinforcement is required if both the meeting edges are continuous.

Design Negative Moments at Continuous Supports

Often it will be observed that design negative moments in the two slabs on either side of support are different. Now the question arises:

"For which moment, the negative reinforcement should be designed?"

As stated earlier, the moment coefficients given in table 26 of IS 456: 2000 are based on inelastic analysis (Yield Line theory) and NOT on elastic method. So **no moment distribution will be applicable** and we design the negative reinforcement for maximum negative moment only.

11.8 Shear in Two Way Uniformly Loaded Slabs

Slab, in general are not designed for shear forces since it is not a governing design criterion for uniformly loaded two way slabs. Distribution of shear forces at the slab edges is quite complex. However Cl.24.5 of IS 456: 2000 recommends the distribution of load in short edge as triangular and on long edge, as trapezoidal with lines forming the boundaries of the contributing areas making an angle of 45 degrees with either edge of the slab. Shear is to be considered at a distance '*d*' away from the face of the support.

The nominal shear stress in slab $\tau_v = \frac{V_u}{bd}$ should be less than '$k \cdot \tau_c$' where $b = 1000$ mm.

Here 'd' should be taken as average of 'd_x' and 'd_y' i.e. $\left(\frac{d_x + d_y}{2}\right)$.

The maximum factored shear force per unit width (V_u) is obtained as:

$V_u = w_u(0.5l_{xn} - d)$ where, l_{xn} = Clear short span

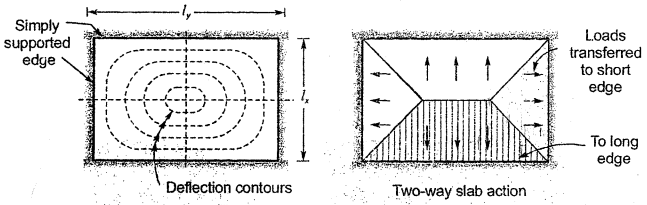


Fig. 11.8 *Assumed load distribution on slab edges in uniformly loaded slab*

Example 11.1 Calculate the effective depth required for a slab of span 5000 mm, when the area of steel provided is exactly same as the area of steel required (=0.4%) with Fe 415 steel.

Solution:

Since area of steel provided = Area of steel required

$\therefore \quad \frac{\text{Area of steel required}}{\text{Area of steel provided}} = 1$

$$f_s = 0.58 \times f_y \times \frac{\text{Area of steel required}}{\text{Area of steel provided}}$$

$\therefore \quad f_s = 0.58 \times 415 \times 1 = 240.7 \text{ N/mm}^2$

Percentage of steel = 0.4%

$\therefore$ Modification factor as per figure 4 of IS 456: 2000 = 1.2

$$\text{Minimum effective depth } (k_t) = \frac{5000}{7 \times 1.2} \text{mm} = 595.24 \text{ mm}$$

If area of steel provided is doubled then

$$f_s = 0.58 \times 415 \times \frac{1}{2} = 120.35 \text{ N/mm}^2$$

$\therefore$ Modification factor for 0.8% steel ($2 \times 0.4\%$) = 1.6

$$\text{Minimum effective depth } (k_t) = \frac{5000}{7 \times 1.6} \text{mm} = 446.43 \text{ mm}$$

If 0.5% compression steel is also provided then

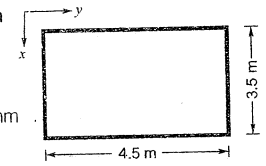
Modification factor as per figure 5 of IS 456: 2000 = 1.15

$$\text{Minimum effective depth } (d) = \frac{5000}{7 \times 1.6 \times 1.15} \text{mm} = 388.2 \text{ mm} \simeq 390 \text{ mm (say)}$$

Example 11.2 Design a two way slab for a room of size 3.5 m x 4.5 m subjected to live load of 4 kN/m^2. Use M30 concrete and Fe415 steel. Take into account the following two cases: (a) edges of slab are simply supported and corners are not held down, (b) edges of slab are simply supported and corners are held down.

Solution:

(a) Edges are simply supported and corners are not held down



Let overall thickness of slab = 150 mm

$\therefore$ Effective depth of slab assuming 15 mm as clear cover (d)

$$= 150 - 15 - \frac{10}{2} = 130 \text{ mm}$$

Effective span along y (l_y) = clear span + depth

= 4500 + 130 mm = 4630 mm

Effective span along x (l_x) = clear span + depth

= 3500 + 130 = 3630 mm

$$\frac{l_y}{l_x} = \frac{4630}{3630} = 1.28 < 2 \Rightarrow \text{Two way slab}$$

Short span moment (M_x) = $\alpha_x w l_x^2$

Short span moment (M_y) = $\alpha_y w l_x^2$

For $\frac{l_y}{l_x} = 1.28$,

$\alpha_x = 0.0912$ } (for simply supported slab)

$\alpha_y = 0.0558$ } (Table 27 of IS 456: 2000)

Loads

Self weight of slab = $0.15 \times 25 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2$

Live load = 4 kN/m^2

Assume miscellaneous loads like finishes etc = 1 kN/m^2

Total load = $3.75 + 4 + 1 = 8.75 \text{ kN/m}^2$

Factored load (w) = $1.5 \times 8.75 = 13.125 \text{ kN/m}^2$

$\therefore \quad M_x = \alpha_x w l_x^2 = 0.912 (13.125) 3.63^2 = 15.77 \text{ kNm/m}$

$M_y = \alpha_y w l_x^2 = 0.0558 (13.125) 3.63^2 = 9.65 \text{ kNm/m}$

Calculation of effective depth required

Ultimate moment of resistance for Fe415 is given by $0.138 f_{ck} bd^2$

$\therefore \quad 15.77 \times 10^6 \text{ Nmm/m} = 0.138 f_{ck} bd^2$

$\Rightarrow \quad 15.77 \times 10^6 = 0.138 (30) (1000) (d)^2$

$\Rightarrow \quad d = 61.72 \text{ mm}$

Adopt overall depth (D) = 100 mm

$\therefore$ Effective depth (d) = $100 - 15 - \frac{10}{2} = 80 \text{ mm}$

Self weight of slab ($= 0.1 \times 25 = 2.5 \text{ kN/m}^2$) need not to be received since already a higher design load is taken.

Reinforcement in short direction

$$R_x = \frac{M_x}{bd^2} = \frac{15.77\times10^6}{1000\times80^2} = 2.464$$

$$\therefore \quad \frac{p_{tx}}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-\frac{4.598R_x}{f_{ck}}}\right]$$

$$= \frac{30}{2(415)}\left[1-\sqrt{1-4.598\times\frac{2.464}{30}}\right] = 7.63\times10^{-3}$$

$$\therefore \quad p_{tx} = 0.763\%$$

$$\Rightarrow \quad A_{stx} = \frac{0.763}{100}\times1000\times80 = 610.432\ \text{mm}^2/\text{m}$$

Using 10 Φ bars, $\text{spacing} = \frac{1000A}{A_{stx}} = \frac{1000\times\frac{\pi}{4}\times10^2}{610.432} = 128.66$ mm c/c

Provide 10 Φ bars @ 120 c/c in short direction.

Reinforcement in long direction

$$R_y = \frac{M_y}{bd^2} = \frac{9.65\times10^6}{1000\times80^2} = 1.5078$$

$$\frac{p_{ty}}{100} = \frac{A_{sty}}{bd} = \frac{30}{2(415)}\left[1-\sqrt{1-4.598\frac{(1.5078)}{30}}\right] = 4.45\times10^{-3}$$

$$\therefore \quad p_{ty} = 0.445\%$$

$$\Rightarrow \quad A_{sty} = \frac{0.445}{100}\times1000\times80 = 356\ \text{mm}^2/\text{m}$$

Using 10 Φ bars, $\text{spacing} = \frac{1000\times\frac{\pi}{4}\times10^2}{356} = 220.62$ mm c/c

Provide 10 Φ bars @ 200 c/c in long direction.

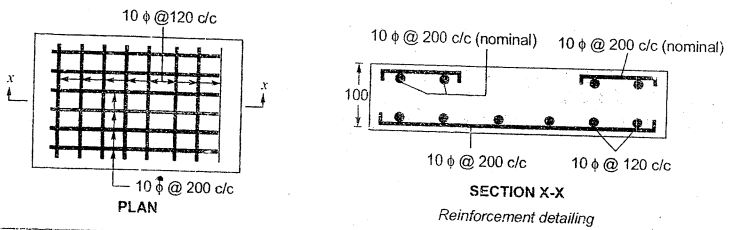
Check for minimum reinforcement

As per IS:456, minimum reinforcement should be 0.12%

$$= \frac{0.12}{100}\times1000\times80 = 96\text{mm}^2/\text{m}$$

$< A_{stx}$ (= 610.432 mm²/m)

$< A_{sty}$ (= 356 mm²/m) (OK)



Reinforcement detailing

(b) Edges are simply supported with corners held down

Let overall thickness of slab (D) = 130 mm

$\therefore$ Effective depth (d) = $130 - 15 - \frac{10}{2} = 110$ mm

$\therefore$ Effective short span (l_x) = 3.5 + 0.11 = 3.61 m

and, effective long span (l_y) = 4.5 + 0.11 = 4.61 m

Assuming that the slab is resting on 230 mm thick supports (one brick wall) centre to centre distance in short direction,

$$= 3.5 + \frac{0.23}{2} + \frac{0.23}{2} = 3.73\text{m}$$

Centre to centre distance in long direction = $4.5 + \frac{0.23}{2} + \frac{0.23}{2} = 4.73$ m

Now, Effective span = [Clear span + depth / c/c distance between the supports (which ever is small)]

Thus, l_x = [3.61 m / 3.73 m (whichever is less)] = 3.61 m

l_y = [4.61 m / 4.73 m (whichever is less)] = 4.61 m

$\frac{l_y}{l_x} = \frac{4.61}{3.61} = 1.28 < 2 \Rightarrow$ Two way slab

Loads

Self weight of slab = 0.13 × 25 = 3.25 kN/m²

Live load = 4 kN/m²

Assume miscellaneous loads = 1 kN/m²

Total load = 3.25 + 4 + 1 = 8.25 kN/m²

Factored load (w) = 1.5 × 8.25 = 12.375 kN/m²

Since all the four edges of the slab are discontinuous

This refers to case 9 of Table 26 of **IS 456: 2000**

$\therefore \quad \alpha_x^+ = 0.0776$

$\alpha_y^+ = 0.056$

$\therefore \quad M_x = \alpha_x w l_x^2 = 0.776\,(12.375)\,3.61^2 = 12.51$ kNm/m

$M_y = \alpha_y w l_x^2 = 0.056\,(12.375)\,3.61^2 = 9.03$ kNm/m

Calculation of effective depth required

$\therefore \quad 12.51\times10^6 = 0.138\,f_{ck}\,bd^2$

$\Rightarrow \quad 12.51\times10^6 = 0.138\,(30)\,(1000)\,(d)^2$

$\Rightarrow \quad d = 54.97$ mm

Adopt overall depth as (D) = 100 mm

$\therefore$ Effective depth (d) = $100 - 15 - \frac{20}{2} = 80$ mm

Reinforcement in short direction

$$R_x = \frac{M_x}{bd^2} = \frac{12.51\times10^6}{1000\times80^2} = 1.955$$

$$\therefore \quad \frac{p_{tx}}{100} = \frac{A_{stx}}{bd} = \frac{30}{2(415)}\left[1-\sqrt{1-\frac{4.598(1.955)}{30}}\right] = 5.8\times10^{-3}$$

$$\therefore \quad p_{tx} = 0.58\%$$

$$\Rightarrow \quad A_{stx} = 471.68\ \text{mm}^2/\text{m}$$

Using 10 mm diameter bars, $\text{spacing} = \dfrac{1000\times\frac{\pi}{4}\times10^2}{471.68} = 166.51\ \text{mm} \quad c/c$

Provide 10 Φ bars @ 150 c/c in short direction.

Reinforcement in long direction

$$R_y = \frac{M_y}{bd^2} = \frac{9.03\times10^6}{1000\times80^2} = 1.411$$

$$\therefore \quad \frac{p_{ty}}{100} = \frac{A_{sty}}{bd} = \frac{30}{2(415)}\left[1-\sqrt{1-4.598\frac{(1.411)}{30}}\right] = 4.146\times10^{-3}$$

$$\therefore \quad p_{ty} = 0.4146\%$$

$$\Rightarrow \quad A_{sty} = 331.69\ \text{mm}^2/\text{m}$$

Using 10 Φ bars, $\text{spacing} = \dfrac{1000\times\frac{\pi}{4}\times10^2}{331.69} = 236.79\text{mm c/c}$

Provide 10 Φ bars @ 200 c/c in long direction.

Check for minimum reinforcement

As per IS:456, minimum reinforcement = 0.12%

$$= \frac{0.12}{100}\times1000\times20 = 96\ \text{mm}^2/\text{m}$$

$$< A_{stx}$$

$$< A_{sty} \qquad \text{(OK)}$$

Shear check for slab

$$\text{Maximum shear force} = \frac{1}{2}wl_x = \frac{1}{2}(12.375)3.5 = 21.66\ \text{kN/m}$$

$$\text{Nominal shear stress } (\tau_v) = \frac{21.66\times10^3}{1000\times80} = 0.27\ \text{N/mm}^2$$

$$\text{Percentage of tension steel provided } (p_t) = \frac{A_{st}}{bd}\times100$$

where, $A_{st} = \dfrac{1000\times\frac{\pi}{4}\times10^2}{150} = 523.6\ \text{mm}^2/\text{m}$

$$= \frac{523.6}{1000\times80}\times100 = 0.6545\%$$

Design shear strength of M30 concrete for $p_t = 0.6545\%$ as per table 19 of IS:456 $(\tau_c) = 0.556\ \text{N/mm}^2$

For 130 mm thick slab, $k = 1.3$

$\therefore \quad k\tau_c = 1.3\,(0.556) = 0.7228\ \text{N/mm}^2 > \tau_v\ (= 0.27075\ \text{N/mm}^2)$

Thus slab is safe in shear.

Corner reinforcement

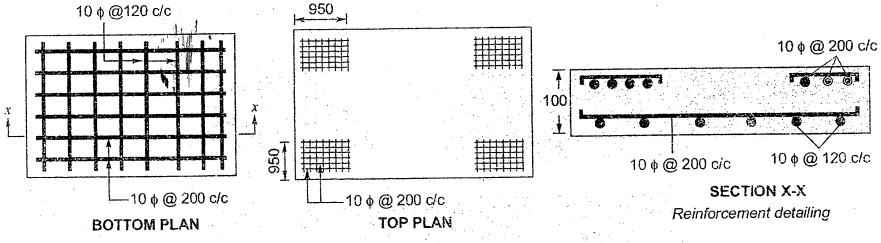
Area of each layer of corner reinforcement = 75% of maximum mid span reinforcement

$$= 0.75\times523.6 = 392.7\ \text{mm}^2$$

Using 10 Φ bars @ 100 mm c/c, $A_s = \dfrac{\frac{\pi}{4}\times10^2\times1000}{100} = 785.4\text{mm}^2 > 392.7\text{mm}^2$ along both short and long directions in four layers.

$$\text{Length of corner reinforcement} = \frac{1}{5}(4610) = 922\text{mm} = 950\text{mm (say)}$$

Provide corner reinforcement for 950 mm length.



Reinforcement detailing

Example 11.3 Design a two way slab of size 4.2 m x 5.50 m clear span carrying a live load 10 kN/m², and load of water proofing of 70 mm thick brick bats treatment. The slab is continuous over 2 adjacent edges and other two edges are discontinuous. Use M 25 concrete and Fe 415. Width of support is 250 mm. (Use IS code methods)

Solution:

Slab depth required

It is a continuous slab

Consider overall depth of slab, $D = 150$ mm

Effective depth of slab, $d = 120$ mm

Effective span

$$\frac{\text{span}}{12} = \frac{4200}{12} = 350\ \text{mm}$$

Width of the supports, $= 250\ \text{mm} < \dfrac{\text{span}}{12}$

Effective span along X: $l_x = l_0 + d = 4.2 + 0.12 = 4.32$ m

or $l_x = l_0$ + support width $= 4.2 + 0.25 = 4.45$ m

} whichever is less.

Effective span along Y: $l_y = l_0 + d = 5.5 + 0.12 = 5.62$ m

$$\frac{l_y}{l_x} = \frac{5.62}{4.32} = 1.3 < 2$$

$\therefore$ It is a two way slab.

Design bending moment

Bending moment coefficients as per Appendix-D of **IS 456: 2000**, for panel no. 4 and $\frac{l_y}{l_x} = 1.3$ are

$\alpha_x^- = 0.065$
$\alpha_x^+ = 0.049$
$\alpha_y^- = 0.047$
$\alpha_y^+ = 0.035$

Live load = $10 \text{ kN/m}^2 \times 1 \text{ m} \times 1 \text{ m} = 10 \text{ kN}$
Self weight of slab = $0.15 \times 1 \times 1 \times 25 = 3.75 \text{ kN}$
Dead load of water proofing = $0.07 \times 1 \times 1 \times 24 = 1.68 \text{ kN}$
Total load $W = 10 + 3.75 + 1.68 = 15.43 \text{ kN/m}^2$
Factored load $W_u = 1.5 \times 15.43 = 23.145 \text{ kN/m}^2$
Design moment $M_u = \alpha w_u l_x^2$
$= \alpha \times 23.145 \times 4.32^2 = 432.03\,\alpha$

	α	Design Moment (kNm)	Depth (mm)	A_{st} (mm²)	Spacing Required (mm)	Bar dia. (mm)	Spacing provided (mm)
M_x^-	0.065	28.08	120	720.2	109	10	100
M_x^+	0.049	21.17	120	527.3	148	10	140
M_y^-	0.047	20.30	120	503.9	155	10	150
M_y^+	0.035	15.12	110	367.9	213.5	10	210

Depth required, $d = \sqrt{\frac{M_u}{QB}} = \sqrt{\frac{28.08 \times 10^6}{0.138 \times 25 \times 1000}} = 90.22 \text{ mm} < 120 \text{ mm provided (OK)}$

Area of steel required

$$A_{st1} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.598 M_u}{f_{ck} B d^2}}\right] Bd$$

$$= \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.598 \times 28.08 \times 10^6}{25 \times 1000 \times 120^2}}\right] 1000 \times 120 = 720.28 \text{ mm}^2$$

Similarly $A_{st2} = 527.3 \text{ mm}^2$, $A_{st3} = 503.9 \text{ mm}^2$, $A_{st4} = 367.9 \text{ mm}^2$

Spacing of reinforcement

Spacing of 10 mm dia. bars $= \frac{1000}{A_{st}} \times \frac{\pi}{4}(10)^2 = \frac{78539.8}{A_{st}}$

Torsion reinforcement

At corner (1)

Size of the mesh $= \frac{l_x}{5} = \frac{4320}{5} = 864 \text{ mm}$

Area of steel required $= \frac{3}{4} \times 527.3 = 395.47 \text{ mm}^2$

Spacing of 10 mm dia. bars $= \frac{864}{A_{st}} \times \frac{\pi}{4} \times 10^2 = \frac{864}{395.47} \times \frac{\pi}{4} \times 10^2 = 171.6 \text{ mm} \approx 170 \text{ mm}$

At corners (2) and (3)

Size of the mesh = 864 mm

Area of steel $= \frac{3}{8} \times A_{st} = \frac{3}{8} \times 527.3 = 197.73 \text{ mm}^2$

Spacing of 10 mm dia. bars $= \frac{864}{197.73} \times \frac{\pi}{4} \times 10^2 = 343.18 \text{ mm say } 340 \text{ mm}$

Check for shear

Shear force along shorter edge $= \frac{1}{3} w_u l_x = \frac{1}{3} \times 23.15 \times 4.32 = 33.34 \text{ kN}$

Nominal shear stress $\tau_v = \frac{V_{ux}}{Bd} = \frac{33.34 \times 10^3}{1000 \times 120} = 0.277 \text{ N//mm}^2 < 0.28 \text{ N/mm}^2 (= \tau_{c\,max})$ (OK)

Shear force along long edge $V_{uy} = w_u l_x \left(\frac{r}{2+r}\right) = 23.15 \times 4.32 \left(\frac{1.3}{2+1.3}\right) = 39.397 \text{ kN}$

Nominal shear stress $\tau_v = \frac{V_{uy}}{Bd} = \frac{39.397 \times 10^3}{1000 \times 120} = 0.33 \text{ N/mm}^2$
$= 0.277 \text{ N//mm}^2 < 0.28 \text{ N/mm}^2 (= \tau_{c\,max})$ (OK)

Percentage of steel, $p_t = \frac{527.3}{1000 \times 120} \times 100 = 0.44\%$

$\tau_c = 0.36 \text{ N/mm}^2 < \tau_v$ (OK)

Check for bond

For reinforcement in x-direction, if curtailment is done 10 mm dia. @ 280 c/c

$$\tau_{bd} = \frac{V_{uy}}{\Sigma o.jd} = \frac{39.397 \times 10^3}{\frac{1000}{280} \times \pi \times 10 \times 0.8 \times 120} = 3.67 \text{ N/mm}^2$$

$\tau_{bd(per)} = 1.4 \times 1.6 = 2.24 \text{ N/mm}^2 < 3.67 \text{ N/mm}^2$
Failed

∴ Curtailment is not possible in this case.

Check for y-direction $V_{ux} = 33.34 \text{ kN}$

∴ $\tau_{bd(dev.)} = \frac{33.34 \times 10^3}{\frac{1000}{210} \times \pi \times 10 \times 0.8 \times 120} = 2.32 \text{ N/mm}^2$

Using 8 mm dia. bars, Spacing $= 210 \times \frac{8^2}{10^2} = 134.4 \text{ mm}$

Provide 8 mm dia. bars @ 130 mm c/c

∴ $\tau_{bd} = \frac{33.34 \times 10^3}{\frac{1000}{130} \times \pi \times 8 \times 0.8 \times 120} = 1.79 \text{ N/mm}^2 < 2.24 \text{ N/mm}^2$ (OK)

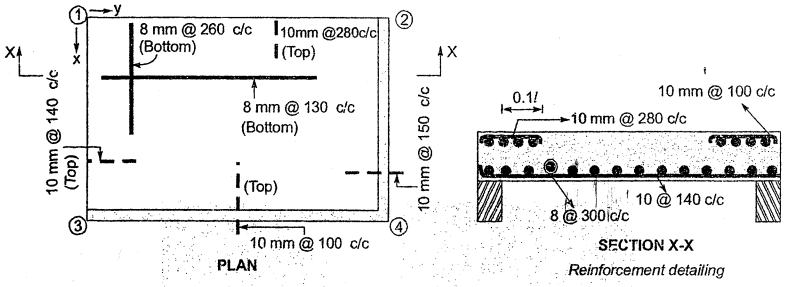
	Reinforcement	Cor. 1	Cor. 2	Cor. 3
M_x^-	10-100	10-140	10-280	10-100
M_x^+	10-140	10-140	10-140	10-140
M_y^-	10-150	8-86	10-150	8-130
M_y^+	8-130	8-65	8-130	8-130

Distribution bars

$$= \frac{0.12}{100}\times 1000\times 120 = 140 \text{ mm}^2$$

Spacing of 8 mm dia. bars $= \frac{1000}{144}\times\frac{\pi}{4}\times 8^2 = 349$ mm

Use 8 mm dia. bars @ 300 c/c



Reinforcement detailing

Example 11.4 Design a slab size 3.5 m × 5.0 m simply supported over 250 mm thick walls. The slab corners are prevented from lifting up. Design the slab for

Live load on slab = 4.5 kN/m²

Floor finishing = 50 mm thick c.c flooring

Use M 20 concrete and Fe 415 steel. Use WSM.

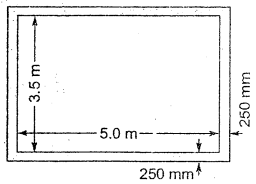
Solution:

Depth of slab

Let, overall depth of slab $= \frac{3500}{28} = 125$ mm

Effective depth of slab, $d = 100$ mm

Overall of depth of slab, $D = 130$ mm



Loads

Live load = 4.5 × 1 × 1 = 4.5 kN

Floor finish = 0.05 × 1 × 1 × 24 = 1.20 kN

Self weight of slab = 0.13 × 1 × 1 × 25 = 3.25 kN

Total load = 4.5 + 1.20 + 3.25 = 8.95 kN

Effective span

Effective span along x

$l_x = l_0 + d = 3.5 + 0.1 = 3.6$ m
$= l_0 +$ support width
$= 3.5 + 0.25 = 3.75$ m
$= 3.6$ m

} whichever is less.

Effective span of along y

$l_y = l_0 + d = 5 + 0.1 = 5.10$ m
$= l_0 +$ support width
$= 5 + 0.25 = 5.25$ m
$= 5.1$ m

} whichever is less.

$$\frac{l_y}{l_x} = \frac{5.10}{3.6} = 1.42 = r < 2$$

∴ It is two way slab.

Using Rankine formula with Marcus correction

$$M_x = cw_x\frac{l_x^2}{B} = \left[1-\frac{5}{6}\frac{r^2}{1+r^4}\right]\times\left(\frac{r^4}{1+r^4}\right)w\frac{l_x^2}{8}$$

$$= 0.668\times 0.802\times 8.95\times\frac{(3.6)^2}{8} = 7.78 \text{ kNm}$$

$$M_y = \frac{7.78\times l_y^2}{r^4\times l_x^2} = \frac{7.78\times 5.10^2}{1.42^4\times 3.6^2} = 3.85 \text{ kN}$$

Depth of slab required,

$d = 93.0$ mm < 100 m (OK)

$c = 7$, $t = 230$, m = 13

$k = 0.2835$, $j = 0.905$, $Q = 0.898$

$$d = \sqrt{\frac{M}{QB}} = \sqrt{\frac{7.76\times 10^6}{0.898\times 1000}}$$

Area of steel required

$$A_{stx} = \frac{7.78\times 10^6}{230\times 0.905\times 100} = 375.8 \text{ mm}^2 \approx 376 \text{ mm}$$

Spacing of 10 mm dia. bars

$$\text{Spacing} = \frac{1000}{376}\times\frac{\pi}{4}\times 10^2 = 208 \text{ mm} = 200 \text{ mm c/c (say)}$$

$$A_{sty} = \frac{3.85\times 10^6}{230\times 0.905\times 100} = 186 \text{ mm}^2$$

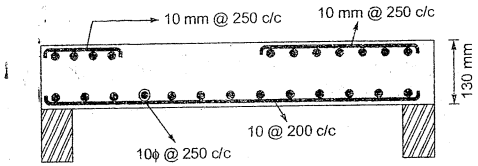
Using 8 mm dia. bars, spacing $= \frac{1000}{186}\times\frac{\pi}{4}(8)^2 = 270$ mm c/c

Distribution reinforcement @ 0.12%

$$= \frac{0.12}{100}\times 1000\times 130 = 156 \text{ mm}^2$$

Using 10 mm dia. bars, spacing $= \frac{1000}{156}\times\frac{\pi}{4}\times 10^2 = 503$ mm c/c

Provide 10 mm dia bars @ 250 c/c



Reinforcement Detailing

Objective Brain Teasers

Q.1 In two way slabs, a set of orthogonally placed reinforcement is capable of developing the moment resistance:
(a) In long span direction only
(b) In short span direction only
(c) In both long and short span direction
(d) In any direction

Q.2 As per IS Code method of two way slab design, if positive moment coefficient is 0.3 then the corresponding negative moment coefficient is:
(a) 0.4 (b) 0.3
(c) 0.25 (d) 0.55

Q.3 A two way continuous slab of short span 3.0 m and subjected to a live load of 2.5 kN/m^2 is reinforced with Fe 250 steel. The minimum depth of the slab as per deflection control criteria is:
(a) 100 mm (b) 150 mm
(c) 75 mm (d) 125 mm

Q.4 The moment coefficients given in IS: 456 for simply supported torsionally unrestrained slab are based on:
(a) Westergaard method
(b) Rankine Grashoff theory
(c) Yield line theory
(d) Marcus method

Q.5 In two way slab, designed as per IS method, moment distribution:
(a) Is applicable
(b) Is not applicable
(c) Depends on the type of end restraints in the slab
(d) Cannot be said

Q.6 The governing criterion in the design of two way slabs is:
(a) Flexural moments (b) Shear forces
(c) Cracking (d) Deflection

Q.7 The negative moment coefficient at discontinuous edge of a two way slab is:
(a) (4/3) times the positive moment coefficient
(b) (3/4) times the positive moment coefficient
(c) Zero
(d) None of the above

Q.8 The maximum bar diameter that can be used in a slab of overall depth (D) is:
(a) $D/5$ (b) $D/6$
(c) $D/8$ (d) $D/12$

Q.9 The Rankine Grashoff method of two way slab design:
(a) Converts the highly indeterminate problem in determinate one.
(b) Deals with determinate problems of slab design only.
(c) Gives the areas of reinforcement required in a slab.
(d) None of the above

Q.10 State true or false.
Two way slabs are designed for shear forces also.

Q.11 The limiting deflection in two way slab is
(a) primarily a functionof short span
(b) primarily a function of long span
(c) independent of long and short span
(d) None of the above

Q.12 A uniformly loaded slab is supported on all of its four edges with corners held down. The Marcus correction factor will
(a) always be > 1
(b) always be equal to 1
(c) always be < 1
(d) can't say with the provided data

Q.13 A two-way RC slab is provided with 500 mm^2 steel in the short direction. The slab corners are restrained. The torsional reinforcement required at the corner is
(a) 375 mm^2 (b) 188 mm^2
(c) 300 mm^2 (d) 200 mm^2

Q.14 A RC slab is exposed to mild exposure condition having main longitudinal bar of 12 mm. The minimum allowable nominal cover to satisfy durability requirement is
(a) 20 mm (b) 40 mm
(c) 50 mm (d) 15 mm

Q.15 Yield line theory results in
(a) upper bond solution
(b) lower bond solution
(c) unique solution
(d) data insufficient

Answers

1. (d)	2. (a)	3. (c)	4. (b)	5. (b)
6. (d)	7. (c)	8. (c)	9. (a)	10. false
11. (a)	12. (c)	13. (a)	14. (d)	15. (a)

Hints:

2. (a)
As per IS Code method, negative support moment is (4/3) times the corresponding positive mid span moment.

3. (c)
For continuous slab reinforced with Fe 250 steel and span upto 3.5 m and LL upto 3 kN/m^2, depth of slab is short span/40 = 3000/40 = 75 mm

11. (a)
Marcus connection factor = $\left(1-\frac{5}{6}\frac{\lambda^2}{1+\lambda^4}\right)$,
where, $\lambda = \frac{\text{long span}}{\text{short span}}$

Conventional Practice Questions

Q.1 A square slab of side 4.5 m is simply supported on all the four edges and is required to support a live load of 4 kN/m^2. Design the slab using M25 concrete and Fe415 steel.

Q.2 Design a two way slab 4 m x 6 m which is continuous on all the four edges and is supported on one brick thick masonry wall. The slab is required to sieve as a floor slab of a building. Use M30 concrete and Fe500 steel.

Q.3 Design a slab panel of size 6.5 m x 5.4 m (centre to centre) of an office building. The one short edge of the slab panel is discontinuous and all other three edge are continuous. Use M30 concrete and Fe500 steel.

■■■■

CHAPTER 12

Design of Compression Members/Columns

12.1 Introduction

In this chapter, we will define what effective length of a column is; classify the columns on the basis of slenderness ratios, types of reinforcement, loading type etc. We will look in to the IS 456: 2000 provisions of maximum and minimum percentage of steel in columns, minimum number and diameter of bars, type of lateral ties, their pitch etc. Also we will look into the aspect of minimum eccentricity in columns and why it should be considered.

Further we will derive the governing equations for strength of the columns under axial loading, axial loading with both uniaxial and biaxial bending and for slender/long columns. We will look into the interaction diagrams and will define the regions of tension failure, balanced failure and compression failure on the basis of location of neutral axis and will learn to design the columns accordingly.

The concept of analysis and design of long/slender columns is covered at last. The various design philosophies of long columns, their behaviour under the action of loads, additional eccentricities and associated additional moments etc. will be covered.

12.2 Compression Member/Column in a Structure

It is a type of structural element which is mainly subjected to axial forces (predominantly compressive forces) and/or occasionally tensile forces and often flexural moments. Cl. 25.1.1 of IS 456: 2000 defines 'column' as a compression member when effective length of the compression member exceeds three times the least lateral dimension. When effective length of the compression member is less than three times the least lateral dimension then it is referred to as pedestal as per Cl. 26.5.3.1(h) of IS 456: 2000.

12.3 Classification of Columns

Columns are classified on numerous bases like on the basis of reinforcement, on the basis of type of loading, on the basis of slenderness ratio and so on.

12.3.1 Classification of columns based on the type of reinforcement

Based on the type of column reinforcement, columns are classified as:

1. **Tied Columns:** In this type of column, the main reinforcement bars of the column are confined by lateral ties (Fig.12.1).
2. **Spiral Columns:** The reinforcing bars of the column are confined by spiral ties running continuously throughout the length of the column (Fig.12.2).
3. **Composite Columns:** Instead of main reinforcing bars, structural steel sections (like I-section, channel section, pipes etc.) are used. Alternatively, it can be said that this type of column is reinforced with structural steel section with or without longitudinal bars (Fig.12.3).

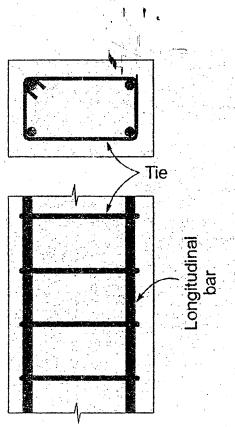


Fig. 12.1 *Tied Column*

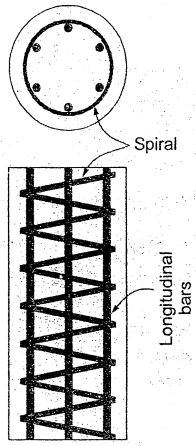


Fig. 12.2 *Spiral Column/spirally reinforced column*

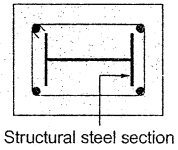


Fig. 12.3 *Composite Column*

12.3.2 Classification of Columns based on the Type of Loading

Based on the type of loading, columns are classified as:

1. **Axially loaded column:** The axial load on the column acts at zero eccentricity w.r.t. geometric centroidal axis of the column. However pure axial case in practical situations is very rare. Axial load on a column is invariably accompanied by flexure and sometimes shear (Fig. 12.4).
2. **Column with axial load and uniaxial bending:** Here the line of action of load is eccentric w.r.t. only one principal axis and zero eccentricity w.r.t. other principal axis. Moments get generated w.r.t. one principal axis only (Fig. 12.5).
3. **Column with axial load and biaxial bending:** This is the most general type of column. Here the line of action of axial load is eccentric w.r.t. two major axes of the column section. Here, moments gets generated in the two principal axes. Biaxial eccentricities in a column are very significant if it is located at the corners of a building (Fig. 12.6).

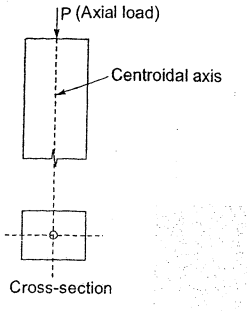


Fig. 12.4 *Column with pure axial load*

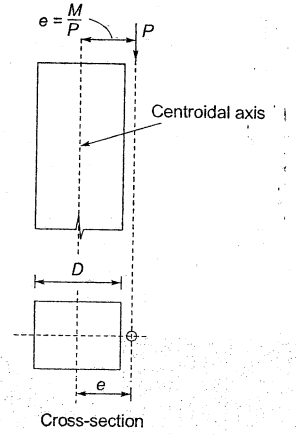


Fig. 12.5 *Column with axial load and uniaxial bending*

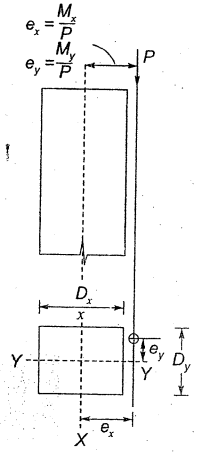


Fig. 12.6 *Column with axial load and biaxial bending*

12.3.3 Classification of Columns based on Slenderness Ratio

Slenderness ratio is a property which is associated with the geometry of the column section and is defined as:

$$\text{Slenderness ratio} = \frac{\text{Effective length of the column}}{\text{Least lateral cross-sectional dimension}}$$

Slenderness ratio describes the vulnerability of failure/collapse of a column by elastic buckling in the plane that is most vulnerable w.r.t. slenderness ratio. Columns with low slenderness ratio fail by material failure (i.e. crushing of concrete or steel) and not buckling but columns with high slenderness ratio fail by elastic buckling.

Thus based on slenderness ratio, columns are classified as:

1. **Short Column:** The slenderness ratio of the column w.r.t. major principal axis (l_{ex}/D_x) and w.r.t. minor principal axis (l_{ey}/D_y) both are **less** than 12 as per **Cl. 25.1.2 of IS 456: 2000.**
2. **Long/slender column:** The slenderness ratio w.r.t. major principal axis (l_{ex}/D_x) and w.r.t. minor principal axis (l_{ey}/D_y) both are **more** than 12 as per **Cl. 25.1.2 of IS 456: 2000.**

Table 28 (IS 456) should be provided here as it is required to calculate effective length.

Do You Know?

How to define slenderness ratio for non-rectangular and non-circular sections?

For such type of sections, instead of lateral dimension (in the expression of slenderness ratio), we use radius of gyration (r) of the section i.e.

$$\text{Slenderness ratio} = \frac{\text{Effective length of the column}}{\text{Radius of gyration } (r)}$$

12.4 Shear Consideration in Columns

Shear in columns is generally neglected owing to the fact that in columns, imposed shear stresses are quite low and shear resistance is high due to the presence of axial compressive force and also due to the lateral ties.

12.5 IS 456: 2000 Recommendations for the Design of Columns

12.5.1 Slenderness Ratio

Higher the slenderness ratio of a column lesser will be the strength because higher slenderness ratio induces secondary moments in the column (like in long/slender columns). Very slender columns fail by buckling without giving any warning prior to its failure. **IS 456: 2000** check this type of failure in a structure by limiting slenderness ratio.

Cl. 25.3.1 of IS 456: 2000 specifies the maximum value of ratio of unsupported length to lateral dimension as 60 i.e.

$$\frac{\text{Unsupported length}}{\text{Lateral dimension}} = \frac{l}{D} \le 60$$

Further, if one end of the column is free (in any direction) i.e. a **cantilever, Cl. 25.3.2 of IS 456: 2000** limits the unsupported length as:

$$\text{Unsupported length } (l) \le \frac{100b^2}{D}$$

where D is the depth of the section in the plane of cantilever and b is the width of the section.

12.5.2 Minimum Eccentricity

It is often observed that eccentricities do not come into picture while doing calculations i.e. *Structural Analysis* of the section but even then the column is subjected to occasional eccentricities. These occasional eccentricities arise due to one or more of the following reasons:

1. Lateral load not taken into account while designing the column.
2. Under estimation of slenderness effects.
3. Accidental eccentric or lateral loads coming on the column.
4. Placement of live load on the structure not certain.
5. Construction errors due to misalignment etc.

For these reasons, it is always tried that accidental eccentricities coming on the column are taken care of even though the column is designed as pure axial column. **Cl. 25.4 of IS 456: 2000** recommends that every column (be it axial column, axial column with uniaxial and/or biaxial eccentricity, slender/long column) must be designed for a **minimum eccentricity** in any plane which is given as:

$$e_{x,\,min} = \begin{cases} \dfrac{l_{unsupported}}{500} + \dfrac{D_x}{30} \\ 20\,mm \end{cases} \text{ (whichever is greater)}$$

$$e_{y,\,min} = \begin{cases} \dfrac{l_{unsupported}}{500} + \dfrac{D_y}{30} \\ 20\,mm \end{cases} \text{ (whichever is greater)}$$

where, l = Unsupported length of the column, D = Lateral dimension of the column

For non-rectangular and non-circular sections, minimum eccentricity is given as:

$$e_{min} = \begin{cases} \frac{l_e}{300} \\ 20\text{mm} \end{cases} \quad \text{(whichever is greater)}$$

where, l_e = Effective length of the column

12.5.3 Longitudinal Reinforcement

Cl. 26.5.3.1 of IS 456: 2000 specifies that the total area of longitudinal bars in a column section must NOT be less than **0.8% of gross column area.** This limit on minimum reinforcement is imposed because of the following reasons:

1. In order to ensure that a minimum flexural resistance of the column exists due to unexpected eccentricities in the column loading.
2. In compression members, creep under sustained loading is very predominant, especially at low percentages of steel. Thus, the resulting creep stress (due to the creep strain) tries to yield the bars.

Minimum Reinforcement in very large sized columns: In very large sized columns (which are provided due to architectural or other considerations), the provision of minimum reinforcement of 0.8% of gross column area results in a large amount of steel reinforcement that is actually not required from structural considerations. In such cases, the minimum area of steel reinforcement is **0.8% of actual area of concrete required to resist the direct stress and NOT the gross column area.**

For pedestals ($l_e/D < 3$), a minimum reinforcement of **0.15% of gross pedestal area** is provided.

For reinforced concrete walls, Cl. 32.5 of IS 456: 2000 recommends a minimum vertical reinforcement of **0.15% of gross area** which can be reduced to **0.12% of gross area in case steel wire fabric or deformed bars (Fe 415, Fe 500) are used but bar diameter must NOT be less than 6 mm. However** in all cases, the **spacing of bars** must **NOT exceed three times the wall thickness or 450 mm, whichever is less.**

Maximum Reinforcement: The maximum area of cross section of longitudinal bars must NOT exceed **6% of gross column area.** However, in practice a maximum of 4% is recommended.

Diameter and Number of Bars: The diameter of longitudinal bars in columns and in pedestal should **NOT be less than 12 mm.** These bars must **NOT be spaced more than 300 mm apart on the column perimeter.**

For **rectangular columns,** a minimum of four (4) bars be provided, one at every corner of the column and for **circular columns,** a minimum of six (6) bars be provided spaced equally on the perimeter of the column.

Cover to reinforcement in columns: Cl. 26.4.2.1 of IS 456: 2000 recommends a **minimum clear cover of 40 mm or bar diameter (whichever is more) to the column ties** which can be reduced to 25 mm in case column size is small (D < 200 mm). For **walls,** this minimum clear cover requirement is **15 mm or bar diameter (whichever is greater).**

This minimum clear cover can be increased in extreme climatic regions but preferably not greater than 75 mm.

12.5.4 Transverse Reinforcement

Cl. 26.5.3.2 of IS 456: 2000 states that all longitudinal bars of column must be enclosed by lateral ties with internal angle not exceeding 135 degrees or the spirals. Ties are provided in columns because of the following reasons:

1. Ties prevent the buckling of longitudinal bars.
2. These improve the ductility and strength by confining the concrete in a core.
3. Ties hold the longitudinal bars in position.
4. Ties provide resistance against possible shear and torsion.

Lateral Ties: The diameter and spacing of ties (i.e. pitch) as per the recommendations of IS 456: 2000 are as follows:

$$\text{Diameter of tie } (\phi_t) \geq \begin{cases} \text{Dia of main bar} / 4 \\ 6\text{mm} \end{cases}$$

$$\text{Spacing of tie i.e., pitch } (s_t) \leq \begin{cases} \text{Least lateral dimension of the column} \\ 16 \times \text{Dia of smaller diameter of main bar} \\ 300\text{mm} \end{cases}$$

Spirals: Helical reinforcement i.e. spirals provide very good confinement to concrete. The diameter and pitch requirement of spirals is same as that of ties. However, when columns are designed to carry 5% extra load, then pitch of the spiral ties should be as given below:

$$\text{Maximum spacing of spiral tie i.e., pitch } (s_t) < \begin{cases} \text{Core diameter} / 6 \\ 75\text{mm} \end{cases}$$

$$\text{Minimum spacing of spiral tie i.e., pitch } (s_t) > \begin{cases} 25\text{mm} \\ 3 \text{ times the diameter of tie} \end{cases}$$

12.5.5 IS: 456-2000 Recommendations for Detailing in Columns

Following figures depict the reinforcement requirements in columns.

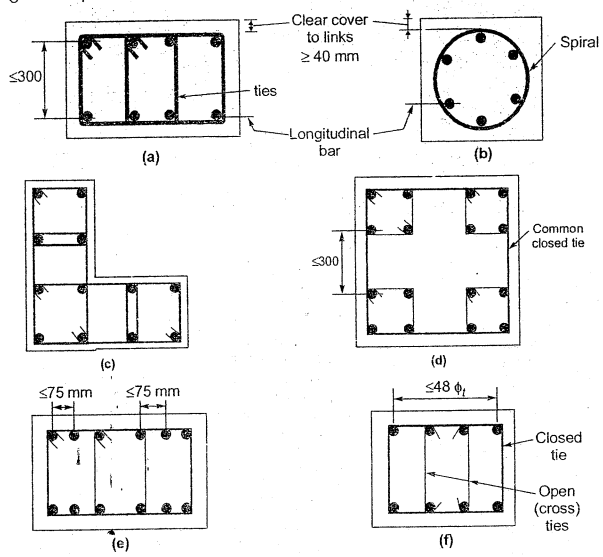


Fig. 12.7 *Code recommendations for detailing in columns*

It is found that the ultimate strength of a column is independent of its age and the history of loading.

Cl. 46.1 of IS 456: 2000 permits "uncracked section" analysis when the eccentricity in the loading is so small that the consequent flexural tension can be taken up by concrete.

Remember: IS 456: 2000 permits the enhancement of design strength by 5% for spirally reinforced columns. This is applicable for short columns only.

12.6 Design of Short Columns under Axial Compression

12.6.1 Columns under Service Loads

As per the working stress method of design, the strength of an axial column is given by:

$$P_0 = \sigma_{cc} A_g + (\sigma_{sc} - \sigma_{cc}) A_{sc}$$

In the above expression, σ_{sc} is taken equal to 1.5 m σ_{cc}, where σ_{sc} and σ_{cc} are permissible stresses in steel and concrete respectively.

The assumption that σ_{sc} = 1.5 m σ_{cc} makes the stress in steel independent of the grade of steel. This is not the case in actual practice. Cl. B-2.2 of IS 456: 2000 removes this anomaly by specifying:

$$\sigma_{sc} = \begin{cases} 130 \text{N/mm}^2 & \text{for Fe250} \\ 190 \text{N/mm}^2 & \text{for Fe415 and Fe500} \end{cases}$$

The allowable stress in concrete for various grades is as given Table 12.1.

Table 12.1: *Allowable compressive stress in concrete in direct compression*

Concrete Grade	Allowable compressive stress in direct compression (N/mm²)
M15	4.0
M20	5.0
M25	6.0
M30	8.0
M35	9.0

Now a days, most of the countries have discontinued the use of *working stress method* due to the advent of more rational *limit state method of design*.

12.6.2 Columns under Ultimate Loads

Assumptions:

- All assumptions discussed for beam/slab for flexure case are also valid for column.
- Maximum compressive strain in axial compression = 0.002
- Maximum compressive strain in bending compression = 0.0035.
- Maximum compressive strain in outermost compression fibre in concrete when there is not tension in the section = 0.0035 – 0.75 × strain in least compressed fibre.

The behaviour of a column can be predicted with higher level of certainty under ultimate loads because of the fact that the ultimate strength of a column is independent of the age and history of loading. As the axial load on a column is increased, axial shortening of the column occurs up to 80% of the ultimate load (P_{uo}) (*OA* in the Fig. 12.8). Behaviour of the column up to this point (point *A*) is independent of the type of ties. However, after this point, ultimate load point *B* is reached beyond which, the behaviour of column depends on the transverse reinforcement i.e. ties as shown in the Fig. 12.8.

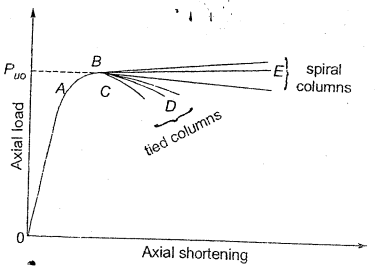


Fig. 12.8 *Axially loaded column with ties and spirals*

In case of spirally reinforced columns, sufficient ductility is provided by the spirals to the core of concrete. Collapse of spirally reinforced columns occurs by the yielding of spiral ties.

The load carrying capacity of spirally reinforced columns after the spalling of outer shell concrete can exceed the ultimate load in case the amount of spiral reinforcement is such that the load carrying capacity contributed by the spiral ties is more than the make-up of loss in the load carrying capacity of the column due to spalling of outer shell concrete. Based on this, Cl. 39.4 of IS 456: 2000 permits to increase the load carrying capacity of the column beyond ultimate load by 5% provided the following condition is satisfied:

The pitch of the helical reinforcement should be such that following criteria should be satisfied.

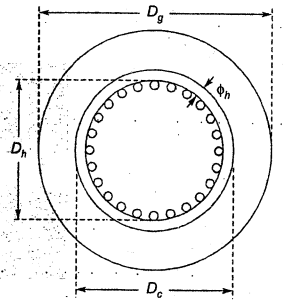
$$\frac{\text{Volume of spiral reinforcement per unit column length}(V_b)}{\text{Volume of core}(V_c)} \geq 0.36\left(\frac{f_{ck}}{f_y}\right)\left(\frac{A_g}{A_{core}} - 1\right)$$

D_g = Gross diameter

Gross area = $\frac{\pi}{4}(D_g)^2$

D_c = Core diameter

Core area = $\frac{\pi}{4}(D_c)^2$



Core volume for unit length (i.e., 1000 mm)

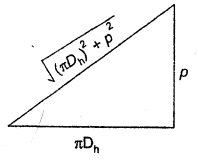
$V_c = 1000\, A_c$

V_h = Volume of helical reinforcement for same unit length

$D_h = D_c - \phi_h$

V_h = (No. of turns) × (Length of one turn) × c/s area of helical reinforcement

Length of helix is one turn = $\sqrt{(\pi D_h)^2 + p^2} \simeq \pi D_h$



Volume of helical reinforcement in 1000 mm

$$\boxed{V_h = \left(\frac{1000}{p}\right) \times (\pi D_h) \times \frac{\pi}{4}\phi_h^2}$$

12.6.3 Design Strength of Axially Loaded Short Columns

Cl. 39.1(a) of IS 456: 2000 limits the maximum axial compressive strain in concrete to 0.002 and the corresponding design stress in concrete is $0.67 f_{ck}/1.5 = 0.446 f_{ck}$ and in steel is $0.87 f_y$ in case of Fe 250, $0.79 f_y$ in case of Fe 415 and $0.746 f_y$ in case of Fe 500. Thus design strength of a short column under axial load is given by:

$$\tilde{P}_{uo} = 0.447\, f_{ck} A_g + (f_{sc} - 0.447\, f_{ck}) A_{sc}$$

where

$$f_{sc} = \begin{cases} 0.87 f_y & \text{for Fe250} \\ 0.79 f_y & \text{for Fe415} \\ 0.746 f_y & \text{for Fe500} \end{cases}$$

But as discussed earlier, **IS 456** recommends that every column must be designed for a minimum eccentricity (e_{min}). Thus the above expression for the design strength of column cannot be applied directly and needs to be modified. **Cl. 39.3 of IS 456: 2000** states that where the calculated minimum eccentricity does not exceed 0.05 times the lateral dimension in the plane under consideration, there the following expression can be used for determining the design strength of axially loaded short column:

$$\tilde{P}_{uo} = 0.4\, f_{ck} A_g + (0.67\, f_y - 0.47 f_{ck}) A_{sc}$$

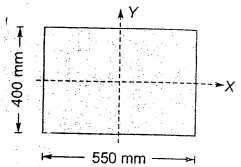
This load carrying capacity can be enhanced by 5% if spiral ties are provided.

Example 12.1 Design a short column of size 400 mm x 550 mm using M 20 grade of concrete and Fe 415 steel. The column is subjected to an axial load of 1500 kN under service and usual conditions of live loads. The effective length of the column is 3.1 m and is braced against side sway in both the directions.

Solution:

$f_{ck} = 20\ \text{N/mm}^2$

$f_y = 415\ \text{N/mm}^2$



Step-1: Checking the column as short or long

$l_{ex} = l_{ey} = 3100\ \text{mm}$

$D_x = 550\ \text{mm},\ D_y = 400\ \text{mm}$

Slenderness ratio along X-direction $= \lambda_x = \dfrac{l_{ex}}{D_x} = \dfrac{3100}{550} = 5.636 < 12$

Slenderness ratio along Y-direction $= \lambda_y = \dfrac{l_{ey}}{D_y} = \dfrac{3100}{400} = 7.75 < 12$

Thus, slenderness ratio in both the directions is less than 12 so the column is short.

Step-2: Calculation of minimum eccentricities

$$e_{xmin} = \frac{l_{ex}}{500} + \frac{D_x}{30} = \frac{3100}{500} + \frac{550}{30} = 24.53\ \text{mm}$$

$$e_{ymin} = \frac{l_{ey}}{500} + \frac{D_y}{30} = \frac{3100}{500} + \frac{400}{30} = 19.53\ \text{mm}$$

$0.05 D_x = 0.05 \times 550\ \text{mm} = 27.5\ \text{mm} > e_{xmin}\ (= 24.53\ \text{mm})$

$0.05 D_y = 0.05 \times 400\ \text{mm} = 20\ \text{mm} > e_{ymin}\ (= 19.53\ \text{mm})$

Since, the minimum eccentricities are less than 0.05 times the lateral dimension in both the directions, the following formula given by IS 456: 2000 can be used for the design of axially loaded short columns:

Step-3: Use of column design formula

$$P_u = 0.4\, f_{ck} A_g + (0.67\, f_y - 0.4\, f_{ck})\, A_{sc}$$

Now axial service load $(P) = 1500\ \text{kN}$

Factored axial load $(P_u) = 1.5 \times 1500\ \text{kN} = 2250\ \text{kN}$

Now,

$$P_u = 0.4\, f_{ck} A_g + (0.67\, f_y - 0.4\, f_{ck})\, A_{sc}$$

$$2250 \times 10^3 = 0.4 \times 20 \times (400 \times 550) + (0.67 \times 415 - 0.4 \times 20)\, A_{sc}$$

$$A_{sc} = 1814.48\ \text{mm}^2$$

Step-4: Reinforcement Design

Provide 4-16 mm + 4-20 mm diameter bars with 20 mm diameter bars at the corners and 16 mm diameter bars at the face.

Thus, $A_{scprovided} = 2060.9\ \text{mm}^2 > 1814.48\ \text{mm}^2$

Percentage of reinforcement provided $(p_t) = 2060.9 \times \dfrac{100}{(400 \times 550)} = 0.937\,\%$

$>$ Minimum reinforcement required $(= 0.8\%)$

$<$ Maximum reinforcement permissible $(= 6\%)$ (OK)

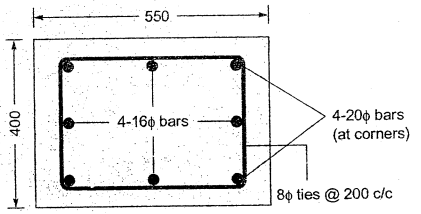
Step-5: Lateral ties

Diameter of the tie: $\phi_t \geq \begin{cases} \dfrac{\phi_{main}}{4} = \dfrac{20}{4} = 5\,\text{mm} \\ 6\,\text{mm} \end{cases}$

Adopt tie diameter as 8 mm

Spacing of the ties: $s_t \leq \begin{cases} \text{Least lateral dimension} \\ \text{of column} = 400\,\text{mm} \\ 16 \times 16 = 256\,\text{mm} \\ 300\,\text{mm} \end{cases}$

Adopt spacing of ties as 200 mm c/c.



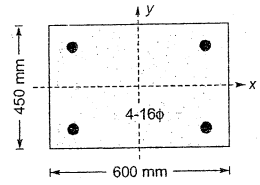
Reinforcement detailing

Example 12.2 A short column of size 450 mm x 600 mm is casted with M 25 grade of concrete and reinforced with 4-16 mm diameter bars of Fe 415 steel. Determine the load carrying capacity of the column assuming that the minimum eccentricity is less than 0.05 times the lateral dimension in both the directions.

Solution:

$f_{ck} = 25\ \text{N/mm}^2$

$f_y = 415\ \text{N/mm}^2$



Since it is given that the minimum eccentricity is less than 0.05 times the lateral dimension in both the directions of bending, thus the formula for short columns under axial loading can be applied.

i.e. $P_u = 0.4\, f_{ck} A_g + (0.67\, f_y - 0.4\, f_{ck})\, A_{sc}$

Now, Gross area, $A_g = 450 \times 600\ \text{mm}^2 = 270000\ \text{mm}^2$

Area of reinforcing steel, $A_{sc} = 4 \times \left(\dfrac{\pi}{4}\right) 16^2 = 804.25\ \text{mm}^2$

$P_u = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}$

Thus, $P_u = 0.4 \times 25 \times 270000 + (0.67 \times 415 - 0.4 \times 25)\, 804.25$

$$= 2700000 + 215579.21\ \text{N}$$

$$= 2915579.213\ \text{N} = 2915.6\ \text{kN} \simeq 2915\ \text{kN}$$

Thus capacity of the column under working loads

$$= \frac{2915}{1.5}\ \text{kN} = 1943.3 \simeq 1940\ \text{kN}$$

Example 12.3 Design a short circular column using M30 grade of concrete and Fe500 steel which is subjected to an axial load of 1000 kN under service load conditions. The column is to be provided with spiral ties.

Solution:

For a short column, load carrying capacity is given by,

$$P_u = 0.4\, f_{ck} A_g + (0.67 f_y - 0.4 f_{ck})\, A_{sc}$$

Axial load under working conditions (P) = 1000 kN

Factored axial load (P_u) = 1.5 x 1000 kN = 1500 kN

For preliminary design, let 1% steel is provided (> 0.8% which is minimum steel required).

Thus, $A_{sc} = 0.01 A_g$

Therefore, $P_u = 0.4\, f_{ck} A_g + (0.67 f_y - 0.4\, f_{ck})\, A_{sc}$

$$1500 \times 1000 = 0.4 \times 30 \times A_g + (0.67 \times 500 - 0.4 \times 30)\, 0.01\, A_g$$

$$1500 \times 1000 = 15.23\, A_g$$

$$A_g = 98.489823 \times 1000\ \text{mm}^2$$

$$\Rightarrow \left(\frac{\pi}{4}\right) D^2 = 98.489823 \times 1000\ \text{mm}^2$$

$$D = 354.12\ \text{mm}$$

Provide D = 400 mm so that $A_g = 125663.71\ \text{mm}^2 > 98.489823 \times 10^3\ \text{mm}^2$ (OK)

Therefore, Area of reinforcement, $A_{sc} = 0.01\, A_g = 0.01 \times \frac{\pi}{4}(400)^2 = 1256.637\ \text{mm}^2$

For spiral columns, minimum 6 numbers of bars are to be provided.

Using 6-20 mm diameter bars,

$$A_{sc\ provided} = 1884.96\ \text{mm}^2 > 1256.637\ \text{mm}^2 \quad \text{(OK)}$$

Provide 6 nos. - 20 mm diameter bars equally spaced along the periphery of the circular column.

Provision of spiral reinforcement:

In columns, minimum clear cover to reinforcement is 40 mm.

Assuming a clear cover of 40 mm, diameter of core of the column = 400 – 2 × 40 mm = 320 mm

Minimum diameter of spirals as per IS 456: 2000 is 6 mm.

Using 6 mm diameter of spirals at a pitch of 's_t'.

Now, for spirally reinforced columns,

$$\left(\frac{\text{Volume of spiral}}{\text{Volume of core}}\right) \text{ per unit length of the column} \geq 0.36\left(\frac{A_g}{A_{core}} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

$$\text{LHS of above relation} = \frac{\left(\frac{\pi}{4}\right)6^2 \frac{\pi(320-6)}{s_t}}{\left(\frac{\pi}{4}\right)320^2} = \frac{0.3468}{s_t}$$

$$\text{RHS of above relation} = 0.36\left(\frac{\frac{\pi}{4}\times 400^2}{\frac{\pi}{4}\times 320^2} - 1\right)\left(\frac{30}{500}\right) = 0.01215$$

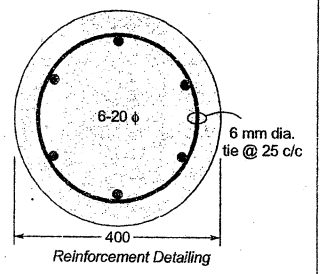
Now L.H.S. ≥ R.H.S.

$$\frac{0.3468}{s_t} \geq 0.01215$$

$$s_t \leq 28.5432\ \text{mm}$$

As per IS 456: 2000,

$$s_t \leq \begin{cases} 75\ \text{mm} \\ \text{core dia}/6 = 320/6 = 53.33\ \text{mm} \end{cases}$$



Reinforcement Detailing

and $s_t > \begin{cases} 25\ \text{mm} \\ 3\phi_t = 3 \times 6 = 18\ \text{mm} \end{cases}$

Thus provide 6 mm spiral ties @ a pitch of 25 mm c/c. (≥ 25 mm and < 53.33 mm)

Example 12.4 Design a reinforced concrete column carrying an axial compression of 1200 kN along with an axial tension of 375 kN. The effective length of the column is 4.45 m. Use Fe 415 steel and M 25 concrete.

Solution:

Given compressive load (P) = 1200 kN

Tensile load (T) = 375 kN

l_{eff} = 4.45 m = 4450 mm

Factored compressive load (P_u) = 1.5 × 1200 = 1800 kN

Factored tensile load (T_u) = 1.5 × 375 = 562.5 kN

Let percentage reinforcement in the column = 1%

$$\therefore A_{sc} = \frac{A_g}{100}$$

and $A_c = A_g - A_{sc} = \frac{99}{100} A_g$

$$\therefore P_u = 0.4\, f_{ck} A_C + 0.67\, f_y A_{sc}$$

$$\Rightarrow 1800 \times 10^3 = 0.4(25)\frac{99}{100} A_g + 0.67(415)\frac{A_g}{100} = 12.6805\, A_g$$

$$\Rightarrow A_g = 141950.24\ \text{mm}^2$$

$$\therefore \text{Side of square column} = \sqrt{A_g} = 376.76\ \text{mm}$$

Provide a square column of 400 × 400 mm

$$\Rightarrow A_g = 160000\ \text{mm}^2 > 141950.24\ \text{mm}^2 \quad \text{(OK)}$$

$$\frac{l_{eff}}{d} = \frac{4450}{400} = 11.125 < 12$$

∴ Column is a short column.

$$\therefore P_u = 0.4\, f_{ck} A_g + (0.67\, f_y - 0.4\, f_{ck})\, A_{sc}$$

$$\Rightarrow \quad 1800\times10^3 = 0.4(25)(400)^2 + 0.67\times415 - 0.4\times25)A_{sc}$$

$$\Rightarrow \quad A_{sc} = 746.13\ mm^2$$

Provide 4 - 20 Φ bars, $A_{sc\ provided} = 4\times\frac{\pi}{4}\times20^2 = 1256.64\ mm^2 > 746.13\ mm^2$

$$\text{Factored tension } (T_u) = 0.87\ f_y A_{st}$$

$$\Rightarrow \quad A_{st} = \frac{562.5\times10^3}{0.87(415)} = 1557.96\ mm^2 > 1256.64 mm^2$$

Provide 4 – 16 Φ bars + 4 – 20 Φ bars,

$$A_{s\ provided} = 4\times\frac{\pi}{4}\times(16)^2 + 4\times\frac{\pi}{4}\times(20)^2$$

$$= 2060.88\ mm^2 > 1557.88\ mm^2 \text{ (for tension)} \quad \text{(OK)}$$

$$> 1256.64\ mm^2 \text{ (for compression)} \quad \text{(OK)}$$

$$\therefore \quad p_{t\ provided} = \frac{2060.88}{400\times400}\times100 = 1.288\% < 6\% \quad \text{(OK)}$$

Check for cracking at working load,

At working load condition, $T = 375$ kN

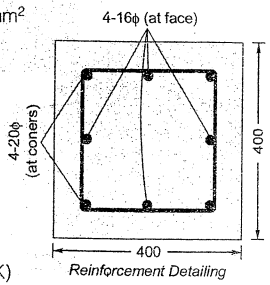
$$\text{Modular ratio} = \frac{280}{3\sigma_{cbc}} \quad \text{where } \sigma_{cbc} = 8.5\ N/mm^2$$

$$= \frac{280}{3(8.5)} = 10.98$$

$$\therefore \quad \text{Tensile stress developed} = \frac{T}{A_g + (m-1)A_{st}}$$

$$= \frac{375\times10^3}{400^2 - (10.98-1)2060.88}$$

$$= 2.689\ N/mm^2 < 3.2\ N/mm^2 \quad \text{(OK)}$$



Reinforcement Detailing

Example 12.5 Calculate the load carrying capacity of a short axially loaded column which is circular in section of diameter 360 mm and is reinforced with 6-25 ϕ bars of Fe 415. The helical reinforcement is of 8 ϕ of Fe 415 @ 50 c/c. Assume M 20 concrete.

Solution:

Let Clear cover to reinforcement = 40 mm

$$\therefore \quad \text{Core diameter } (D_{core}) = 360 - 2\times40 = 280\ mm$$

Thus $\text{Area of core } (A_{core}) = \frac{\pi}{4}D^2_{core} = \frac{\pi}{4}(280)^2 = 61575.22\ mm^2$

Cross-section area of helix (helical reinforcement)

$$(A_h) = \frac{\pi}{4}\times8^2 = 50.27\ mm^2$$

Gross area of column $(A_g) = \frac{\pi}{4}(360)^2 = 101787.602\ mm^2$

$$\text{Total reinforcement area } (A_{sc}) = 6\times\frac{\pi}{4}\times25^2 = 2945.24\ mm^2$$

$$\therefore \quad \text{Percentage reinforcement } (p) = \frac{A_{sc}}{Ag}\times100 = \frac{2945.24}{101787.602}\times100 = 2.89\ \%$$

For spirally reinforced columns,

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core}} \geq 0.36\frac{f_{ck}}{f_y}\left(\frac{A_g}{A_{core}} - 1\right)$$

$$\Rightarrow \quad \frac{A_h \pi D_{core}}{A_{core}\, s} \geq 0.36\frac{f_{ck}}{f_y}\left(\frac{A_g}{A_{core}} - 1\right)$$

$$\Rightarrow \quad \frac{(50.27)\pi(280)}{(61575.22)50} \geq 0.0113$$

$$\Rightarrow \quad 0.0144 \geq 0.0113 \quad \text{Which is true.}$$

Thus load carrying capacity of the column can be increased by 5%.

$$\therefore \quad P_u = 1.05[0.4\ f_{ck} A_g + (0.67\ f_y - 0.4\ f_{ck})\ A_{sc}]$$

$$= 1.05[0.4(20)101787.602 + (0.67\times415 - 0.4\times20)2945.24]N$$

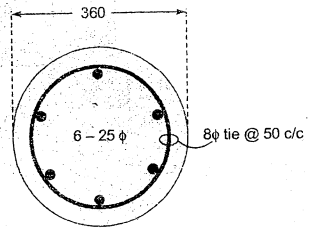
$$= 1690.15\ kN$$

$$\therefore \text{ Safe working load } (P) = \frac{P_u}{1.5}$$

$$= \frac{1690.15}{1.5}\ kN$$

$$= 1126.77\ kN$$

$$\simeq 1126\ kN$$



Reinforcement Detailing

12.7 Design of short columns with Axial Load and Uniaxial Bending

This involves short columns subjected to axial load and bending about one principal axis. This situation is equivalent to application of axial load at an eccentricity (e) from the geometric centroidal axis, where $e = \frac{M_u}{P_u}$.

A limiting case of uniaxial eccentric compression is the condition of zero eccentricity i.e. $e = 0$ which implies that $M_u = P_u \cdot e = 0$. This corresponds to pure axial loading condition. For this condition, strain across the section is uniform and is limited to 0.002. (Limit state of collapse of concrete in compression).

Other limiting case of uniaxial eccentric compression is infinite eccentricity, i.e. e = infinity. This implies that $P_u = \frac{M_u}{e} \to 0$. This is the case of pure flexure which is used in the design of beams. Corresponding to this condition, the strains are linearly distributed across the concrete cross section with a value of zero at neutral axis. One side of the neutral axis is in compression and the other side is in tension. The maximum value of strain in flexural compression is limited to 0.0035. (Limit state of collapse of concrete in flexure).

Consider a general case of uniaxial compression, $M_u \neq 0$, $P \neq 0$ and $0 \leq e \leq$ infinity. For this general condition, the strain variation is non-uniform and is assumed to vary linearly across the section with maximum strain of concrete in compression between 0.002 and 0.0035 at the ultimate limit state.

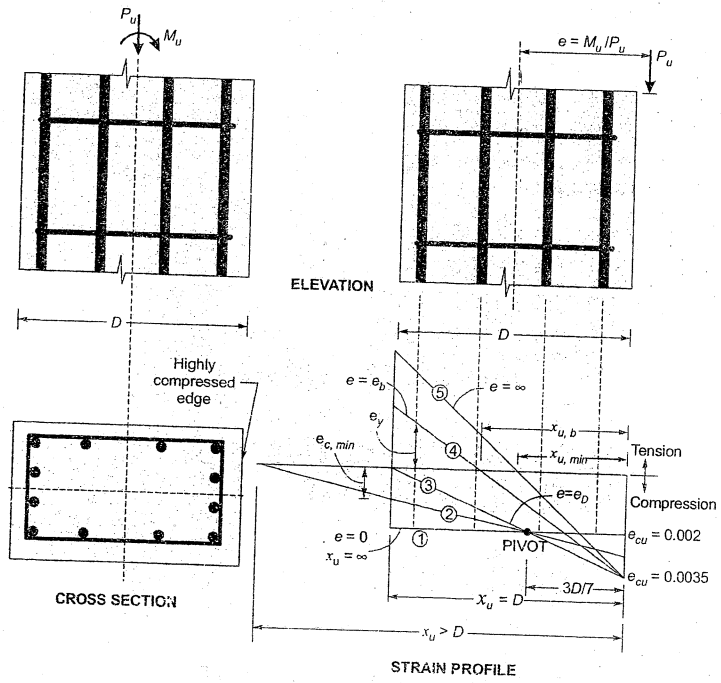


Fig. 12.9 *Strain profile under ultimate flexural eccentric compression*

For eccentric compressed members, the depth of neutral axis w.r.t. highly compressed edge can vary from minimum value $x_{u\,min}$ (corresponding to e = infinity) to a maximum value of x_u = infinity (corresponding to $e = 0$).

When loading eccentricity is small such that the entire cross section is under non-uniform compression and neutral axis lies outside the section ($x_u > D$), IS 456: 2000 limits the strain in the highly compressed edge to a value between 0.002 and 0.0035 which is given by:

$$\varepsilon_c = 0.0035 - 0.75\,\varepsilon_{c,\,min} \text{ for } x_u >= D$$

where, $\varepsilon_{c,\,min}$ = strain in the least compressed edge.

From the above equation, when $\varepsilon_{c,\,min} = 0$ then $\varepsilon_c = 0.0035$ i.e. $x_u = D$ or $e = e_D$

And when $\varepsilon_{c,\,min} = 0.002$ then $\varepsilon_c = 0.002$ i.e. x_u = infinity or $e = 0$.

The point of intersection of these two limiting strain profiles (corresponding to $e = 0$ and e = infinity) occurs at a distance of $\frac{3D}{7}$ from the highly compressed edge and this point of intersection acts like a pivot through which all the strain profiles with $x_u >= D$ will pass. It can be shown that for all $x_u >= D$,

$$\varepsilon_{cu} = 0.002\left[1 + \frac{\frac{3D}{7}}{x_u - \frac{3D}{7}}\right]$$

12.8 Failure Modes in Eccentric Compression

The term 'limit state of collapse' is used to describe the ultimate limit state of compression members (whether axially loaded or eccentrically loaded). But the actual failure may or may not occur in compression since an eccentrically loaded column is subjected to axial load along with moment and the failure mode depends upon the eccentricity. For small eccentricity, axial compression behaviour dominates thereby making the subsequent failure as compression failure whereas for large eccentricity, flexural behaviour predominates thereby making the failure as tension failure. Thus depending upon the eccentricity, we can predict in advance that a compression failure or a tension failure will take place.

12.8.1 Balanced Failure

Between the two extreme failure conditions i.e. compression failure and tension failure, there exists a critical failure condition which is called as 'balanced failure'. This failure condition is characterized by the yielding of outermost row of reinforcing steel (tension side) and attainment of maximum compressive strain in concrete in flexure which is 0.0035 at the highly compressed edge of the column simultaneously. Thus both crushing of concrete (at highly compressed edge) and yielding of steel (outer most row of reinforcing steel) occurs simultaneously. At this stage, yield strain for Fe250 and Fe415 / Fe500 is defined respectively as:

$$\varepsilon_y = \begin{cases} \dfrac{0.87 f_y}{E_s} & \text{(for Fe250)} \\ \dfrac{0.87 f_y}{E_s} + 0.002 & \text{(for Fe 415 and Fe 500)} \end{cases}$$

12.8.2 Compression Failure

When the loading eccentricity is less than the eccentricity corresponding to balanced failure condition then yielding of reinforcing steel does not take place and failure occurs at ultimate state by crushing of concrete at the highly compressed edge. The compression steel may or may not yield depending upon the grade of steel and its nearness to the highly compressed edge.

12.8.3 Tension Failure

When the eccentricity of loading is more than that corresponding to balanced failure condition then yielding of tension steel at the outermost row of reinforcing steel takes place. The outermost row of reinforcing bars yield first followed by yielding of inner row of bars with increasing strain. But the collapse occurs when concrete at the highly compressed edge gets crushed.

12.9 Design Strength of Axially Loaded Short Columns with Uniaxial Bending

The design strength of an axially loaded short column depends on eccentricity of loading. The design strength of these types of columns has two components viz. the axial compression component P_{uR} and the corresponding uniaxial moment $M_{uR} = P_{uR}\,e$.

For a given eccentricity, there exists a unique strain profile and thus a unique location of neutral axis at the ultimate limit state. For this distribution of strains, the corresponding stresses in concrete and steel and

hence their respective resultant forces C_c and C_s can be determined. Applying the condition of static equilibrium,

$$P_{uR} = C_c + C_s$$
$$M_{uR} = M_c + M_s$$

where, M_c and M_s are resultant moments due to C_c and C_s respectively w.r.t. the centroidal axis of the column section.

For a given location of neutral axis $\left(\frac{x_u}{D}\right)$, the design strength values P_{uR} and M_{uR} can be determined and the corresponding eccentricity can be computed as $e = \frac{M_{uR}}{P_{uR}}$.

But for an arbitrary value of eccentricity 'e', it is also possible to compute design strength values P_{uR} and M_{uR} but only after first locating the neutral axis. The location of neutral axis can be determined by considering the moments of the forces C_c and C_s about the eccentric line of action of P_{uR}. But the relationship between x_u and C_c, C_s does not give any explicit solution of x_u in terms of eccentricity e. This relationship is highly nonlinear requiring a trial and error procedure.

12.9.1 Interaction Curve

It is a graphical representation of the design strength of uniaxial eccentric column for a given proportions. Each point on the curve represents the design strength values P_{uR}, M_{uR} for a particular eccentricity 'e'. Thus if a short column is subjected to an eccentric load P at an eccentricity e and if this load P is increased gradually till the ultimate limit state is reached, then the ultimate load at failure is given by $P_u = P_{uR}$ and corresponding moment by $M_u = M_{uR} = P_{uR}e$. The co-ordinates (M_{uR}, P_{uR}) form a unique point on the interaction curve. Interaction curve defines the different combinations of (M_{uR}, P_{uR}) for all possible values of eccentricity e ($0 \le e \le$ infinity). If a typical point (M_u, P_u) falls within the interaction curve, then the column is safe otherwise not. Thus the design interaction curve acts as a failure envelop.

12.9.2 Salient Points of the Interaction Curve

In interaction curve, axial load P is along x-axis and moment M is along y-axis.

The salient points of the interaction curve are as follows:

1. The point '1' on the above curve corresponds to zero eccentricity i.e. pure axial load. For this case, $M_{uR} = 0$ and $P_{uR} = P_{uo}$.
2. Point '1'' on the curve corresponds to the condition of minimum eccentricity (e_{min}) as per Cl. 25.4 and 39.3 of IS 456: 2000. The corresponding ultimate resistance is $\tilde{P}_{uo}$.
3. Point '3' on the curve depicts the condition when $x_u = D$ (i.e. $e = e_D$). Now when $e < e_D$ then entire column section is under compression and the corresponding neutral axis lies outside the section i.e. $x_u > D$ and ultimate compressive strain in concrete follows $0.002 < \varepsilon_{cu} < 0.0035$.

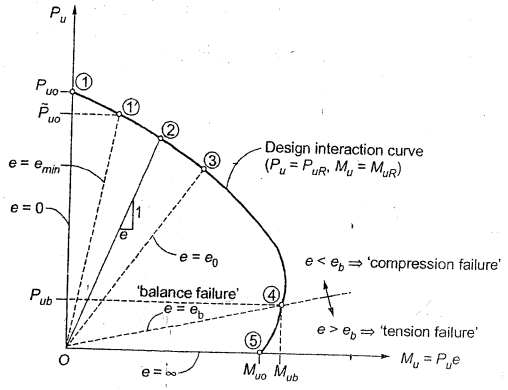


Fig.12.10 *A typical $P_u - M_u$ interaction diagram*

When $e > e_D$ then neutral axis lies within the section i.e. $x_u < D$ with $\varepsilon_{cu} = 0.0035$ at the highly compressed fiber of concrete.

4. Point '2' represents a general case when neutral axis lies outside the section i.e. $x_u > D$ and $e < e_D$.
5. Point '4' represents the balanced condition of failure. Here $e = e_b$ and $x_u = x_{u,b}$. The design strength values of the column section for this condition i.e. at ultimate limit state are denoted as P_{ub} and M_{ub}. When $P_{uR} < P_{ub}$ then tension failure of the column section occurs. M_{ub} is the maximum value of ultimate moment of resistance of the column section and this value is higher than the corresponding ultimate moment resisting capacity of the section under pure flexure. This is due to the fact that the presence of axial compression delays the yielding of steel thereby increasing the moment resisting capacity of the section.

   However presence of axial compression also fastens the compression of concrete thereby increasing the compressive stress in concrete which nullifies the gain in moment resisting capacity from tension side.
6. Point '5' on the curve corresponds to the condition of pure bending/flexure with $P_{uR} = 0$ and $e \to$ infinity. Here the depth of neutral axis is minimum i.e. $x_u = x_{u\,min}$.

12.10 Interaction Charts as Analysis Aids

The terms P_{uR} and M_{uR} denote the inherent resistance of the column section while P_u and M_u denote the external load effects i.e. factored axial load and factored moment about one axis respectively. Analysis of column section subjected to a given loading (P_u, M_u) involves the determination of strength of the section i.e. (P_{uR}, M_{uR}) along with the factor of safety. The point (P_{uR}, M_{uR}) lies on the interaction curve but the point (P_u, M_u) can lie anywhere on the curve i.e. either within the curve, on the curve or outside the curve depending on the values of P_u and M_u. Thus analysis of a column section for a given loading is to assess whether the column is safe for the given loading or not. There are two approaches to assess this viz.:

APPROACH–I

Determine the design strength of the column (P_{uR}, M_{uR}) for the given eccentricity (e) = M_u/P_u. Check if $P_{uR} > P_u$ and $M_{uR} > M_u$ then the column is safe under the given set of loading (P_u, M_u).

APPROACH–II

Here it is assumed that the ultimate limit state has been reached at the given axial load P_u i.e. $P_u = P_{uR}$. After that compare the moment of resistance of the column section M_{uR} with the applied moment M_u. If $M_{uR} > M_u$ then the column is safe for the given set of loading (P_u, M_u).

Irrespective of the approach that we follow, it involves the trial and error procedure if we go by first principle which is quite cumbersome and not recommended for routine design practice. In order to overcome this, Cl. 39.5 of IS 456: 2000 states that:

This design of a member subjected to combined axial load and uniaxial bending will involve lengthy calculations. These difficulties can be overcome by the use of interaction diagrams.

12.11 Design Aids as Non-dimensional Interaction Diagrams

Design of a column involves fixing the column dimensions and determining the area of reinforcement required to resist the superimposed factored loads.

Do you know? Provision of ties in columns is NOT a design in itself and it is merely the provisions of IS 456: 2000 that has to be complied with.

The interaction diagram as discussed above refers to a particular type of column section that is usually indicated at the inset of each diagram.

These interaction diagrams can be converted into more versatile diagrams by making the co-ordinates viz. P_u and M_u independent of the dimensions of the column cross-section. This is done by the following procedure:

$$p_u = \frac{P_u}{f_{ck} bD}; \qquad m_u = \frac{M_u}{f_{ck} bD^2}$$

The altogether new $p_u - m_u$ interaction diagram now becomes applicable to all the types of column cross-section which are geometrically similar to and has the same material properties (steel) as the one that is shown on the insert of the figure. Now the revised co-ordinates will be (p_u, m_u) instead of earlier one i.e. (P_u, M_u).

12.12 Design Charts of SP-16

The non-dimensional interaction diagrams $p_u - m_u$ given in the design aid SP-16 covers the following three cases:

Case I:

Rectangular sections with reinforcement distributed equally on the two sides of the column section. (Charts 27 – 38). These two sides are parallel to the axis of bending. Each row of bar has an area of $0.5A_s$.

Case II:

Rectangular sections with reinforcement distributed equally on all the four sides of the column section (Charts 39-50).

Case III:

Circular column sections with at least six bars of equal diameter, uniformly spaced circumferentially (Charts 51-62).

Corresponding to each of the above three cases, each case has as many as 12 charts covering the three grades of steel viz. Fe 250, Fe 415 and Fe 500 and four types of d'/d ratios viz. 0.05, 0.1, 0.15 and 0.2. For values of d'/d other than this, linear interpolation has to be performed.

12.13 Design of Short Columns under Axial Compression and Biaxial Bending

It is the most general case that we mainly encounter in reality. Cl. 25.4 of IS 456: 2000 states that where biaxial bending is considered, it is quite sufficient to ensure that the eccentricity exceeds the minimum about one axis at a time.

Now the bending moments about the two principal axes are M_{ux} and M_{uy}. These can be reduced to a single bending moment (M_u) as:

$$M_u = \sqrt{M_{ux}^2 + M_{uy}^2}$$

But

$$M_u = P_u.e$$

$$M_{ux} = P_u.e_y$$

$$M_{uy} = P_u.e_x$$

Therefore, substituting these values in the above expression gives:

$$e = \sqrt{e_x^2 + e_y^2}$$

12.14 Interaction Surface

In the previous sections, we discussed interaction curves. This is an advancement of the interaction curve wherein, now we have three axes viz. P_u, M_{ux} and M_{uy}. The resulting surface is called as interaction surface (like interaction curve).

Now, let $M_{uR,x}$ = Design flexural strength of the column section w.r.t. major axis in biaxial bending

$M_{uR,y}$ = Design flexural strength of the column section w.r.t. minor axis in biaxial bending

M_{ux1} = Design flexural strength of the column section w.r.t. major axis under uniaxial bending ($e_x = 0$)

M_{uy1} = Design flexural strength of the column section w.r.t. minor axis under uniaxial bending ($e_y = 0$)

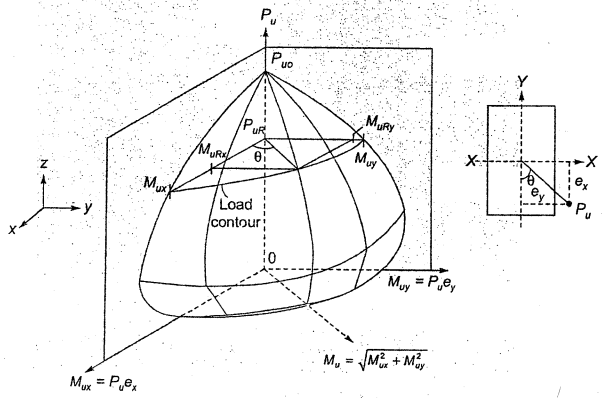


Fig. 12.11 *Interaction surface for a column subjected to bi-axial bending*

12.15 IS 456: 2000 Procedure for the Design of Columns with Axial Load and Bi-axial Bending

Cl. 39.6 of IS 456: 2000 gives simplified procedure for the design of columns subjected to axial load and bi-axial bending. This method is based on Bresler's formulation and is expressed as:

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

where M_{ux} and M_{uy} are the factored biaxial moments on the column and M_{ux1} and M_{uy1} are the uniaxial moment capacities of the column w.r.t. major and minor axes respectively. α_n is a constant which depends on the factored axial load P_u which is expressed as:

$$\alpha_n = \begin{cases} 1.0 & for \frac{P_u}{P_{uz}} < 0.2 \\ 2.0 & for \frac{P_u}{P_{uz}} > 0.8 \\ 0.667 + 1.667\frac{P_u}{P_{uz}} & otherwise \end{cases}$$

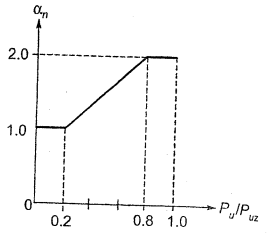


Fig.12.12 *Variation of α_n with P_u/P_{uz}*

Cl. 39.6 of IS 456: 2000 defines P_{uz} as:

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$\Rightarrow \quad P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

NOTE: M_{ux1}, M_{uy1} and P_{uR} are the inherent capacities of the column and M_{ux}, M_{uy} and P_u are due to the external loading on the column.

12.16 Design Steps for the Design of Column with Axial Load and Bi-axial Bending

Step1. For the given factored loads i.e. P_u, M_{ux} and M_{uy} determine the eccentricities as:

$$e_y = \frac{M_{ux}}{P_u}, \quad e_x = \frac{M_{uy}}{P_u}$$

Verify that the computed eccentricities are more than the minimum eccentricities.

Step 2. Assume a trial section for the column i.e. assume width and depth of the column section.

Step 3. Determine M_{ux1} and M_{uy1} corresponding to the given factored load P_u from the design charts of SP-16.

Step 4. Determine P_{uz} from the expression as given above and again reproduced below as:

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$\Rightarrow \quad P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

Step 5. Determine α_n as explained above and again reproduced below as:

$$\alpha_n = \begin{cases} 1.0 & for \frac{P_u}{P_{uz}} < 0.2 \\ 2.0 & for \frac{P_u}{P_{uz}} > 0.8 \\ 0.667 + 1.667\frac{P_u}{P_{uz}} & otherwise \end{cases}$$

Step 6. Check the adequacy of the column section as given below and redesign if required.

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

Example 12.6 Design a short column of size 500 mm × 500 mm using M 30 grade of concrete and Fe 500 steel. The column is subjected to an axial load of 900 kN and moments of 52 kNm and 45 kNm along 'x' and 'y' axes respectively. Assume reinforcement to be equally distributed on all the four sides of the column.

Solution:

As per IS 456: 2000 requirements, minimum area of compression steel in column is 0.8%.

Thus, let p = 0.8% and effective cover = 50 mm on all the sides

So, $$\frac{p}{f_{ck}} = \frac{0.8}{30} = 0.02667$$

and, $$\frac{d'}{D} = \frac{50}{500} = 0.1$$

Now since the column is square in section and is symmetrically reinforced and thus the uniaxial moment capacity of the column will be same in both the directions i.e. along 'x' and 'y' directions.

Axial load (P) = 900 kN

Factored axial load (P_u) = 1.5 × 900 kN = 1350 kN

$$\frac{P_u}{f_{ck} b D} = \frac{1350 \times 10^3}{30 \times 500 \times 500} = 0.18$$

From Chart 48 of SP: 16, for $\frac{d'}{D} = 0.1$

and $$\frac{P_u}{f_{ck} b D} = 0.18$$

$$\frac{M_u}{f_{ck} b D^2} = 0.04$$

or $$M_u = M_{ux_1} = M_{uy_1}$$

$$= 0.04 \times 30 \times 500 \times 500^2 \text{ Nmm}$$

$$= 150 \text{ kNm}$$

Load carrying capacity of the column under pure axial load (P_z)

$$P_z = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$= 0.45 f_{ck} (A_g - A_{sc}) + 0.75 fy A_{sc}$$

$$= 0.45 \times 30 \times (A_g - 0.8\% \text{ of } A_g) + 0.75 \times 500 \times (0.8\% \text{ of } A_g)$$

$$= 0.45 \times 30 \times 0.992 \times A_g + 0.75 \times 500 \times 0.008 \times A_g$$

$$= 0.45 \times 30 \times 0.992 \times 5002 + 0.75 \times 500 \times 0.008 \times 500^2$$

$$= 4098 \text{ kN}$$

So, $$\frac{P_u}{P_z} = \frac{1350}{4098} = 0.32943$$

Also, $$\frac{M_{ux}}{M_{ux_1}} = \frac{52}{150} = 0.346667$$

and, $$\frac{M_{uy}}{M_{uy_1}} = \frac{45}{150} = 0.3$$

Value of α_n for $\frac{P_u}{P_z} = 0.32943$ comes out to be 1.21572 (Cl. 39.6 of IS: 456)

Checking the strength of the section:

$$\left(\frac{M_{ux}}{M_{ux_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} = (0.346667)^{1.21572} + (0.3)^{1.21572} = 0.507 < 1 \quad \text{(OK)}$$

So, the reinforcement assumed initially is O.K.

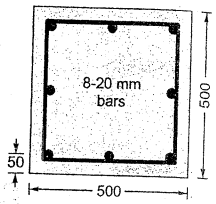
i.e. p = 0.8% of gross column area.

i.e. A_{sc} = 0.8% of A_g

$= 0.008 \times 500 \times 500 \text{ mm}^2$

$= 2000 \text{ mm}^2$

Provide 8 - 20 mm diameter bars so that A_{sc} provided is



Reinforcement detailing

$$A_{sc\,provided} = 8 \times \frac{\pi}{4}(20)^2 = 8 \times 314.16 \text{ mm}^2$$

$$= 2513.28 \text{ mm}^2 > 2000 \text{ mm}^2 \quad \text{(OK)}$$

Example 12.7 Design a short column to carry a working axial load of 450 kN. Use M 20 concrete and Fe 250 steel. The unsupported length of the column is 4.0 m and effective length factor is 0.85. Use SP-16.

Solution:

Axial load, $P = 450$ kN

Factored load $P_u = 1.5 \times 450 = 675$ kN

$f_y = 250 \text{ N/mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$

Effective length of the column $l_{eff} = 0.85 \times 4000 \text{ mm} = 3400 \text{ mm}$

For column to be short, $\frac{l_{eff}}{D} \le 12$

$\Rightarrow$ $D \ge \frac{l_{eff}}{12} = \frac{3400}{12} = 283.33$ mm

Provide $D = 300$ mm

Minimum eccentricity, $e_{min} = \frac{\text{Unsupported length}}{500} + \frac{D}{30} = \frac{4000}{500} + \frac{300}{30} = 18 \text{ mm} < 20 \text{ mm}$

$\therefore$ $e_{min} = 20$ mm

Also $0.05D = 0.05(30) = 15$ mm

$\therefore$ $e_{min} > 0.05D$

$\therefore$ Column is designed as short column carrying axial load and moment

$$M_{u\,min} = P_u e_{min} = 675\left(\frac{20}{1000}\right) = 13.5 \text{ kNm}$$

Here, $b = D = 300$ mm

$$\frac{P_u}{f_{ck}bD} = \frac{675 \times 10^3}{20(300)^2} = 0.375$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{13.5 \times 10^6}{20(300)^3} = 0.025$$

Using 20 mm diameter bars and 6 mm dia. ties, effective cover (d')

$$= 40 + 6 + \frac{20}{2} = 56 \text{ mm}$$

$\therefore$ $\frac{d'}{D} = \frac{56}{300} = 0.1867$

From chart 53 and 54 of SP-16

for $\frac{P_u}{f_{ck}bD} = 0.375$ and $\frac{M_u}{f_{ck}bD^2} = 0.025$

and $\frac{d'}{D} = 0.15$, $\frac{p}{f_{ck}} = 0.07$

and for $\frac{d'}{D} = 0.2$, $\frac{p}{f_{ck}} = 0.07$

$\therefore$ when $\frac{d'}{D} = 0.1867$, $\frac{p}{f_{ck}} = 0.07$

$\Rightarrow$ $p = 0.07 f_{ck} = 0.07(20) = 1.4\%$

$\therefore$ Area of steel required, $A_{sc\,required} = \frac{1.4}{100} \times \frac{\pi}{4}(D)^2 = \frac{1.4}{100} \times \frac{\pi}{4}(300)^2 = 989.6 \text{ mm}^2$

$\therefore$ Number of 20 mm diameter bars $= \frac{989.6}{\frac{\pi}{4}(20)^2} = 3.15 \simeq 4$ bars

But for circular columns, a minimum of 6 nos. of bars is to be provided

$\therefore$ Provide 6-20 mm diameter bars

$\Rightarrow$ $A_{sc\,provided} = 6 \times \frac{\pi}{4} \times 20^2 = 1885 \text{ mm}^2$

Minimum reinforcement @ 0.8% of gross column area

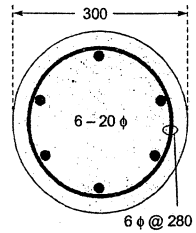
$$= \frac{0.8}{100} \times \frac{\pi}{4}(300)^2$$

$$= 565.5 \text{ mm}^2 < A_{sc\,required} \quad \text{(OK)}$$

Diameter of tie $\ge \begin{cases} \text{Dia. of main bar}/4 \\ 6 \text{ mm} \end{cases} = \begin{cases} 20/4 = 5 \text{ mm} \\ 6 \text{ mm} \end{cases} = 6$ mm

Pitch, $s_t \le \begin{cases} \text{Least lateral dimension of column} = 300 \text{ mm} \\ 16 \times \text{dia. of main bar} = 16 \times 20 = 320 \text{ mm} > 300 \text{ mm} \\ 300 \text{ mm} \end{cases}$

Provide 6 mm ties @ 280 mm c/c.



Reinforcement detailing

12.17 Design of Long/Slender Columns

In long/slender columns, slenderness effect of the column cannot be ignored. The behaviour of long columns deviates quite significantly from that of short columns.

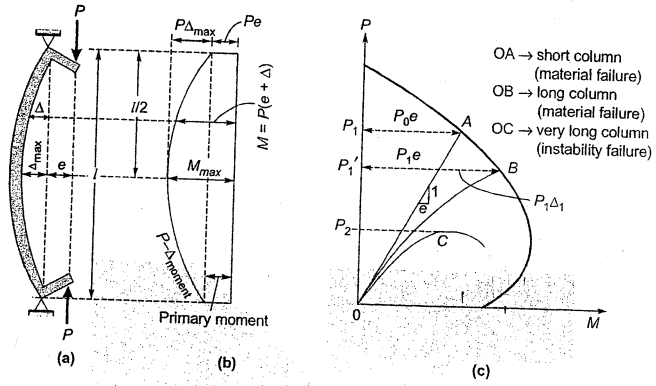


Fig. 12.13 *Behaviour of long columns*

Let a load '*P*' is applied on a long column of effective length '*l*' and at an eccentricity '*e*' from the undeflected longitudinal axis of the column. Due to the applied eccentricity '*e*', primary moments are,

$$M_{pr} = Pe$$

The primary moments are developed not only at the ends but all along the length of the column. The bending of long column causes additional load dependent eccentricities in the column.

Let Δ = Lateral deflection of the longitudinal axis of the column.

Thus, total eccentricity = $e + \Delta$

Thus, total moment = $M = P\cdot(e + \Delta) = M_{pr} + P\cdot\Delta$

Here $P\cdot\Delta$ = Secondary moment (also known as P – Δ moment) which varies along the length of the column in the same fashion as Δ varies along the column length. The maximum value of Δ ($= \Delta_{max}$) occurs at mid - length and thus maximum value of total moment is:

$$M_{max} = P\cdot(e + \Delta_{max})$$

Here Δ_{max} includes both the deflections produced due to primary moment, M_{pr} and also due to P – Δ effect. Thus M_{max} varies non-linearly with *P*. Axial load '*P*' decreases the flexural stiffness of column and for long columns; it may even reduce to zero thereby causing lateral buckling of column. For short columns, flexural stiffness is so high that lateral deflection Δ is very small resulting in negligible *P* – Δ moment.

For short columns, $\Delta_{max} = 0$

Thus $M_{max} = P\cdot e$ which results in *P-M* path to be linear. Here the failure occurs by crushing of concrete.

For long columns, Δ_{max} is significant and also $M_{max} = P\cdot(e + \Delta_{max})$ will vary non-linearly with *P*.

At ultimate limit state, secondary moment (*P* – Δ moment) becomes significant as compared to primary moment M_{pr}. For the same cross-sectional areas of the columns, long columns cause a reduction in the load carrying capacity of the column.

For very long columns, Δ_{max} may be so large that $\frac{dP}{dM}$ reaches zero. Thus such columns fail due to instability/buckling.

12.18 Analysis of Slender/Long Columns

The major problem in the design of slender or long columns is the determination of factored moments including the *P* – Δ effect. Thus the problem is mainly of analysis and not the design. Principle of design of column section subjected to factored axial load and factored moments M_{ux}, M_{uy} remains same for both short and slender columns but in the later case, the factored moments must include the effect of secondary moments. These secondary moments are negligible in short columns.

12.18.1 Rigorous Analysis of Slender Columns

Cl.39.7 of IS 456: 2000 recommend the use of 'second-order' structural analysis for determining the moments and axial forces. This requires a significant computational effort. Also 'principle of superposition' is not valid for second - order analysis.

12.19 Provisions for the Design of Slender Columns as per IS 456: 2000

Since second order structural analysis is quite complex and time consuming, first-order analysis based on linear-elastic theory is preferred. Thus IS: 456 recommends highly simplified design procedure for the design of slender columns. This Codal (IS: 456) procedure tries either to increase the design moments or reduction in strength due to secondary moments.

12.19.1 Strength reduction coefficient method

This is a simplified procedure given in IS 456: 2000 and is applicable for working stress method of design. As per this method, the permissible stresses in concrete and steel are reduced by a factor called strength reduction coefficient (C_r) given by:

$$\boxed{C_r = 1.25 - \frac{l_e}{48d}}$$

Where *d* = Least lateral dimension of the column.

Alternatively and more precisely,

$$C_r = 1.25 - \frac{l_e}{160\, r_{min}}$$

Where r_{min} = Least radius of gyration of the column

However, instead of applying the strength reduction coefficient to the permissible stresses, it can be applied directly to the load carrying capacity of the column.

12.19.2 Additional moment method

This method is prescribed by IS: 456 for Limit State Method of design. Accordingly, all slender columns must be designed for biaxial eccentricities which must include the *P* – Δ moment (which is in fact the additional moment) i.e.

$$M_{ux}' = P_u(e_x + e_{ax}) = M_{ux} + M_{ax}$$

$$M_{uy}' = P_u(e_y + e_{ay}) = M_{uy} + M_{ay}$$

Where M_{ux}' and M_{uy}' are total design moments and M_{ux}, M_{uy} are primary factored moments. M_{ax} and M_{ay} are additional moments due to bending about major and minor axes respectively.

The main feature of this method is the determination of additional eccentricities e_{ax} and e_{ay}. For a hinged slender column, additional eccentricity e_a is Δ_{max} which is in fact a function of curvature to which a column is subjected. Let maximum curvature at mid-height of the slender column is φ_{max}, then Δ_{max} lies between $\frac{\varphi_{max} l^2}{12}$ to $\frac{\varphi_{max} l^2}{8}$. Considering the average of these,

$$e_a = \Delta_{max} = \frac{\varphi_{max} l^2}{10}$$

Column failure is expected to occur at the section corresponding to φ_{max}. However, on the basis of certain assumptions, φ_{max} can be expressed in terms of failure strain ε_{cu} and ε_{st} in concrete and steel respectively. But values of these strains ε_{cu} and ε_{st} depends on factored axial load P_u.

Now let, $\varepsilon_{cu} = 0.0035$ and $\varepsilon_{st} = 0.002$, $d' = 0.1D$ and assuming that additional moments are about 80% of total moment which is a quite conservative assumption, then,

$$\varphi_{max} = \frac{0.8 \times (0.0035 + 0.002)}{0.9D} = \frac{1}{200D}$$

Also,

$$e_a = \frac{\varphi_{max} l^2}{10}$$

$$\frac{e_a}{D} = \frac{(l/D)^2}{2000}$$

Thus additional moments are,

$$M_{ax} = P_u \cdot e_{ax} = P_u . D_x \frac{\left(\frac{l_{ex}}{D_x}\right)^2}{2000}$$

$$M_{ay} = P_u \cdot e_{ay} = P_u . D_y \frac{\left(\frac{l_{ex}}{D_y}\right)^2}{2000}$$

Thus $\frac{e_a}{D}$ ratio increases with the square of slenderness ratio $\frac{l}{D}$. Now for slender columns, $\frac{l}{D} \geq 12$. Therefore,

$$\left(\frac{e_a}{D}\right)_{min} = \frac{12^2}{2000} = 0.072$$

These additional moments M_{ax} and M_{ay} are to be considered in the design of slender columns in addition to factored primary bending moments M_{ax} and M_{ay}. In case of eccentric loadings, these primary moments should not be less than the moments corresponding to minimum eccentricities as specified by IS: 456. In case of axial loading where primary moments are not considered in design, it must be ensured that the corresponding additional moments are not less than those corresponding to minimum eccentricities.

In the above derivations, it was assumed that the column bent symmetrically in a single curvature and was braced. However, some modification is required to account for unequal moments at the ends; be in magnitude or sign or both. Also it was assumed that the axial load corresponds to the balanced failure i.e. $P_u = P_{ub}$. IS 456 recommends some modifications for this which are as follows:

1. For $P_u > P_{ub}$, the failure mode is compression failure and the corresponding e/D ratio is low. At very high axial loads, the entire column section may be under compression thereby giving low curvatures. In this case, the use of above equations will result in highly conservative results. The additional moments are reduced by a factor as per Cl. 39.7.1.1 of IS: 456 as:

$$k_{ax} = \frac{(P_{uz} - P_u)}{(P_{uz} - P_{ub,x})} \text{ for } P_u > P_{ub,x}$$

$$k_{ax} = \frac{(P_{uz} - P_u)}{(P_{uz} - P_{ub,y})} \text{ for } P_u > P_{ub,y}$$

where, P_{uz} = Maximum pure compression strength of the column.

$P_{ub,x}$ and $P_{ub,y}$ = Axial strength of the column corresponding to balanced failure w.r.t. major and minor axis respectively.

The value of the factor 'k' varies from zero (for $P_u = P_{uz}$) to unity (for $P_u = P_{ub}$). The above relationship for the factor 'k' is not applicable when $P_u < P_{ub}$ i.e. $k = 1$ for $P_u < P_{ub}$.

2. For braced columns which are subjected to unequal primary moments M_1, M_2 at the two ends of the column, the value of M_u is given by:

$$M_u = 0.4\, M_1 + 0.6\, M_2 > = 0.4\, M_2$$

where, M_2 = Higher column end moment.

Do You Know?

Approximation Involved in the Design Charts of SP-16

The following approximations are involved in the design charts of SP -16.

1. **Grade of concrete:** All design charts of SP : 16 assume a uniform grade of M 20 concrete. But each chart has 14 plots with different values of 'p/f_{ck}' ranging from 0 to 0.26 at an interval of 0.02. Thus we can make use the actual grade of concrete by multiplying with p/f_{ck} obtained from the plot with actual 'f_{ck}'.
2. **d'/D ratio:** The four fixed values of d'/D is SP : 16 are 0.05, 0.1, 0.15 and 0.20. In actual design, d'/D may not match with any of the four d'/D values as given in SP : 16. For such cases, linear interpolation has to be done.
3. **Reinforcement bars in circular columns:** Although the design charts are prepared assuming eight bars uniformly placed circumstantially, they can be used for any number of bars greater than six but placed uniformly around the circumference.

12.20 Limitations of Design Charts of SP-16

1. **Unsymmetrical arrangement** of longitudinal bars in rectangular cross-section is not at all covered in SP-16.
2. **Apart from rectangular and circular cross-section,** other cross-sections like *L*, *T*, *H*, *X*, octagon, pentagon etc. are not covered in SP-16.

Example 12.8 What is the maximum factored load that a column of size 300 x 500 mm can carry when it is braced against sideways and has an unsupported length of 7 m? (Assume effective length ratios, $k_x = k_y = 0.85$, M25 concrete, Fe415 steel, $A_{sc} = 2950$ sq. mm) P_{ubx} and P_{uby} as determined from interaction curves are 445 kN and 470 kN respectively.

Solution:

$D_x = 500$ mm, $D_y = 300$ mm, $A_{sc} = 2950$ mm^2, $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2

Unsupported length (l) = 7 m = 7000 mm

$k_x = k_y = 0.85$

Determination of slenderness ratios

$$l_{ex} = 0.85 \times 7000 = 5950 \text{ mm}$$

$$l_{ey} = 0.85 \times 7000 = 5950 \text{ mm}$$

$$\frac{l_{ex}}{D_x} = \frac{5950}{500} = 11.9 \simeq 12$$

$$\frac{l_{ey}}{D_y} = \frac{5950}{300} = 19.83 > 12$$

Thus, the given column is long/slender.

Method-1: Method of reduction in strength coefficient

$$C_r = 1.25 - \frac{\left(\frac{l_e}{D}\right)_{max}}{48} = 1.25 - \frac{19.83}{48} = 0.8369$$

Axial load carrying capacity considering it as a short column

$$\tilde{P}_{uo} = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_s$$
$$= 0.4(25)(300 \times 500) + (0.67 \times 415 - 0.4 \times 25) 2950$$
$$= 2290.75 \text{ kN}$$

Thus for slender column, $P_{umax} = C_r \tilde{P}_{uo}$

$$= 0.8369 \times 2290.75 \text{ kN} = 1917.13 \text{ kN} \sim 1917 \text{kN}$$

Method-2: Additional Moment Method

Minimum eccentricities

$$e_{xmin} = \frac{l}{500} + \frac{D_x}{30} = \frac{7000}{500} + \frac{500}{30} = 30.67 \text{ mm} > 20 \text{ mm}$$

$$e_{ymin} = \frac{l}{500} + \frac{D_y}{30} = \frac{7000}{500} + \frac{300}{30} = 24 \text{ mm} > 20 \text{ mm}$$

Primary Moments

Since the column is axially loaded and thus $M_{ux} = M_{uy} = 0$. But it should always be checked that the total moments are not less than that due to minimum eccentricities.

Additional Moments

Without modification factor, $e_{ax} = D_x \dfrac{\left(\frac{l_{ex}}{D_x}\right)^2}{2000} = \dfrac{500(11.9)^2}{2000} = 35.4025 \simeq 35.4$ mm

$$e_{ay} = D_y \frac{\left(\frac{l_{ey}}{D_y}\right)^2}{2000} = \frac{300(19.83)^2}{2000} = 58.98 \text{ mm}$$

Modification factors k_{ax}, k_{ay}

$$k_{ax} = \frac{P_{uz} - P_u}{P_{uz} - P_{ub,x}}$$

$$k_{ay} = \frac{P_{uz} - P_u}{P_{uz} - P_{ub,y}}$$

where, $P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_z$

$$= 0.45(25)(300 \times 500) + (0.75 \times 415 - 0.45 \times 25) 2950$$
$$= 2572.5 \text{ kN}$$

$P_{ub,x} = 445$ kN

$P_{ub,y} = 470$ kN

$\therefore$ $k_{ax} = \dfrac{2572.5 - 1935}{2572.5 - 445} = 0.2996$

$k_{ay} = \dfrac{2572.5 - 1935}{2572.5 - 470} = 0.3032$

$\therefore$ $M_{ax} = P_u(k_{ax} \cdot e_{ax}) = 1935\left(0.2996 \times \dfrac{35.4}{1000}\right) = 20.522$ kNm

$M_{ay} = P_u(k_{ay} \cdot e_{ay}) = 1935\left(0.3032 \times \dfrac{58.98}{1000}\right) = 34.603$ kNm

Total moments:

$M_x = M_{ux} + M_{ax} = 0.0 + 20.522$ kNm $= 20.522$ kNm

$M_y = M_{uy} + M_{ay} = 0.0 + 34.603$ kNm $= 34.63$ kNm

$P_u \cdot e_{x\,min} = 1935 \times \dfrac{30.67}{1000} = 59.346 \text{ kNm} > M_x$

$P_u \cdot e_{y\,min} = 1935 \times \dfrac{24}{1000} = 46.44 \text{ kNm} > M_y$

For $P_u = 1935$ kN

and $M_{ux1} = 130$ kNm, $M_{uy1} = 76$ kNm } (from SP: 16)

Now, $M_{ux1} > 59.346$ kNm

and $M_{uy1} > 46.44$ kNm

$\dfrac{P_u}{P_{uz}} = \dfrac{1935}{2572.5} = 0.7522$

$\therefore$ $\alpha_n = 1 + \left(\dfrac{0.7522 - 0.2}{0.8 - 0.2}\right)(2 - 1) = 1.92033$

$$\therefore \quad \left(\frac{M_{ux}}{M_{ux_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} = \left(\frac{59.346}{130}\right)^{1.92033} + \left(\frac{46.44}{76}\right)^{1.92033}$$

$$= 0.22183 + 0.38833$$

$$= 0.61016 < 1 \qquad \text{(OK)}$$

The factored load carrying capacity of the column is 1935 kN.

Example 12.9 A column of size 200 × 300 mm has an effective length of 3 m and is reinforced with 6 nos. 20 mm dia. bars. Determine the safe load on the column using WSM. Use M 20 concrete and Fe 415 steel.

Solution:

Slenderness ratio, $\lambda = \frac{l_{eff}}{B} = \frac{3000}{200} = 15 > 12$

∴ It is a long column.

Load carrying capacity of the column

$$P = C_r(A_c\sigma_{cc} + A_{sc}\sigma_{sc})$$

$$= \left(1.25 - \frac{l_{eff}}{48B}\right)((A_g - A_{sc})\,\sigma_{cc} + A_{sc}\sigma_{sc})$$

$$= \left(1.25 - \frac{3000}{48\times 200}\right)\left(\left(200\times 300 - 6\times\frac{\pi}{4}\times 20^2\right)\times 5 + 6\times\frac{\pi}{4}\times(20)^2\times 100\right)$$

$$= 608.17 \text{ kN}$$

Example 12.10 Design a square column for a load of 1500 kN using WSM. The column is 9 m long and is fixed at one end and other end is held in position but not restrained against rotation. Use M 20 concrete Fe 415 steel.

Solution:



Axial load (P) = 1500 kN

Length of the column (L) = 9 m

Effective length of the column (l_{eff}) = 0.8 L = 0.8 × 9 m = 7.2 m

Check whether the column is short or long?

Let area of steel (A_{sc}) = 2%

Let side of the square column = B

∴ Area of the column section (A) = B^2

A_{sc} = 0.02 A = 0.02 B^2

Area of concrete (A_c) = $A - A_{sc} = A - 0.02\,A = 0.98\,A = 0.98\,B^2$

$P = A_c\sigma_{cc} + A_{sc}\sigma_{sc}$

$1500 \times 10^3 = 0.98\,B^2 \times 5 + 0.02\,B^2 \times 190$

B = 415.22 mm

Slenderness ratio (λ) = $\frac{l_{eff}}{B} = \frac{7200}{415.22} = 17.3 > 12$

∴ The column is long

$$C_r = 1.25 - \frac{l_{eff}}{48B} = 1.25 - \frac{7200}{48B} = 1.25 - \frac{150}{B}$$

$$P = C_r(A_c\sigma_{cc} + A_{sc}\sigma_{sc})$$

$$1500 \times 10^3 = \left(1.25 - \frac{150}{B}\right)(0.98\,B^2 \times 5 + 0.02\,B^2 \times 190)$$

B = 436 mm ≃ 440 mm (say)

Area of column section, $A = B^2 = 440^2 = 193600 \text{ mm}^2$

Area of steel, $A_{sc} = 0.02 \times B^2 = 0.02 \times 440^2 = 3872 \text{ mm}^2$

Using 25 mm dia. bars, no. of bars required

$$= \frac{3872}{\frac{\pi}{4}\times 25^2} = 7.88 \simeq 8 \text{ bars (say)}$$

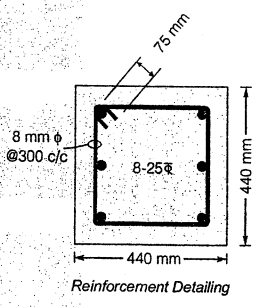
Design of transverse reinforcement (ties)

Dia of tie ≥ $\begin{cases} \frac{\phi_{main}}{4} = \frac{25}{4} = 6.25 \text{ mm} \\ 6\text{mm} \end{cases}$

Provide 8 mm dia. tie

Tie spacing ≤ $\begin{cases} \text{Least lateral dimension} = 440 \text{ mm} \\ 16\phi_{main} = 16\times 25 = 400 \text{ mm} \\ 300 \text{ mm} \end{cases}$

Provide 8 mm dia. ties @ 300 mm c/c.



Reinforcement Detailing

Example 12.11 Design a circular column with helical reinforcement for a load of 2400 kN. Diameter of the column is 600 mm. Use M 25 concrete and Fe 415 steel. The effective length of the column is 9 m. Use WSM.

Solution:

Given: Dia. of the column (D) = 600 mm

Axial load (P) = 2400 kN

Effective length of the column (l_{eff}) = 9000 mm

Slenderness ratio,

$$\lambda = \frac{l_{eff}}{D} = \frac{9000}{600} = 15 > 12$$

∴ It is a long column.

Load carrying capacity of the column

$$P = 1.05 \times C_r\,(A_s\sigma_{cc} + A_{sc}\sigma_{sc})$$

$$2400 \times 10^3 = 1.05 \times \left(1.25 - \frac{9000}{48\times 600}\right)\left(\left(\frac{\pi}{4}\times 600^2 - A_{sc}\right)\times 6 + A_{sc}\times 190\right)$$

$$A_{sc} = 4030.63 \text{ mm}^2$$

Using 25 mm dia. bars, no. of bars required

$$= \frac{40330.63}{\frac{\pi}{4}\times 25^2} = 8.2 \simeq 10 \text{ nos. (say)}$$

Design of helical reinforcement

Dia. of core (D_c) = 600 – 40 – 40 = 600 – 80 mm = 520 mm

$$\text{Area of core } (A_c) = \frac{\pi}{4}\times D_c^2 = \frac{\pi}{4}\times 520^2 = 212371.66 \text{ mm}^2$$

Volume of core per meter column length (V_c) = $A_c \times 1000 = 212371660 \text{ mm}^3$

Gross area of the column section

$$(A_g) = \frac{\pi}{4}\times 600^2 = 282743.3 \text{ mm}^2$$

Dia. of helix (D_h) = 520 – 10 mm = 510 mm (using 10 mm dia. ties)

$$\text{Volume of helix } (V_h) = \left(\frac{1000}{p}\right)\times(\pi D_h)\times\frac{\pi}{4}(\phi_h)^2$$

$$= \left(\frac{1000}{p}\right)\times 510\pi\times\left(\frac{\pi}{4}\right)\times 10^2$$

$$= \frac{125837456.1}{p} \text{ mm}^3$$

As per the requirement of helical reinforcement

$$0.36\frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c}-1\right) \le \frac{V_h}{V_c}$$

$$0.36\times\frac{25}{415}\left(\frac{282743.3}{212371.66}-1\right) = \frac{125837456.1}{212371660p}$$

$$p = 82.46 \text{ mm}$$

Pitch, $p \not> 75$ mm

$p \not> \frac{D_c}{6} = \frac{520}{6} = 86.67$ mm } max. pitch

$p \not< 3\phi_h = 3 \times 10 \text{ mm} = 30 \text{ mm}$

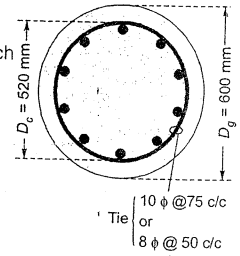
$p \not< 25$ mm } min. pitch

If 8 mm dia. ties are used then,

pitch $p = \frac{82.46\times 8^2}{10^2} = 52.77$ mm

Option 1: Provide 10 mm dia ties @ 75 mm c/c

Option 2: Provide 8 mm dia. ties @ 50 mm c/c



Reinforcement Detailing

Objective Brain Teasers

Q.1 In case of pedestal, the effective length shall not exceed the lateral dimension by _____ times.
(a) 3.0 (b) 2.0
(c) 1.5 (d) 4.0

Q.2 The length of a column is 3 m and its size being 300 mm x 300 mm. For this column to be designed as an axially loaded column, the minimum eccentricity is:
(a) 16 mm (b) 18 mm
(c) 20 mm (d) 25 mm

Q.3 In a spirally reinforced axially loaded short column, the concrete inside the CORE is subjected to:
(a) Axial tension
(b) Axial compression and biaxial bending
(c) Axial compression and uniaxial bending
(d) Tri-axial compression

Q.4 The main purpose of column ties is to resist:
(a) Shear stress in column
(b) Flexural stresses induced in a column
(c) Buckling of main reinforcing bars
(d) Axial compression and flexure

Q.5 For the design of an axially loaded short square column, a minimum eccentricity of 20 mm is considered. The size of the column will be:
(a) 350 mm (b) 450 mm
(c) 500 mm (d) 250 mm

Q.6 The minimum percentage of longitudinal reinforcement in a short column is:
(a) 0.6% (b) 0.8%
(c) 6% (d) 8%

Q.7 In case of rectangular column which is being subjected to bi- axial bending, the neutral axis is
(a) Parallel to column depth
(b) Parallel to column width
(c) Inclined to the column cross section
(d) Data insufficient

Q.8 The strength of a column subjected to bi-axial bending is a function of:
(a) Ultimate moment capacity about X-axis
(b) Ultimate moment capacity about Y-axis
(c) Axial load carrying capacity
(d) All of the above

Q.9 In a rectangular column subjected to axial compression and uniaxial bending, if neutral axis lies within the section then:
(a) Whole of the column section is under compression
(b) Whole of the column section is under tension
(c) Part of column section is under compression and part under tension
(d) Cannot be predicted with the present data available

Q.10 A RC wall is designed as a column carrying vertical loads. The ratio of minimum reinforcement is the vertical and horizontal directions is
(a) 3 : 5 (b) 5 : 3
(c) 1 : 1 (d) 2 : 1

Q.11 A short RC column of size 300 mm × 300 mm is made of M20 concrete and is reinforced with 4-20 mm diameter of Fe415 grade. The ultimate load carrying capacity of the column is _____ carrying axial load only. Ignore reduction of concrete area due to bars.
(a) 1200 kN (b) 1500 kN
(c) 1650 kN (d) None of these

Q.12 A short RC concrete column of size 250 mm × 400 mm is reinforced with 5-20 mm Fe500 steel bars. The column is casted with M30 concrete. The column carries axial load with minimum eccentricity as per IS 456: 2000. The load carrying capacity of the column is _____ kN.
(a) 1200 (b) 1500
(c) 1707 (d) 2205

Q.13 The lateral ties in RC column of rectangular section under axial load is used to

(a) provide shear capacity to the column
(b) avoid axial deformation of the column
(c) avoid buckling of reinforcing bars under compression
(d) All of the above

Q.14 The increase in flexural strength along with the increase in axial strength in a RC column occurs
(a) when yielding of tension reinforcement is the deciding factor for strength
(b) beyond ultimate load for the column
(c) at ultimate load for the column
(d) beyond crushing of concrete in the compression zone

Q.15 The effective length of a column in a RC building frame as per IS 456: 2000 is independent of
(a) span of the beam
(b) loads acting on the frame
(c) height of the column
(d) type of frame i.e., braced or unbraced

Q.16 A RC column is having 1% steel of cross-sectional area of column. Assume a modulus ratio of 10. The loads carried by longitudinal steel is P_s and that by net concrete area is P_c. The ratio P_s/P_c in terms of percentage is
(a) 10 (b) 10.1
(c) 10.5 (d) 10.7

Q.17 For a reinforced concrete pedestal, its length should _____
(a) exceed three times the lateral dimension of pedestal
(b) exceed five times the lateral dimension of pedestal
(c) not exceed five times the lateral dimension of pedestal
(d) not exceed three times the lateral dimension of pedestals

Q.18 The reduction coefficient of a RC column with 4800 mm effective length and size 250 mm × 300 mm is
(a) 0.85 (b) 0.9167
(c) 0.767 (d) 0.4377

Q.19 In an axially loaded short RC column, the concrete inside the core is subjected to
(a) tri-axial compression
(b) bi-axial compression
(c) axial compression
(d) flexural compression

Q.20 The maximum spacing of vertical reinforcement in RCC wall should not exceed
(a) the wall thickness
(b) three times the wall thickness
(c) twice the wall thickness
(d) 1.5 times the wall thickness

Answers

1. (a)	2. (c)	3. (d)	4. (c)	5. (a)
6. (b)	7. (c)	8. (d)	9. (c)	10. (a)
11. (a)	12. (c)	13. (c)	14. (a)	15. (b)
16. (b)	17. (d)	18. (a)	19. (a)	20. (b)

Hints:

5. (a)

For short columns, $(l/D)_{max} = 12$

Thus $l = 12D$

Now $e_{min} = 20$ mm

$$= \left(\frac{l}{500}\right) + \left(\frac{D}{30}\right) = \frac{12D}{500} + \frac{D}{30}$$

$$= 0.05733333\,D$$

$D = 348.83$ mm = 350 mm

10. (a)

As per IS 456: 2000, minimum reinforcement in vertical and horizontal directions is 0.12% and 0.2% respectively for HYSD bars upto 16 mm diameter and 0.15% and 0.25% otherwise

$$\therefore \frac{\text{Min. vertical reinforcement}}{\text{Min. horizontal reinforcement}} = \frac{0.12}{0.2}$$

or $$\frac{0.15}{0.25} = \frac{3}{5}$$

11. (a)

∵ The column carries axial load only without eccentricity

$\therefore P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

$$= 0.45(20)(300)^2 + 0.75(415)4 \times \frac{\pi}{4} \times 16^2$$

$$\simeq 1200 \text{ kN}$$

12. (c)

With minimum eccentricity,

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

where, $A_c = A_g - A_{st}$

$$= 250 \times 400 - 5 \times \frac{\pi}{4} \times 20^2$$

$$= 98429.20 \text{ mm}^2 \simeq 1707 \text{ kN}$$

16. (b)

$A_{st} = 0.01 A_g$

$A_c = A_g - A_{st} = 0.99 A_g$

$$\therefore \frac{P_s}{P_c} = \frac{mA_{st}f_c}{A_c f_c} \times 100$$

$$= \frac{10 \times 0.01 A_g f_c}{0.99 A_g f_c} \times 100 = 10.1\%$$

18. (a)

$$C_1 = 1.25 - \frac{l_{eff}}{48D}$$

$$= 1.25 - \frac{4800}{48 \times 300}$$

$$= 1.25 - \frac{100}{300} = 0.9167$$

$$C_2 = 1.25 - \frac{l_{eff}}{48b} = 1.25 - \frac{4800}{48 \times 250} = 0.85$$

(Lesser of C_1 and C_2)

∴ Reduction coefficient = 0.85.

Conventional Practice Questions

Q.1 A rectangular column section of size 600 × 450 mm in reinforced with 6-28ϕ bars of Fe415. Determine the ultimate load carrying capacity of the column assuming it a short column. Use M25 concrete.

Ans. [3690 kN]

Q.2 What is the ultimate load carrying capacity of a circular short column of 400 mm diameter which is reinforced with 6-25ϕ bars of Fe415 and effectively tied with (a) lateral ties (b) spirals? Use M20 concrete.

Ans. [1800 kN, 1890 kN]

Q.3 Design an axially loaded short column for an ultimate load of 1090 kN. The ultimate moment on the column is 19 kNm. Use M20 and Fe415. Keep width of column as 230 mm.

Ans. [230 × 400 mm size, 1208 mm² steel area]

Q.4 A short column is required to carry an axial load of 1500 kN under service load conditions. The column is also subjected to a service moment of 75 kNm. Using M25 concrete and Fe415 steel, design the column.

■■■■

CHAPTER 13

Design of Reinforced Concrete Shallow Foundations

13.1 Introduction

This chapter on footings will cover the analysis and design aspects of reinforced concrete footings or foundations. The words **'footing'** and **'foundation'** are often used synonymously but there is a slight difference between these two terms. By the term **footing**, we mean the actual surface which comes in direct contact to the soil but the term **foundation** implies that sub-structure that actually transmits the superimposed load of the super-structure and its self-weight plus soil backfill to the soil underneath.

In this chapter, we will define shallow and deep foundations, identify the situations where isolated or combined footing will be benefficial, the gross and net bearing capacity of the soil and the minimum depth of foundation required. Further, we will look into the critical locations for moments and shear in foundations for analysis and design purposes, the base pressure distribution in soil just below the footing.

In the design of footings, we will consider **rigid footings**. The analysis and design of **flexible footings** involves the concept of **soil-structure interaction** which is beyond the scope of present context. The design portion will cover the design of plain concrete footings, isolated reinforced concrete footings, wall footings and combined footings.

13.2 Footings

Proportioning of footing dimension should be such that, more or less a **uniform base pressure** is **obtained** from the soil underneath. Highly non-uniform soil pressures (due to sustained eccentric loads) are hazardous due to possible tilting of the footing leading to partial or complete collapse of the structure.

Cl. 20.1 of IS 456: 2000 permits a reduced factor of safety of **1.2** against overturning if the over turning moment is entirely due to the dead loads. However, a uniform factor of safety of **1.4** should invariably be used in all loading cases.

13.3 Footing as a Structural Element

Any structure that is built on ground consists of two parts viz. part of the structure located above the ground which is generally referred to as the **superstructure** and the part which lies below ground which is

referred to as the **substructure.** The elements of the superstructure transfer the loads and moments to its adjacent elements through a definite **load path** which finally comes down to the foundation. As per **Cl. 34.1** of **IS 456: 2000**, footings shall be designed to fulfil the following requirements:

1. Foundation structures must be able to sustain the applied loads, moments, and induced reactions without exceeding the safe bearing capacity of soil.
2. The settlement of the structure should be as uniform as possible and must be within the permissible limits.

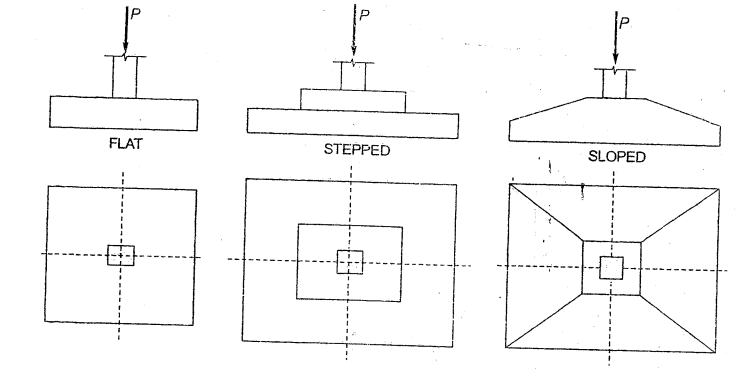
Further, in addition to the above requirements, (as per **Cl. 20 of IS 456: 2000**) the foundation structure must provide adequate safety against the stability of the structure due to overturning and sliding. The design of foundation structure is little bit different from the design of other super structure elements in the following ways:

1. Foundation structures involve soil-structure interaction. Thus, the behavior of foundation depends on the properties of soil and the structural elements. Determination of soil properties is altogether a different field (Geotechnical Engineering). Understanding the interaction of soil with structure is quite difficult and hence many simplifying assumptions are made in the foundation design.
2. Precise estimation of all the types of loads, moments, and forces is required for the present and future needs. Foundation once designed and constructed is very difficult to strengthen in future.
3. Foundation structures remain under ground so involves very little architectural consideration. The foundation must be kept within the property line which may induce additional moments due to eccentricity of column coming to the foundation.
4. Foundation structures remain in direct contact with the soil and hence any harmful chemicals if present in soil may lead to deterioration of foundation. Fluctuations in ground water table are also to be taken into account while designing a foundation.
5. While constructing a foundation, adjacent structures may get affected leading from formation of cracks to complete collapse of the adjoining structure particularly during pile driving operations etc. This aspect needs to be taken care of in the design and construction of foundations.

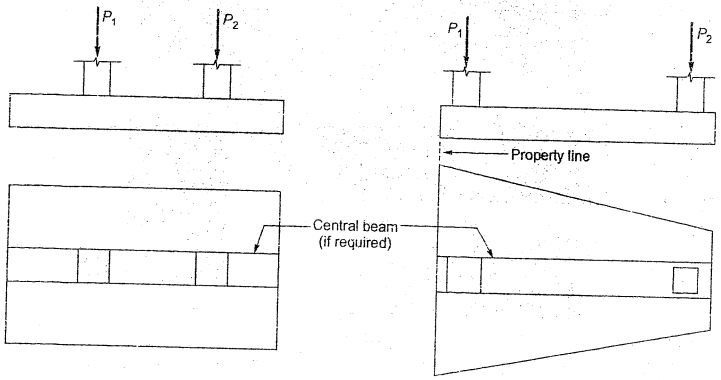
13.4 Types of Footings

Footings are strictly the shallow foundations and NOT the deep foundations like piles and caissons and are used when soil of sufficient strength is available at a shallow depth below the ground surface/level. Shallow foundations consists of footings (which support columns, walls and have a smaller/limited area and width in plan), rafts (which support multiple columns on a large plan area). Shallow foundation has a large plan area as compared to cross-sectional area of the column(s) it supports because:

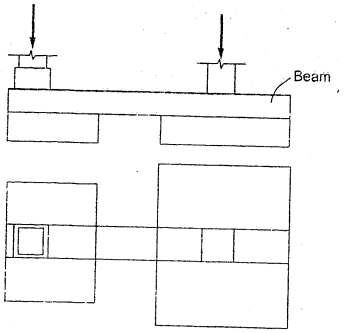
1. The loads on the columns (axial thrust, bending moments) are resisted by concrete under compression and reinforcing steel under tension and/or compression while these load effects are transferred by the footing/raft to a relatively weak supporting soil.
2. The 'safe bearing capacity' of the soil is very low (100 – 400 kPa) as compared to the permissible compressive stresses in concrete (5 – 15 MPa) and steel (130 – 190 MPa) under service loads.



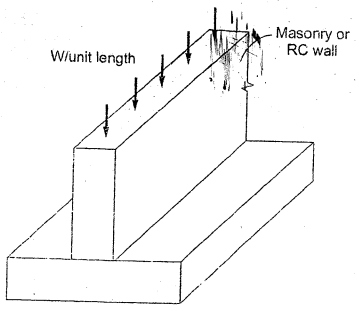
(a) Isolated footings



(b) Combined footings



(c) Strap footing



(d) Wall footing

Fig.13.1 *Various types of footing*

13.4.1 Plain Concrete/Pedestal Footings

These types of footings are very much economical for columns of small loads or pedestals. (Cl. 34.12 and Cl. 34.1.3 of IS: 456-2000) (Fig. 13.2).

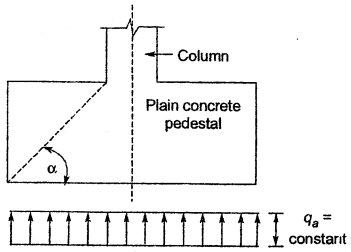


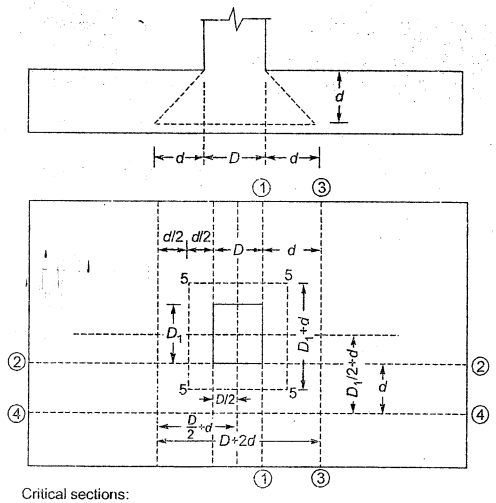
Fig.13.2 *Plain concrete pedestal*

13.4.2 Isolated Footings

For ordinary structures located on relatively firm soil, it is usually sufficient to provide individual footing for every column. Such a footing is called an **isolated footing**. Isolated footings are generally square or rectangular in plan but other shapes can also be adopted in special circumstances. The footing consists of a thick slab which may be flat (of uniform thickness), stepped or sloped (on the upper surface). Cl. 34.1.1 of IS: 456-2000 states that the sloped or stepped footings must be designed as one unit and constructed to ensure to act as a unit.

The soil bearing pressures from below the footing tend to make the base slab of the footing bend upwards and hence the footing is reinforced at the bottom of the slab.

Pedestal refers to the portion of a column below ground level where the cross-sectional dimensions are enlarged. The provision of a pedestal is not compulsory, but is often provided since it reduces the development length requirements. Pedestals are also used to support structural steel columns, the load transfer between the steel column and the concrete pedestal being achieved generally by the use of gusseted steel base plates with 'holding down' bolts.



Critical sections:
(1) For moments 1-1 and 2-2
(2) For one-way shear 3-3 and 4-4
(3) For punching shear, perimeter marked by 5-5-5-5.

Fig.13.3 *Rectangular footing*

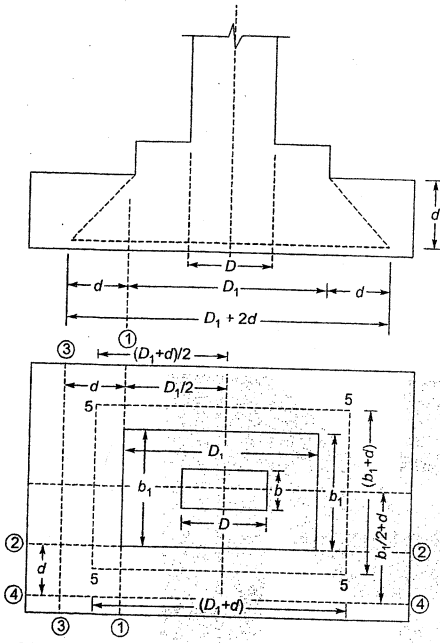


Fig.13.4 *Stepped and Rectangular footing*

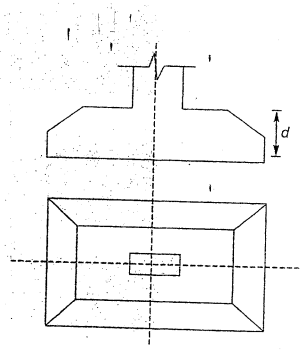


Fig.13.5 *Sloped and Rectangular footing*

13.4.3 Combined Footings

Many a times it is inconvenient to provide separate isolated footings for columns (or walls) due to insufficient areas available in plan.

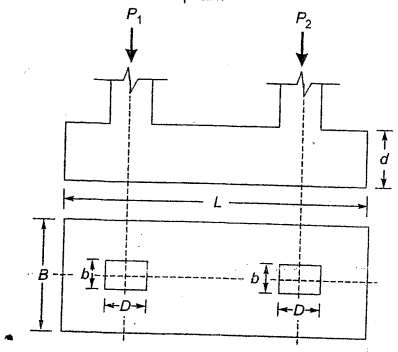


Fig13.6. *Combined footing without central beam*

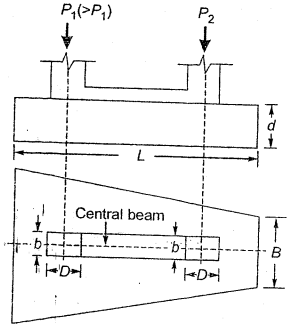


Fig.13.7 *Combined footing with central beam*

This usually occurs when two or more columns (or walls) are located close to each other or if they are relatively heavily loaded or rest on soil with low safe bearing capacity, resulting in an overlap of areas if isolated footings are provided. In such cases, it is preferred to provide a single combined footing for the columns Fig. 13.6.

At locations where there is a 'property line' which restricts the extension of the footing on the side towards the property line, in such cases, the problem of non-availability of space near the exterior column is overcome by providing combined footings Fig. 13.7.

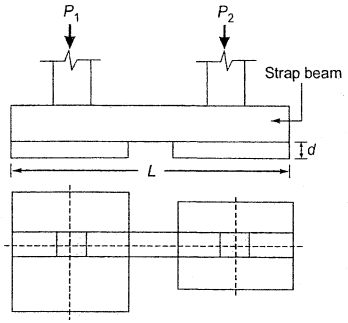


Fig.13.8 *Strap footing*

13.4.4 Strap Footings

If the two isolated footings are joined by a strap beam then it is called strap footing. These types of footings are required where the columns carry heavy loads and individual footing areas are not overlapping. This type of footing reduces the possibility of differential settlements Fig. 13.8.

13.4.5 Wall Footings/Strip Footings

Many a times, reinforced concrete footings are required to support reinforced concrete walls and sometimes load bearing masonry walls. Wall footings distribute the load from the wall to a larger area and runs throughout the length of the wall. The footing slab bends primarily in the direction transverse to the wall (as a one-way slab) and hence it is reinforced mainly in the transverse direction, with only nominal distribution reinforcement in the longitudinal direction Fig. 13.9.

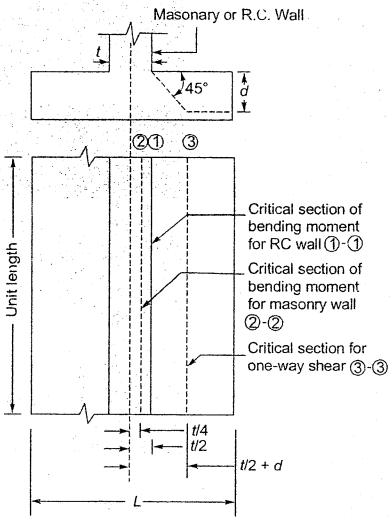


Fig.13.9 *Wall footing*

13.4.6 Raft/Mat Footings

These are special types of combined footings which support a large number of columns. This footing is provided when the column loads are heavy and the soil below is poor. Mat footing minimizes the chances of differential settlements and transfers the column loads to a very large area Fig. 13.10.

13.5 Distribution of Soil Pressure under Isolated Footings

13.5.1 Allowable Soil Pressure

The area in plan of the footing is so selected that the maximum pressure under the footing base due to dead and imposed loads of the structure (as well as weight of soil backfill and self-weight of footing) is well within the safe

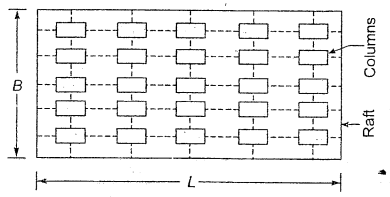


Fig.13.10 *Raft footing*

bearing capacity of the soil. This safe limit of soil pressure i.e. the permissible bearing capacity of the soils determined by the use of principles of soil mechanics (which is dealt in the subject of Geotechnical Engineering). Following are the main considerations in determining the safe soil pressure:

1. The loaded soil should not fail under the applied loads. Soil in fact, generally fails in shear.
2. The settlements, including both the total and differential settlements, should be well within the permissible limits.

Due to uncertainties and approximations involved in the soil mechanics problems, a safety factor in the range of 2 – 6 is generally adopted.

The safe soil bearing capacity (allowable soil pressure), q_a, provided for structural design is applicable for service load conditions since 'q_a' includes the factor of safety. Thus area of footing required must be calculated on the basis of q_a and service loads. Thus, **the partial safety factors for different load combinations will be those applicable to serviceability limit state and NOT the ultimate limit state when used along with q_a.**

The allowable soil pressure 'q_a' includes the existing overburden pressure (soil up to the foundation depth level). It is NOT the net soil pressure which is in excess of existing soil pressure. Thus total load to be calculated for footing design must include the weight of footing and that of soil backfill. Often, this value is taken as about 10-15 percent of axial column load.

Also,

Net Bearing Capacity = Gross Bearing Capacity – Pressure due to overburden soil

13.5.2 Depth of Foundation

As per **IS 1080 : 1980**, all types of foundation must have a **minimum depth of 500 mm.** This minimum depth is required to ensure the availability of soil having the safe bearing capacity assumed in the design. The foundation must be placed well below the natural or finished ground level below so that it will not be affected by swelling and shrinkage of soil. **Rankine's formula** gives a preliminary estimate of depth of foundation which is expressed as:

$$d_{min} \geq \frac{q}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

where, d_{min} = Minimum depth of foundation
q = Gross bearing capacity of soil
γ = Density of soil
ϕ = Angle of internal friction of soil

Rankine's formula does not take into account the loads acting on the foundation.

13.5.3 Base Pressure Distribution

The soil reaction at the base of the footing and its distribution depends on the rigidity of the footing and the soil properties i.e. it falls in the category of 'soil-structure interaction' problem. In fact the distribution of soil pressure below the footing is non-uniform, however a linear variation in soil pressure distribution below the footing is generally assumed in design.

13.5.4 Concentrically Loaded Footing

When the resultant vertical load ($P + \Delta P$) acting on the column passes through the centroid of the footing, the soil pressure is assumed to be uniformly distributed below the footing of plan area 'A' and the uniform soil pressure is given by,

$$q = \frac{P+\Delta P}{A}$$

For calculating the area of footing required,

$$A_{reqd} = \frac{P+\Delta P}{q_a}$$

where, ΔP = load due to foundation and backfill

13.5.5 Eccentrically Loaded Footing

In general, the line of action of column load does not coincide with the centroid of footing i.e. the line of action of load is often eccentric to the footing centroid, which gives rise to moments in the footing. Eccentricity in loading results due to the following reasons/causes:

1. The column transfers a moment in addition to axial load to the footing.
2. The line of action of column load does not pass through the centroid of footing.
3. When the column carries a lateral force located above the foundation level.

In practice, generally bi-axial eccentricities exist i.e. eccentricity in two principal, orthogonal, centroidal axes of the footing.

For finding the base pressure under the footing in eccentric load conditions, the footing is assumed to be rigid and linear contact pressure distribution under the footing is assumed. The magnitude of pressure distribution is determined using the principles of static equilibrium of the footing. This implies that the center of soil pressure (through which the resultant soil reaction R acts) must be collinear with the resultant line of action of eccentrically applied load, i.e. $R = P + \Delta P$. For preliminary design, ΔP, the weight of the footing plus soil backfill is taken generally as 10-15% of P.

Case 1: $|e| \leq L/6$

If the resultant eccentricity 'e' ($= M/(P+\Delta P)$) lies within the one-third of the footing, then entire area of contact of the footing is subjected to a non-uniform base pressure distribution which varies linearly from q_{min} to q_{max}. These are obtained as:

$$q_{max,min} = \frac{P+\Delta P}{A} \pm \frac{(P+\Delta P)e}{Z}$$

where, $A = BL$ and the section modulus $Z = BL^2/6$. L is the length of the footing in the direction of eccentricity 'e'.

From the above expression, it can be deduced that:

$$q_{max,min} = \frac{P+\Delta P}{A}\left(1 \pm \frac{6e}{L}\right)$$

when $e = L/6$, then $q_{min} = 0$ and $q_{max} = 2(P+\Delta P)/A$. This results in a triangular base pressure distribution. Limiting case of $e = L/6$ is valid only for uniaxial bending. For a general case of biaxial bending, the limiting case is taken as:

$$\frac{e_x}{\frac{L_x}{6}} + \frac{e_y}{\frac{L_y}{6}} \leq 1$$

Case 2: $|e| > L/6$

When the eccentricity 'e' is greater than $L/6$, then the expression of Case 1 yields negative pressure distribution i.e. value of q_{min} is negative. This negative pressure in soil i.e., tensile capacity is not possible in soil. In this case, the soil tends to separate from the footing thereby offering no pressure to the footing. For this case, q_{max} is given by:

$$q_{max} = 2(P + \Delta P)/BL'$$

Here, $L' = 3c$ and $c = 0.5L - e$

Thus effective length of contact gets reduced from L to $L' = 3c$ and the maximum soil pressure q_{max} gets increased to $2(P + \Delta P)/BL'$. In order that the value of q_{max} does not exceed allowable soil pressure q_a and also to maximize the effective bearing length, it is essential to design the footing with large base area.

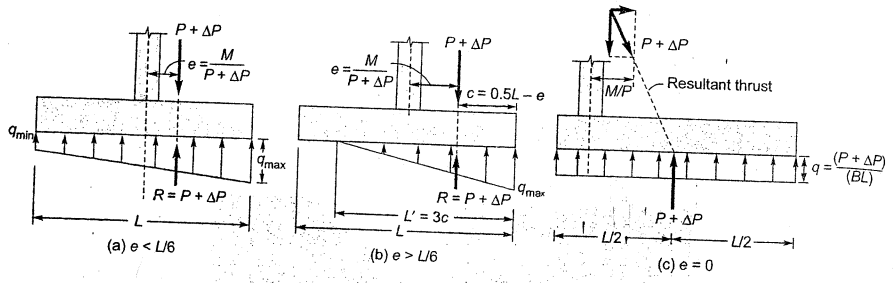


Fig.13.11 *Base pressure distribution under rectangular footing (assumed linear distribution)*

13.6 Footing Design: General Requirements and Codal (IS 456 : 2000) Provisions

13.6.1 Factored Soil Pressure at Ultimate Limit State

The base area of the footing is determined using the value of safe bearing pressure 'q_a' and applied loads and moments under the **service load conditions**. Once we are having the footing area which is in fact fixed now, the subsequent structural design of footing is done on the basis of **factored loads at ultimate limit state**. Thus for computing the values of bending moment, shear force etc. at critical locations under the factored load conditions, an **imaginary factored soil pressure** 'q_u' corresponding to factored loads is computed.

In reality, the moments and shear induced in the footing are due to net soil pressure, q_{net} (excluding the soil over burden pressure). This net pressure arises due to concentrated load carried by the column and the moments at the column base.

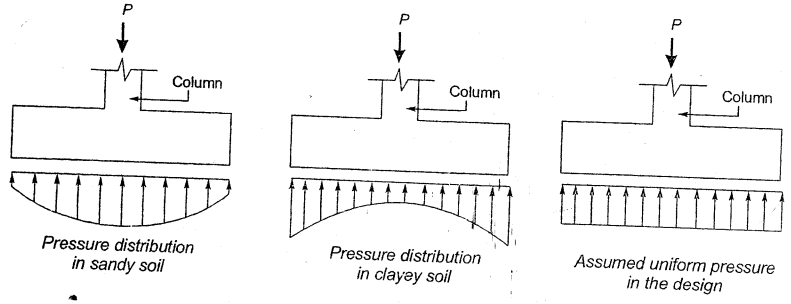


Fig.13.12 *Soil pressure distribution in sandy and clayey soils*

13.6.2 Design Considerations

In footing design, the major design considerations are as listed below:

1. Bending moments
2. Shear force (both one way and two way shear)
3. Bearing stress
4. Bond/development length
5. Other considerations include the transfer of force from column to footing and where horizontal forces are also involved, there; safety against the possible sliding and possible over turning needs to be adequately ensured. Deflection control is not considered in the design of footings which are buried under ground.

13.6.3 Thickness of Footing

The thickness of footing depends on considerations of shear force and bending moments which are critical near the column locations on the footing. Generally, shear consideration decides the thickness of footing. As per **Cl. 34.1.2 of IS: 456**, minimum thickness of footing at the edges must not be less than **150 mm** (300 mm in case of pile caps). This is done to ensure that the footing has sufficiently high rigidity to provide the calculated bearing pressures.

13.6.4 Minimum Nominal Cover

The minimum nominal cover to footings should be more than that of other structural elements of the super structure since footings remain in direct contact with the soil. As per **Cl. 26.4.2.2 of IS: 456-2000**, footings must have a minimum cover of **50 mm**. Actual cover may be more than that depending on soil conditions and presence of harmful chemicals in the soil.

13.6.5 Shear Consideration in Footing Design

The depth of footing is mostly governed by the shear requirement. In the design of footing, usually the design for shear supersedes the flexural design of footing. Both 'one-way shear' and 'two-way shear' need to be considered for footing design. The critical section for one way shear is located at a distance 'd' (effective depth) from the column face.

One way shear: (Cl. 34.2.4 of IS: 456-2000) One way shear has to be checked across the full width of the base slab on a vertical section located from the face of column or pedestal at a distance equal to:

1. Effective depth of footing slab in case of footing slab on soil.
2. One half of the effective depth of footing slab if footing slab is on piles.

The design shear strength of concrete without shear reinforcement is given in **Table 19 of IS: 456-2000.**

Two way shear/Punching shear: (Cl. 31.6 and Cl. 34.2.4 of IS: 456-2000) Two way shear should be checked around the column perimeter at a distance equal to half the effective depth of the footing slab away from the column face or pedestal.

The permissible shear stress when shear reinforcement is not provided shall not exceed $k_s\tau_c$ where, $k_s = (0.5 + \beta_c) \le 1$, β_c being the ratio of short side to long side of the column and $\tau_c = 0.25\sqrt{f_{ck}}$.

In general, the depth of footing is governed by shear considerations and adequate depth of footing must be provided to avoid shear reinforcement.

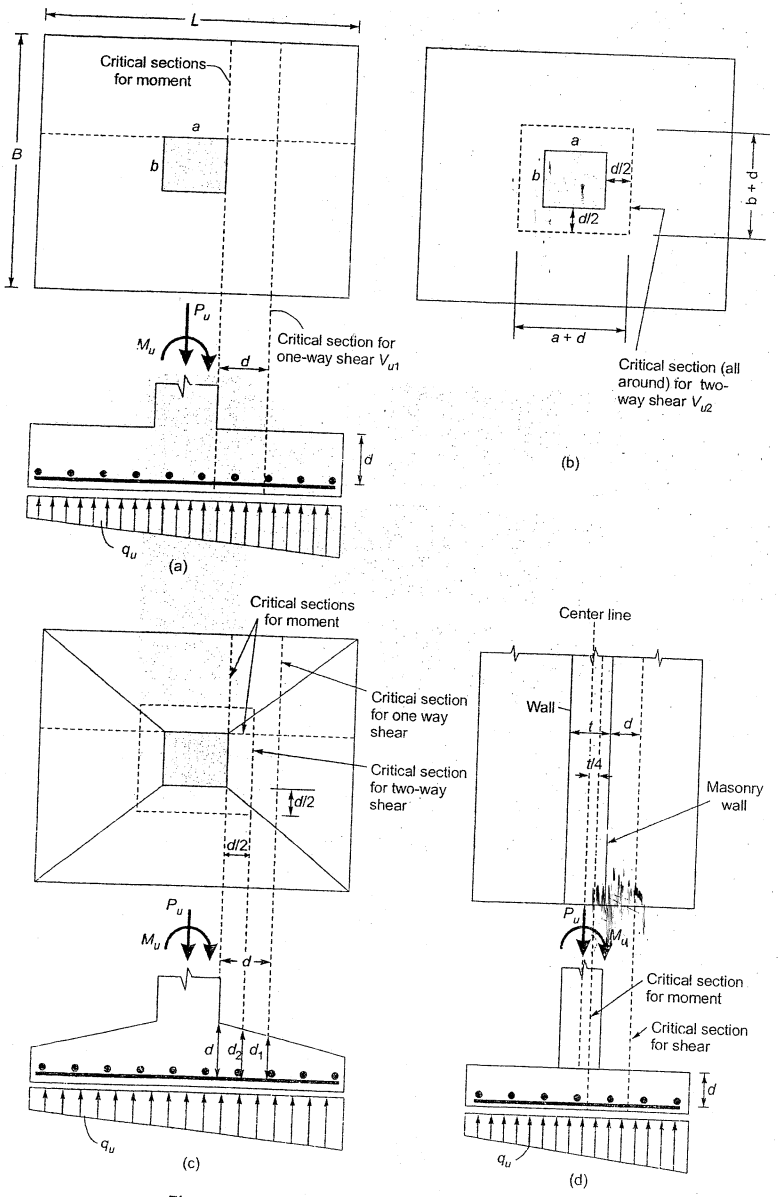


Fig.13.13 *Critical sections for shear and moment in footings*

The behavior of footings in two way shear is identical to flat two way slabs on columns. The critical section for two way shear is taken at a distance 'd/2' from the column face. Shear reinforcement is usually NOT provided in footings and factored shear force (V_u) is kept well below the factored shear resistance (V_{uc}) of concrete by providing the sufficient depth of footing.

Remember

For locating the critical sections for shear and moments in columns having circular/octagonal cross section, an equivalent square section is considered. The equivalent square must be inscribed within the perimeter of the column section.

13.6.6 Flexural Moments Consideration in Footing Design

The footing base slab tends to bend upwards due to net soil pressure from below the footing base as shown in the figure below. In fact the footing base bends like a saucer shaped element. For this, as per Cl. 34.2.3.2 of IS: 456, the footing can be designed for flexure by considering the critical section for moments as a straight section passing through:

1. The face of column, pedestal or wall for footing supporting a concrete column, pedestal or wall;
2. Half-way between the wall face and wall center line for masonry walls;
3. Halfway between the face of the column or pedestal and the edge of the gusseted base for footings under gusseted bases.

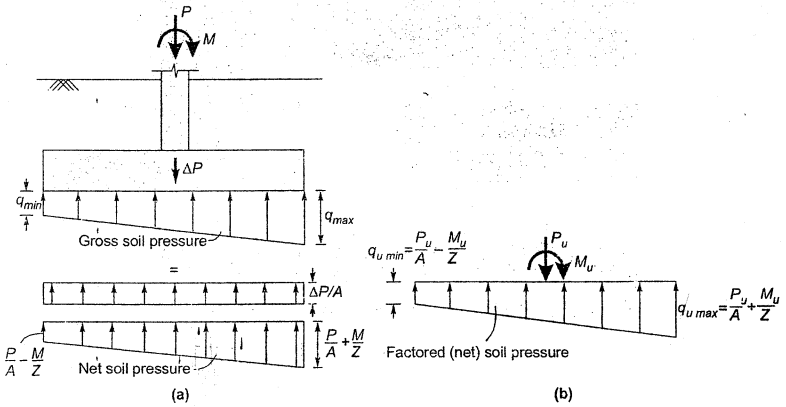


Fig.13.14 *Net soil pressure below footings*

In footings, which are mainly reinforced in one direction only i.e. in wall footings etc., the main reinforcement is placed perpendicular to the wall at a uniform spacing. In the longitudinal direction of the wall, nominal distribution reinforcement is provided generally to account for secondary moments due to Poisson's effect, shrinkage and temperature effects.

In two way reinforced concrete footings (square footing, rectangular footing etc.), flexural reinforcement is provided in both the orthogonal directions at a uniform spacing. For two –way reinforced concrete rectangular footing, the reinforcement in long direction is uniformly spaced but the reinforcement in the short direction is distributed in the central band and the end bands. Central band requires larger concentration of reinforcement than the end bands. The width of this central band is equal to width of the footing (B) as shown in Fig. 13.15.

Also reinforcement in the central band is given by as per Cl. 34.3.1 of IS 456 as,

$$A_{st,short} = \frac{2}{\beta+1}.$$

(Reinforcement in central band)

Where, $A_{st,\ short}$ = Total reinforcement required in the short direction

$\beta = L/B$ i.e. ratio of long to short side of the footing.

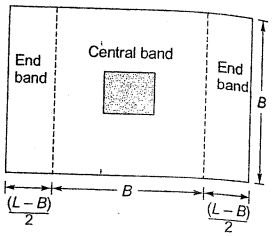


Fig.13.15 *Bands for reinforcement in a rectangular footing*

The remaining reinforcement is distributed in the end bands equally. These reinforcement detailing requirements are invariably intended for footings of uniform thickness. In sloped footings, it is usually sufficient to provide uniformly distributed reinforcement in the short direction. Due to large thickness of footing provided for shear considerations, the consequent required percentage of flexural reinforcement in footings is generally very low. But in any case, the percentage of flexural reinforcement should not be less than that prescribed for slabs.

It is beneficial to use small diameter bars at small spacing to prevent cracks in concrete and also to have sufficient development length. Development length requirement for flexural reinforcement must be satisfied at critical sections of moments. Any short fall (i.e. shortage) in the development length can be made by bending the bars up near the edges of the footing.

Further, the longitudinal reinforcement in the column or pedestal must also have the sufficient development length which is measured from the interface of the column/pedestal and the footing. When the column carries compressive load only (i.e. bars are not in tension), it is possible to have full transfer of forces from column/pedestal to footing by the action of bearing alone. Where the bars are in tension, adequate development length has to be provided at the column/pedestal – footing interface.

13.6.7 Bond Consideration in Footing Design

The critical sections for checking the development length in footings shall be the same as those for flexure/moments. However, development length must be checked at all the critical locations where the section changes abruptly. Cl. 34.2.4.3 of IS 456: 2000 specifies the critical section for checking the development lengths. This further specifies to check the anchorage requirements in case of curtailment of reinforcement. Curtailment shall be done according to Cl. 26.2.3 of IS 456: 2000.

13.6.8 Tensile Reinforcement

Tensile reinforcement should be calculated as per moments at critical locations:

1. For one way reinforced concrete footing slabs like wall footings, reinforcement shall be distributed evenly across the full width of the footing i.e. normal to the direction of wall. Nominal distribution reinforcement shall also be provided as per Cl. 34.5 of IS 456: 2000 along the length of wall in order to account for secondary moments, temperature effects, shrinkage and differential settlements.
2. In two way reinforced concrete square footings, reinforcement should extend in both the directions distributed uniformly across the full length and width of the footing.
3. In two way reinforced concrete rectangular footings, reinforcement in the long direction should be distributed evenly across the full width of the footing. In the short direction, a central band of width equal to width of the footing is marked along the length of the footing. In the central band, reinforcement is distributed as:

$$\text{Reinforcement in central band} = \frac{2}{(\beta+1)} \times \text{Total short direction reinforcement}$$

where, β = Ratio of long side to short side of the footing.

In the other two end bands, half of the remaining reinforcement should be provided uniformly distributed.

13.6.9 Force Transfer at the Column Base

All forces (axial force, lateral force and moments) acting on the column are required to be transferred to the footing either by compression of concrete or by tension/compression of reinforcing steel. But for the case of force transfer by compression of concrete, one cannot transfer a force larger than bearing resistance of concrete for either surface (i.e. supported surface or supporting surface). Under the action of factored loads, as per Cl. 34.4 of IS: 456, the maximum bearing resistance should not exceed the value as given below:

$$f_{br,\ max} = 0.45 f_{ck}\sqrt{\frac{A_1}{A_2}}$$

where, A_2 = loaded area at the column base

A_1 = maximum area of the portion of supporting surface that is geometrically similar to and concentric with the loaded area.

The significance of the factor $\sqrt{\frac{A_1}{A_2}}$ lies in the fact that it allows for increase in concrete strength in the bearing area in the footing due to confinement provided by the surrounding concrete and maximum value of this factor is limited to 2. This limitation is imposed because very high axial compressive stress gives rise to transverse tensile strain (due to Poisson's effect) which leads to spalling of concrete, lateral splitting of concrete or even bursting of concrete. Cl. 34.4 of IS 456 limits the permissible bearing stress of concrete to $0.25f_{ck}$ in case of working stress method and $0.45f_{ck}$ in limit state method of design.

The bearing resistance of concrete in column at the interface (for which $\sqrt{\frac{A_1}{A_2}} = 1$) is the governing factor rather than bearing resistance of concrete in footing at the interface (for which $1 < \sqrt{\frac{A_1}{A_2}} <= 2$). If the actual compressive stress exceeds $f_{br,\ max}$ then the excess force is transferred by the reinforcement, dowels or mechanical connectors (Cl. 34.4.1 and Cl. 34.4.2 of IS 456). For transferring moment at the column base (reinforcement is in tension), it is necessary to provide same reinforcement in footing as in the column. The reinforcement provided at the interface must consists of at least four bars with a total area not less than 0.5% of the cross sectional area of the supported column/pedestal (Cl. 34.4.3 of IS 456). Where dowels are used, the diameter of dowel bars shall not exceed the diameter of column bars by 3 mm.

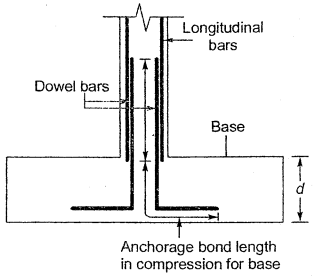


Fig.13.16 *Anchorage length of dowels*

Column bars of diameter greater than 36 mm, in compression only can be doweled at the footing with bars of smaller diameter of required area. The dowel shall extend into the column a distance equal to development length of the column bar and into the footing, a distance equal to the development length of the dowel as per Cl. 34.4.4 of IS 456: 2000.

13.6.10 Nominal Reinforcement

1. Cl. 34.5.1 of IS 456: 2000 gives minimum reinforcement and spacing of bars in footing as per the requirements of solid slab.
2. The nominal reinforcement of concrete sections of thickness greater than 1000 mm shall be 360 sq. mm per meter length of footing in each direction on each face as per Cl. 34.5.2 of IS 456: 2000.

13.7 Plain Concrete Footings

It is also called as pedestal footing and is used where the column load is very low and the footing area required is also low. If the bearing stress at the column base is less than the permissible bearing stress of concrete i.e. $f_{br,max} = 0.45 f_{ck}\sqrt{A_1/A_2}$ then the force transfer from the column base to the footing can be achieved without any need of reinforcement at the interface of column and the footing. Also, if the base area of the footing falls within a certain confined zone of load dispersion through the footing, then the entire column load can be transferred to the footing by 'strut action' (i.e. by compression) and the soil pressure does not induce any bending in the footing. These imaginary struts are inclined to the vertical whose horizontal component will call for a strut-tie like action as shown in Fig. 13.7. To carry the tie forces and to avoid the possible cracking of concrete due to tensile stresses, it is essential to provide some minimum reinforcement to serve as ties.

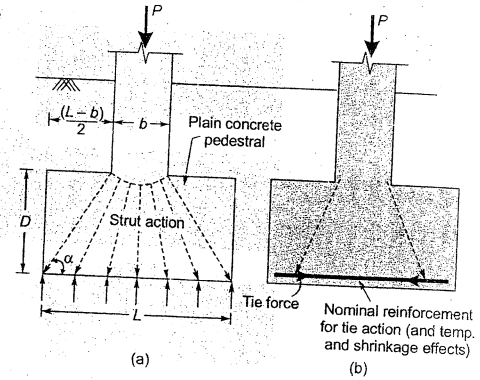


Fig. 13.17 *Tie action in plain concrete footing*

For the purpose of defining this confined zone of load dispersion and thus to determine the thickness of footing required, Cl. 34.1.3 of IS: 456 defines an angle α between the plane passing through the bottom edge of the footing and the corresponding edge of the column at the interface. This angle α is defined as:

$$\tan\alpha > 0.9\sqrt{\frac{100q_{max}}{f_{ck}} + 1}$$

where, q_{max} is the calculated maximum soil pressure at the base of the pedestal. Thus the thickness D of the footing required is: $D = (L - b)\tan\alpha/2$

Example 13.1 Design a plain concrete footing for a column of size 400 mm × 400 mm carrying an axial load of 350 kN as service loads consisting of dead and imposed loads. The allowable soil bearing pressure is 370 kN/m² at a depth of 1 m below the natural ground level (NGL). Use M 25 concrete and Fe 415 steel.

Solution:

Axial force transfer at the column base:

In case of plain concrete footing, full transfer of axial load at the column base must occur without any requirement of reinforcement at the interface of column and footing.

Factored load $P_u = 1.5 \times 350\ \text{kN} = 525\ \text{kN}$

Limiting bearing stress at the column footing interface = $f_{br,max} = 0.45 f_{ck} \cdot \sqrt{(A_1/A_2)}$

Now bearing stress in this case is governed by the column face and not the footing face.

Thus $A_1 = A_2 = 400 \times 400\ \text{mm}^2 = 160000\ \text{mm}^2$

$$f_{br,max} = 0.45 \times 25 \times \sqrt{\frac{160000}{160000}} = 11.25\ \text{N/mm}^2$$

Limiting bearing resistance = $F_{br} = 11.25 \times 400^2\ \text{N} = 1800\ \text{kN} >$ Factored load P_u (= 525 kN)

Thus full transfer of load from column to footing is possible without any reinforcement.

Calculation of the size of the footing required:

Let the weight of the footing and the backfill is about 10% of the column axial load i.e. 10% of 350 kN = 35 kN.

Thus base area required = (350 + 35)/370 = 1.0405 m²

Provide a footing of size 1.25 m × 1.25 m.

Thus footing area provided = 1.25 × 1.25 m² = 1.5625 m² > 1.0405 m²

Calculation of the depth of the footing required:

Footing depth $D = (1/2) \times (1.25 - 0.4) \times \tan\alpha$

where $\tan\alpha \geq 0.9\sqrt{\left(\frac{100q_{max}}{f_{ck}}\right) + 1} = 0.9\sqrt{\frac{100 \times 0.370}{25} + 1} = 1.41732$

(Here $q_{max} = 370\ \text{kN/m}^2 = 0.37\ \text{N/mm}^2$)

Thus $D = (1/2) \times (1.25 - 0.4) \times 1.41732 = 0.602361\ \text{m} = 602.361\ \text{mm}$

Provide a footing depth of 650 mm.

Thus required footing will be of size **1250 mm × 1250 mm × 650 mm**

Minimum reinforcement requirement:

A minimum amount of reinforcement @ 0.12% of gross area is always provided to ensure tie-action in the footing and to account for temperature and shrinkage effects.

Thus $A_{st,min} = 0.0012 \times 1250 \times 650\ \text{mm}^2 = 975\ \text{mm}^2$

Provide 5 nos. 16 mm diameter bars ($A_{st,provided} = 1005.3\ \text{mm}^2 > 975\ \text{mm}^2$) in both the directions with a clear cover of 75 mm.

Thus spacing of bars = (1250 mm − 2 × 75 mm − 5 × 16 mm)/4

= 255 mm < 5d (= 5 × (650 − 75) = 2875 mm) < 450 mm (OK)

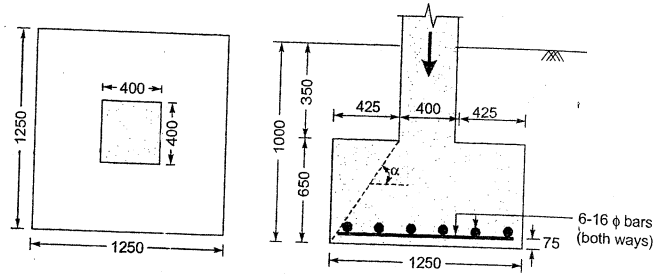
Check for soil base pressure:

Let unit weight of soil as 19 kN/m³ and that of concrete as 25 kN/m³.

Gross soil base pressure = 350/(1.25 × 1.25) + (25 × 0.65) + (19 × (1 − 0.65))

= 224 + 16.25 + 6.65)

= 246.9 kN/m² < 370 kN/m² (OK)



Reinforcement detailing

13.8 Design of Rectangular Isolated Footing

Given: Column load = P

Safe bearing capacity of the soil = q_o

Grade of concrete = f_{ck}

Grade of steel = f_y

Design Steps: For both WSM and LSM

Assume weight of foundation = P_F = 10% of P = $0.1P$

Total weight on soil = $P_T = P + 0.1P = 1.1P$

$$\text{Area of foundation required} = 1.1\frac{P}{q_0} \le L \times B$$

L and B should be decided suitably. (Preferably in the ratio of column size)

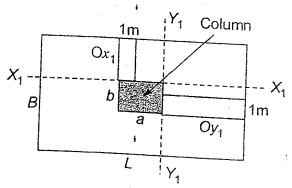
$$\text{Net pressure on soil} = w_o = q_0 - \frac{P_F}{A} = \frac{P_T}{A} - \frac{P_F}{A} = \frac{P_T - P_F}{A}$$

$$= \frac{1.1P - 0.1P}{A} = \frac{P}{A} = \text{Design soil pressure}$$

Factored soil pressure (for LSM) = $1.5w_o = 1.5(P/A)$

Depth of footing required by bending moment requirement:

Critical section for moment is as shown above.



At critical section $X_1 - X_1$: Overhang = $OX_1 = \frac{B-b}{2}$

$$M_X = w_o(OX_1)1\text{m}\,\frac{OX_1}{2}$$

$$= w_o\left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right) = \frac{w_o(B-b)^2}{8} \text{ (For both WSM and LSM)}$$

At critical section $Y_1 - Y_1$: Overhang = $OY_1 = \left(\frac{L-a}{2}\right)$

$$My = w_o(OY_1)\,1\text{m}\left(\frac{OY_1}{2}\right) = w_o\left(\frac{L-a}{2}\right)\left(\frac{L-a}{4}\right)$$

$$= w_o\frac{(L-a)^2}{8} \text{ (For both WSM and LSM)}$$

Design depth of footing = $d = \sqrt{\frac{M_{max}}{QB}}$ where B = 1000 mm

Check for shear: Depth calculated as per moment criteria is checked for one way and two way shear.

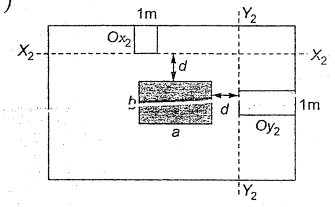
Shear at critical section $X_2 - X_2 = w_o(1\text{ m})\,OX_2 = w_o\left(\frac{B-b}{2} - d\right)$

Shear at critical section $Y_2 - Y_2 = w_o(1\text{ m})\,OY_2 = w_o\left(\frac{L-a}{2} - d\right)$

Nominal shear stress for maximum shear force:

$$\tau_v = \frac{V_y}{Bd} \text{ (For WSM)}$$

$$\tau_{uv} = \frac{V_{uy}}{Bd} \text{ (For LSM)}$$

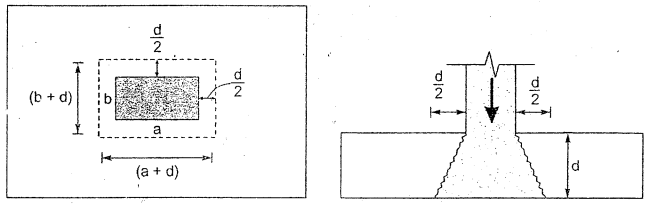


Shear stress developed (τ_v) < $k \cdot \tau_c$ where k is a factor depending on slab thickness

The values of k as per Cl. 40.2.1.1 (LSM) and B.5.2.1.1 (WSM) of IS 456: 2000 are as given below:

Depth of Slab (mm)	≤ 150	175	200	225	250	275	≥ 300
k	1.3	1.25	1.2	1.15	1.1	1.05	1.0

Check for Punching Shear



- The critical section for two way shear is at a distance $\frac{d}{2}$ from the column face.
- The area around the column gets punched through the soil.

  Net punching force = $P - w_o(a + d)(b + d)$

  Resisting cross sectional area = $2[(a + d) + (b + d)]d$

$$\text{Punching shear stress} = \frac{P - w_o(a+d)(b+d)}{2[(a+d)+(b+d)]d}$$

$$\text{Permissible shear stress } (\tau_{vp}) = \begin{cases} k_s(0.25)\sqrt{f_{ck}} & \text{(LSM)} \\ k_s(0.16)\sqrt{f_{ck}} & \text{(WSM)} \end{cases}$$

where, $k_s = 0.5 + \frac{b}{a} \le 1.0$

Area of steel required: Maximum depth required from the above three criteria is adopted as design footing depth (d).

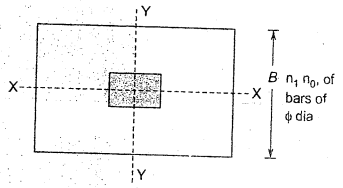
Thus overall footing depth (D) = d + effective cover = d + clear cover + $\frac{1}{2}$(bar dia.)

Area of steel along shorter side $Y - Y = A_{st}$

$$= \frac{M_x}{\sigma_{st} jd} \quad \text{(For WSM)}$$

$$A_{st} = \frac{M_x}{0.87 f_y jd} \quad \text{(For LSM)}$$

$$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - 4.598\frac{M_x}{Bd^2 f_{ck}}}\right]Bd$$



Total area of steel along shorter side = LA_{st}

Number of bars required (n_T) = $\frac{A_{st} L}{\frac{\pi}{4}\phi^2}$

No. of bars to be provided in central band = $\frac{2n_T}{1 + \frac{L}{B}}$

Remaining reinforcement shall be distributed equally in the end bands.

Area of steel along longer side $X - X = A_{st}$

$$= \frac{M_x}{\sigma_{st} jd} \quad \text{(For WSM)}$$

$$A_{st} = \frac{M_x}{0.87 f_y jd} \quad \text{(For LSM)}$$

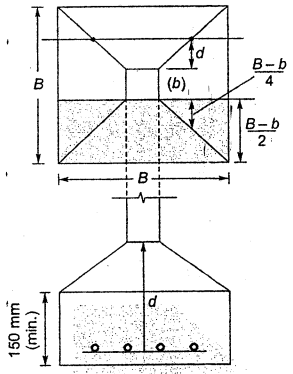
$$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - 4.598\frac{M_x}{Bd^2 f_{ck}}}\right]Bd$$

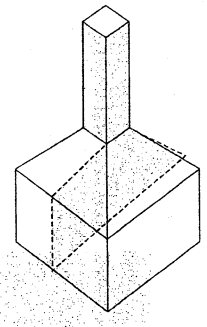
Total area of steel = BA_{st}

No. of bars required = $\frac{A_{st} B}{\frac{\pi}{4}\phi^2}$

This reinforcement is placed uniformly.

13.9 Design of Sloped Isolated Footing

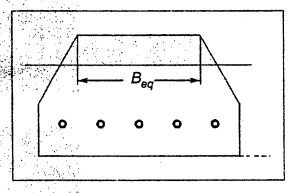




Minimum thickness of footing at the ends = 150 mm

Usually 200 mm is a sufficient value.

Size of foundation: Same as isolated foundation



Check for bending moment:

Critical section is at the face of column.

$$M_x = w_o B \cdot \left(\frac{B-b}{2}\right)\cdot\left(\frac{B-b}{4}\right)$$

$$= w_o B \frac{(B-b)^2}{8}$$

$$d = \sqrt{\frac{M_x}{QB_{eq}}}$$

Approximately, $B_{eq} = b + \frac{B-b}{8}$

Check for one way shear:

Shear force at critical section is given by:

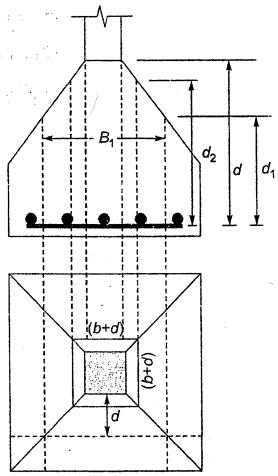
$$V_x = w_o B\left(\frac{B-b}{2} - d\right)$$

Nominal shear stress is given by:

$$\tau_v = \frac{V}{B_1 d_1} \le \tau_c$$

Here B_1 = Width of section at critical section for shear

d_1 = Depth at critical section for shear



Check for two way/punching shear

Punching shear stress developed = Net punching force / Resisting area

$$= \frac{P - w_o(b+d)^2}{4(b+d)d_2}$$

Here, d_2 = Depth of section at critical section for punching shear

τ_v (developed) $\le \tau_{vp}$ (permissible) = $k_s(0.16)\sqrt{f_{ck}}$ (as per WSM)

$= k_s(0.25)\sqrt{f_{ck}}$ (as per LSM)

Area of steel: Same as that for isolated footing.

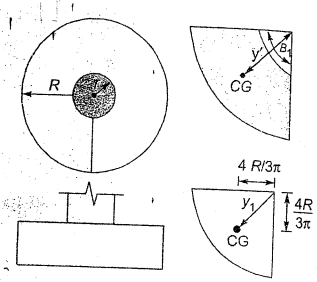
13.10 Design of Circular Isolated Footing of Uniform Thickness

Area of footing: Same as that for isolated footing.

$$A = \frac{P_T}{q_0}$$

Radius of footing (R) = $\sqrt{\frac{A}{\pi}}$

Net soil pressure (w_o) = $\frac{P}{A}$



Check for bending moment: Consider one quadrant of circle. Bending moment is calculated at the face of column for one quadrant. Distance of CG of quadrant from centre is given by:

$$y_1 = \sqrt{2}x\frac{4R}{3\pi} = 0.6R$$

Distance of CG of shaded area from centre is given by:

$$y' = \frac{A_1y_1 - A_2y_2}{A_1 - A_2} = \frac{\frac{\pi}{4}R^2x0.6R - \frac{\pi}{4}r^2x0.6r}{\frac{\pi}{4}R^2 - \frac{\pi}{4}r^2} = 0.6\frac{R^3 - r^3}{R^2 - r^2}$$

Distance of CG from the face of column = $y' - r = 0.6\left(\frac{R^3 - r^3}{R^2 - r^2}\right) - r$

Bending Moment:

$$M = w_o(\text{Shaded area})(y' - r) = w_o\frac{\pi}{4}(R^2 - r^2)\left[0.6\left(\frac{R^3 - r^3}{R^2 - r^2}\right) - r\right]$$

Thus $d = \sqrt{\frac{M}{QB_1}}$

where, $B_1 = \frac{2\pi r}{4}$

Check for one way shear

Shear force = $V = w_o \times$ Area of shaded portion

$$V = w_o x \frac{\pi}{4}[R^2 - (r+d)^2]$$

Nominal shear stress = $\tau_v = \frac{V}{B_2 d}$ where, $B_2 = \frac{2\pi(l+d)}{4}$

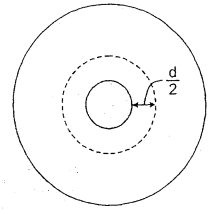
Check for punching/two way shear

Net punching shear force is given by:

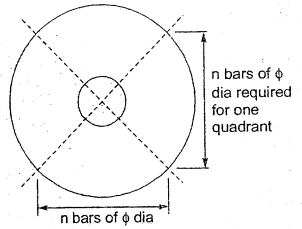
$$P_{net} = P - w_o\pi\left(r + \frac{d}{2}\right)^2$$

Punching shear stress is given by:

$$\tau_{vp} = \frac{P_{net}}{\text{Perimeter} \times d} = \frac{P - w_o\pi\left(r + \frac{d}{2}\right)^2}{2\pi\left(r + \frac{d}{2}\right)d}$$



Critical section for two way shear



Area of steel required

Area of steel for one quadrant is given by:

$$A_{set} = \frac{M_u}{\sigma_{st}jd} = \frac{M_u}{0.87f_yjd}$$

$$= \frac{0.5f_{ck}}{f_y}\left[1 - \sqrt{1 - 4.598\frac{M_u}{f_{ck}B_1d^2}}\right]B_1d$$

13.11 Design of Wall

Two types of reinforcement are provided in walls viz. vertical and horizontal reinforcement.

Vertical Reinforcement: Minimum vertical reinforcement should be as given below:

(a) 0.12%of total gross area, diameter of bars ≤ 16 mm for Fe 415 and higher grades of steel.

(b) 0.15%of the total gross area for other types of steel

(c) 0.12% of total gross area for welded wire fabric

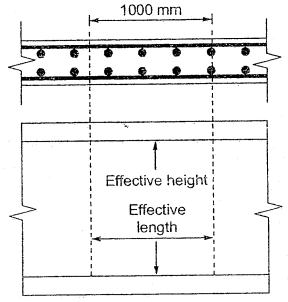
Horizontal Reinforcement: Minimum vertical reinforcement should be as given below:

(a) 0.2% of total gross area for *Fe* 415 and higher grades of steel.

(b) 0.25% of total gross area for mild steel.

(c) 0.2% of total gross area for welded wire fabric

Design is same as that of column.



Example 13.2 Design a spread footing to carry an axial load of 1200 kN through a column of size 450 mm x 450 mm having 4-25Φ bars. Bearing capacity of soil is 105 kN/m². Assume footing to be 1.25 m below ground level and concrete of grade M25 and Fe415 steel.

Solution:

Axial load (P) = 1200 kN

Assuming 10% of column load as weight of footing and soil backfill.

∴ Weight of footing and soil backfill = 10% of 1200 kN = 120 kN

∴ Total axial load = 1200 + 120 = 1320 kN

∴ Area of footing required = $\frac{1320}{105} = 12.57\,m^2$

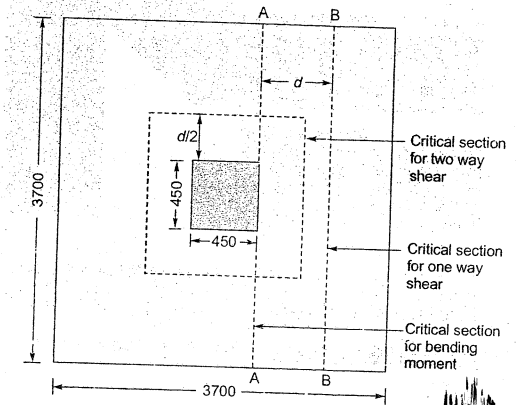
∵ Column section is square of size 450 mm × 450 mm

∴ Provide square footing of side = $\sqrt{12.57} = 3.55$m

i.e., provide square footing of size 3.7 m × 3.7 m.

Calculation of bending moment

Soil pressure acting upwards = $\frac{1.5\times1200}{3.7\times3.7} = 131.48\ kN/m^2$



Section (A) – (A) is the face of column which is critical section for bending moment.

∴ $M_u = 131.48\times3.7\left(\frac{3.7-0.45}{2}\right)\left(\frac{3.7-0.45}{2}\right)\frac{1}{2} = 642.3$ kNm

Depth of footing required

$M_u = 0.138\, f_{ck}\, bd^2$ ... (for Fe415 steel)

⇒ $642.3\times10^6 = 0.138\,(25)\,3700\,d^2$

⇒ $d = 224.315$ mm

Adopt overall depth of footing as 500 mm so that effective depth of footing (d),

$d = 500 - 75 - \frac{20}{2} = 415$ mm

(Assuming 20 mm diameter bars and 75 mm clear cover)

Reinforcement required for footing

$D = 500$ mm

$d = 415$ mm

$R = \frac{M_u}{bd^2} = \frac{642.3\times10^6}{3700(415)^2} = 1.00795$

$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1-\sqrt{1-4.598\left(\frac{1.00795}{25}\right)}\right] = 2.935\times10^{-3}$

∴ $p_t = 0.2935\%$

⇒ $A_{st} = \frac{0.2935}{100}\times3700\times415 = 4506.7\ mm^2$

Using 20 mm diameter bars,

No. of bars required = $\frac{4506.7}{\frac{\pi}{4}(20)^2} = 14.3 \simeq 15$ bars

Spacing of bars = $\frac{3700-2\times75-15\times20}{(15-1)} = 232.14$ mm

Using 16 nos. of 20 mm dia. bars, spacing = $\frac{3700-2\times75-16\times20}{(16-1)} = 215.3$ mm

Provide 16 nos. 20 Φ bars ⇒ $A_{st\,provided} = 16\times\frac{\pi}{4}\times20^2 = 5026.55\,mm^2 > 4506.7\,mm^2$ (OK)

Check for one way shear

Critical section for one way shear is at a distance 'd' from the face of column i.e. at section (B) – (B)

SF at critical section (B) – (B) (V_{u1}) = $131.48\times3.7\left[\left(\frac{3.7-0.45}{2}\right)-0.415\right]$ kN = 588.64 kN

Nominal shear stress (τ_v) = $\frac{588.64\times10^3}{3700\times415} = 0.3834\ N/mm^2$

Shear strength of M25 concrete with p_t = $\frac{5026.55}{3700\times415}\times100 = 0.3274\%$

As per table 19 of IS:415, (τ_c) = 0.4 N/mm² > τ_u (= 0.3834 N/mm²) (OK)

This depth provided is safe in one way shear.

Check for two way shear

Critical section for two way shear is at a distance '$d/2$' from the column face.

SF at critical section (V_{u2}) = $131.48\,(3.7^2-(0.45+0.415)^2) = 1701.6$ kN

Nominal shear stress (τ_v) = $\frac{1701.6\times10^3}{4(450+415)415} = 1.185\,N/mm^2$

Design shear strength of M25 concrete in two way shear

$\tau'_c = k_s\tau_v$

where $k_s = \frac{1}{2}+\beta_c$

where $\beta_c = \frac{\text{Shorter side of column}}{\text{Longer side of column}} = 1 = 0.5 + 1 \not> 1 = 1$

$$\tau'_c = k_s \tau_v = k_s\, 0.25\sqrt{25}$$
$$= (1)\, 0.25\sqrt{25} = 1.25\ \text{N/mm}^2 > \tau_v (= 1.185\ \text{N/mm}^2)$$

Thus depth of footing is safe in two way shear.

Development length

$$\text{Development length } (L_d) = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

where,

$$\tau_{bd} = 1.6 \times 1.4 = 2.24\ \text{N/mm}^2$$

$$L_d = \frac{0.87(415)10}{4(2.24)} = 403\ \text{mm} \qquad \text{(Cl. 26.2.1.1 of IS:456)}$$

Actual development length provided from the face of column

$$= \frac{3700 - 450}{2} - 75(\text{clear cover})$$
$$= 1550\ \text{mm} > 403\ \text{mm} \qquad \text{(OK)}$$

Load transfer at column-footing interface

Bearing pressure in column concrete at column footing interface.

$$= f_{br} = \frac{1.5 \times 1200}{450^2} \times 1000 = 8.89\ \text{N/mm}^2$$

$$\text{Permissible bearing stress} = 0.45 f_{ck}\sqrt{\frac{A_1}{A_2}} \quad \text{where } \frac{A_1}{A_2} = 1 \text{ here}$$

$$\text{Permissible bearing stress} = 0.45(25)$$
$$= 11.25\ \text{N/mm}^2$$
$$> 8.89\ \text{N/mm}^2 \qquad \text{(OK)}$$

Thus column load can be transferred by bearing alone. Even then a minimum 0.5% steel is provided at column footing interface.

$$\therefore \quad \text{Minimum steel } (A_{s\,min}) = 0.5\%\ \text{of column area} = \frac{0.5}{100} \times 450^2 = 1012.5\ \text{mm}^2$$

Using 20 mm diameter bars,

$$\text{No. of bars required} = \frac{1012.5}{\frac{\pi}{4}(20)^2} = 3.22 = 4\ \text{bars}$$

Provide 4-20 Φ bars as dowel bars, at the interface of column and footing.

$$\text{Development length in compression } (L_d) = \frac{0.87(415)20}{4(2.24)1.25} = 644.7\ \text{mm}$$

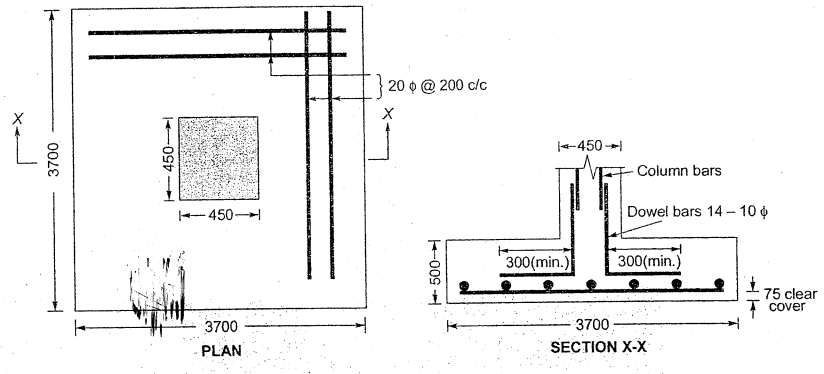
$$\text{Available length} = 500 - 75\ (\text{clear cover}) - 2 \times 20\ (\text{bar diameter})$$
$$= 385\ \text{mm} < 644.7\ \text{mm}$$

Provide small diameter bars as dowel bars.

Using 10 mm diameter bars,

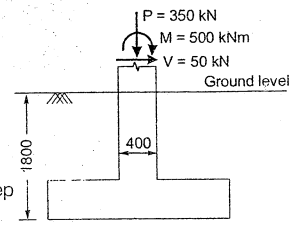
$$\text{No. of dowel bars required} = \frac{1012.5}{\frac{\pi}{4}(10)^2} = 1289 \simeq 14\ \text{bars}$$

$$\text{Development length} = \frac{0.87(415)10}{4(2.24)1.25} = 322.37\ \text{mm} < 385\ \text{mm} \qquad \text{(OK)}$$



Example 13.3 Fix the dimensions of a footing which is required to carry an axial load of 350 kN, a lateral force of 50 kN and a uniaxial moment of 500 kNm. Safe bearing capacity of the soil is 143 kN/m². Consider footing to be at a depth of 1.8 m below the ground level. The water table is at 1.8 m depth from ground level which may use upto the ground level. Unit weight of soil is 16.8 kN/m³. Take column size as 400 mm × 400 mm.

Solution:



$$\text{Moment due to lateral force} = 50 \times 1.8 = 90\ \text{kNm}$$
$$\text{Total moment at footing base } (M) = 500 + 90 = 590\ \text{kNm}$$
$$\text{Eccentricity } (e) = \frac{M}{P} = \frac{590}{350} = 1.686\ \text{m}$$
$$\text{Provide a square footing of size} = 5\ \text{m} \times 5\ \text{m} \times 1.25\ \text{m deep}$$

When water table is at 1.8 m below ground level,

$$\text{Weight of footing} = 5 \times 5 \times 1.25 \times 25$$
$$= 781.25\ \text{kN}$$
$$\text{Weight of soil above footing} = (5^2 - 0.4^2)(1.8 - 1.25)\, 16.8 = 229.52\ \text{kN}$$
$$\text{Axial load} = 350\ \text{kN}$$
$$\text{Total load} = 350 + 229.52 + 781.25 = 1360.77\ \text{kN} \simeq 1361\ \text{kN}$$

When water table is at ground level,

$$\text{Weight of footing} = 5 \times 5 \times 1.25 \times (25 - 9.81) = 474.7\ \text{kN}$$
$$\text{Weight of soil above footing} = (5^2 - 0.4^2)(1.8 - 1.25)(16.8 - 9.81) = 95.5\ \text{kN}$$
$$\text{Axial load} = 350\ \text{kN}$$
$$\text{Total load} = 350 + 95.5 + 474.7 = 920.2\ \text{kN} \simeq 921\ \text{kN}$$
$$\text{Overturning moment about point x } (M_0) = 590\ \text{kNm}$$

Restoring moment about point × (M_R) = $0.9 \times 921 \times \frac{5}{2} = 2072.25$ kNm

Factor of safety against overturning = $\frac{2072.25}{590} = 3.57 > 1.4$ (OK)

Bearing pressure check

Water table at 1.8 m below GL

$$\sigma = \frac{P}{A} \pm \frac{M}{I} y = \frac{1361}{5^2} \pm \frac{590}{5 \times \frac{5^3}{12}} \left(\frac{5}{2}\right)$$

$$= 54.44 \pm 28.32 = 82.76 \text{ kN/m}^2, \quad 26.12 \text{ kN.m}^2$$

Thus there is no uplift any where in the footing base.

Water table at GL

$$\sigma = \frac{P}{A} \pm \frac{M}{I} y = \frac{921}{5^2} \pm \frac{590}{5 \times \frac{5^3}{12}} \left(\frac{5}{2}\right)$$

$$= 36.84 \pm 28.32 = 65.16 \text{ kN/m}^2, 8.52 \text{ kN/m}^2$$

Thus footing size 5 m × 5 m × 1.25 m is adequate.

Example 13.4 Design a spread footing for a column of size 350 mm × 500 mm which is reinforced with 4-20 Φ bars, carrying an axial load of 950 kN at service conditions and service moment of 100 kN. Assume that the moment is reversible and use M 25 concrete and Fe415 steel. The safe bearing capacity of the soil is 190 kN/m² at 1 m depth below the ground level.

Solution:

Proportioning the size of footing

Given,

Axial load at service conditions (P) = 950 kN

Service moment (M) = 100 kNm

Factored load (P_u) = 1.5 × 950 = 1425 kN

Factored moment (M_u) = 1.5 × 100 = 150 kNm

Now since the moment is reversible and thus the fasting must be symmetric with respect to the column.

Let weight of footing and soil backfill as 10% of factored axial load (P_u).

∴ Eccentricity of load P_u at the base of the footing (e),

$$= \frac{M_u}{1.10 P_u} = \frac{150 \times 10^6 \text{ Nmm}}{1.1 \times 1425 \times 10^3 \text{ N}} = 95.694 \text{ mm}$$

For no tension any where in the footing, $e < L/6$. The stresses developed in the footing due to factored axial load and factored moment must be less than the factored bearing capacity of the soil.

i.e. $$\frac{1.1 P_u}{A} \pm \frac{M_u}{z} \le 1.5 (190) \text{ kN/m}^2$$

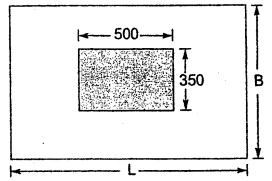
$$\Rightarrow \quad \frac{1.1(1425)}{BL} + \frac{150}{BL^2/6} \le 285$$

$$\Rightarrow \quad \frac{1567.5}{BL} + \frac{900}{BL^2} \le 285$$

$$\Rightarrow \quad 1567.5 L + 900 \le 285 BL^2$$

$$\Rightarrow \quad 5.5 L + 3.1579 \le BL^2$$

$$\Rightarrow \quad BL^2 - 5.5 L - 3.1579 \le 0 \quad \text{... (i)}$$



Now the footing must be symmetric in order to take care of reversible moment. Thus the footing must have equal projections on either side of the column. Also for economy, the footing must have equal projections in the two orthogonal directions i.e.

$$\frac{L - 0.5}{2} = \frac{B - 0.35}{2}$$

$$\Rightarrow \quad L - B = 0.15$$

$$\Rightarrow \quad B = (L - 0.15) \quad \text{...(ii)}$$

Substitute the value of 'B' from (ii) in equation (i),

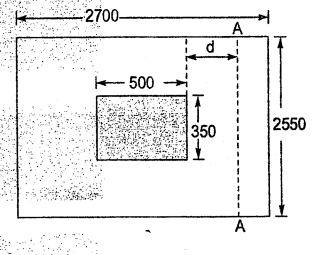
$$(L - 0.15) L^2 - 5.5 L - 3.1579 \ge 0$$

$$\Rightarrow \quad L^3 - 0.15 L^2 - 5.5 L - 3.1579 \ge 0$$

Solving by trial and error,

$$L \simeq 2.665 \text{ m} = 2.7 \text{ m (say)}$$

$$\therefore \quad B = L - 0.15 = 2.7 - 0.15 = 2.55 \text{ m}$$



Thus soil base pressure are,

$$\frac{(1.1)1425}{2.7 \times 2.55} \pm \frac{6 \times 150}{2.55 \times 2.7^2} = 227.67 \pm 48.41 \text{ kN/m}^2$$

$$= 276.08 \text{ kN/m}^2$$

$$< 1.5 \times 190 \ (= 285 \text{ kN/m}^2)$$

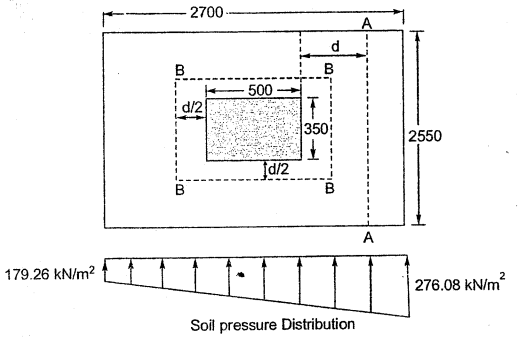
$$= 179.26 \text{ kN/m}^2$$

$$< 285 \text{ kN/m}^2 \quad \text{(OK)}$$

Determining the Depth of footing required

On the basis of one-way shear

Section (A) – (A) as shown in the figure is critical section for one way shear which is at a distance 'd' from the face of column.



$$\text{Soil pressure at sec } (A)-(A) = 276.08-\left[\left(\frac{276.08-179.26}{2700}\right)(1100-d)\right]$$

$$= 276.08 - 39.445 + 0.03586\,d$$

$$= (236.635 + 0.3586\,d)\ \text{kN.m}^2 \quad \text{(where } d \text{ is in mm)}$$

$$\therefore \text{ One way shear force at sec } (A)-(A) = (236.635 + 0.03586d)\,2.55\,(1.1 - 0.001d)$$

(where d is in mm)

Let percentage reinforcement in footing = 0.15%

$\therefore$ For M25 concrete and P_t = 0.15%, design shear strength of concrete as per table 19 of IS 456 = 0.29 N/mm^2

$\therefore$ Shear strength of M25 concrete at sec $(A)-(A) = (0.29)\,2550d\,\text{N} = 2.55d\,(0.29)$ kN

$\therefore\ (236.635 + 0.03586d)\,(2.805 - 0.00255d) \le 2.55\,(0.29)d$

$\Rightarrow\ 663.791 - 0.60342d + 0.1005873d - 0.000091443d^2 \le 0.7395d$

$\Rightarrow\ 7.2587 \times 106 - 5498.86d - d^2 \le 8087.005d$

$\Rightarrow\ d^2 + 13585.865d - 7.2587 \times 10^6 \ge 0$

$\therefore\quad d \ge 514.78$ mm

Let $d = 550$ mm

On the basis of two-way shear

Section $(B)-(B)-(B)-(B)$ around the column is critical for two way shear located at a distance of $(d/2)$ from the column face.

Shear strength of concrete in two way shear,

$$\tau'_c = k_s\tau_c \qquad \text{where } \tau_c = 0.25\sqrt{f_{ck}}$$

$$k_s = 0.5 + \beta_c \not> 1$$

$$\beta_c = \frac{\text{Short side of column}}{\text{Long side of column}} = \frac{350}{500} = 0.7$$

$\therefore$

$$k_s = 0.5 + \beta_c \not> 1$$

$$= 0.5 + 0.7 \not> 1 = 1.2 \not> 1 = 1$$

$\therefore\quad \tau'_c = (1)\,0.25\sqrt{f_{ck}} = 1.25\ \text{N/mm}^2$

Average soil pressure at centre line of column

$$= \frac{276.08 + 179.26}{2}\ \text{kN/m}^2 = 227.67\ \text{kN/m}^2$$

Show force at Section $(B)-(B)-(B)-(B)$ = $(227.67)\,[2.7 \times 2.55 - (0.5 + 0.001d) \times (0.35 + 0.001d)]$

(where d is in mm)

$$= 227.67\,[6.885 - (0.175 + 0.00085d + 1 \times 10^{-6}\,d^2)]$$

Substituting $d = 550$ m in the above expression (where $d = 550$ mm is fixed on the basis of one-way shear),

Shear force at Section $(B)-(B)-(B)-(B)$ = $227.67\,[6.885 - (0.175 + 0.00085 \times 550 + 10^{-6} \times (550)^2)]$

$$= 1352.36\ \text{kN}$$

Two way shear strength of soil = $1.25\,[(350 + d + 500 + d)2\,(600)]$ N

$$= 1.25\,[1200\,(850 + 2 \times 600)]\ \text{N} = 3075\ \text{kN}$$

$$> 1352.36\ \text{kN}$$

Thus footing depth is safe in two way shear also.

Assuming 50 mm clear cover and using 20 Φ bars, overall depth of footing (D)

$$= 600 + 50 + 20 + \frac{20}{2}$$

$$= 600 + 50 + 20 + 10 = 680\ \text{mm} = 700\ \text{mm (say)}$$

$\therefore\quad d = 700 - 50 - 20 - \frac{20}{2} = 620$ mm

Thus size of footing is **2700 x 2550 x 700 mm.**

Check for gross bearing pressure

$$\text{Pressure due to axial load } (P_u) = \frac{1425}{2.7 \times 2.55} = 206.97\ \text{kN/m}^2$$

$$\text{Pressure due to moment } (M_u) = \pm\frac{6 \times 150}{2.55 \times 2.7^2} = \pm 48.41\ \text{kN/m}^2$$

Pressure due to self weight of footing of 700 mm depth = $25 \times 0.7 = 17.5$ kN/m^2

Pressure due to soil backfill of height = $(= 1000 - 700 = 300\ \text{mm}) = 18 \times 0.3 = 5.4$ kN/m^2

$\therefore$ Maximum and minimum pressures are,

$$206.97 \pm 48.41 + 17.5 + 5.4$$

$$= \begin{bmatrix} 278.25\text{kN/m}^2\,(p_{max}) < 1.5(190) = 285\ \text{kN/m}^2\,(\text{ok}) \\ 181.46\text{kN/m}^2\,(p_{min}) < 285\ \text{kN/m}^2\,(\text{ok}) \end{bmatrix}$$

Reinforcement requirement in long direction

The critical section for bending moment is at the face of column itself.

$$\text{Soil pressure at the column face} = 276.08 - \left[\left(\frac{276.08 - 179.26}{2700}\right)1100\right] = 236.64\ \text{kN/m}^2$$

$$\text{BM at column face } (M) = \left[236.64 \times 1.1 \times 2.55 \times \frac{1.1}{2} + (276.08 - 236.64)\frac{1.1}{2}(2.55)\frac{2}{3}(1.1)\right]\text{kNm}$$

$$= (365.08 + 40.56)\ \text{kNm} = 405.64\ \text{kNm}$$

$\therefore\quad R = \frac{M}{bd^2} = \frac{405.64 \times 10^6}{(2550)620^2} = 0.413825\ \text{N/mm}^2$

$\therefore\quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1 - \sqrt{\frac{1 - 4.598(0.413825)}{25}}\right]$

$$= 0.00116892476$$

$\Rightarrow\quad p_t = 0.1169\%$

$\Rightarrow\quad A_{st} = \frac{0.1169}{100} \times 2550 \times 620 = 1848.189\ \text{mm}^2$

$\therefore$ No. of 20 Φ bars required = $\frac{1848.189}{\frac{\pi}{4}(20)^2} = 5.88 \simeq 6$ bars

$$\therefore \quad \text{Spacing} = \frac{2550 - 2\times 50 - 6\times 20}{5} = 466\,\text{mm} > 300\,\text{mm}$$

If 8-20 Φ bars are provided, then spacing $= \frac{2550 - 2\times 50 - 8\times 20}{7} = 327.14 > 300$ mm

If 10-20 Φ bars are provided, then spacing $= \frac{2550 - 2\times 50 - 10\times 20}{9} = 250\,\text{mm} < 300\,\text{mm}$

$$\therefore \quad A_{st\,provided} = 10\times\frac{\pi}{4}\times 20^2 = 3141.6\,\text{mm}^2 > 1848.189\,\text{mm}^2$$

Thus provide 10-20 Φ bars @ 250 c/c.

Reinforcement requirement in short direction

Average soil pressure between edge and column centre line

$$= \frac{227.67 + 276.08}{2} = 251.875\,\text{kN/m}^2$$

$$BM = 251.875\times 2.7\times 1.1\times\frac{1.1}{2} = 411.44\,\text{kNm}$$

$$R = \frac{411.44\times 10^6}{2700\times 620^2} = 0.3964\,\text{N/mm}^2$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1-\sqrt{\frac{1-4.598(0.3964)}{25}}\right]$$

$$= 0.111875\times 10^{-2}$$

$$\therefore \quad p_t = 0.11188\%$$

$$\Rightarrow \quad A_{st} = \frac{0.11188}{100}\times 2700\times 620 = 1872.87\,\text{mm}^2$$

$$\text{No. of 20 Φ bars required} = \frac{1872.87}{\frac{\pi}{4}(20)^2} = 5.96 \simeq 6\,\text{bars}$$

$$\text{Spacing of 6-20 Φ bars} = \frac{2700 - 2\times 50 - 6\times 20}{5} = 496\,\text{mm} > 300\,\text{mm}$$

$$\text{Using 10-20 Φ bars, spacing} = \frac{2700 - 2\times 50 - 10\times 20}{9} = 266.67\,\text{mm} < 300\,\text{mm}$$

Provide 10-20 Φ @ 250 c/c.

Development Length

Development length required for M25 concrete (L_d)

$$= \frac{0.87(415)\Phi}{4\times 1.6\times 1.4} = 40.296\,\Phi = 40.296\,(20) = 805.92\,\text{m}$$

$$\text{Development length available} = 1100 - 50 - \frac{20}{2} = 1040\,\text{mm} > 805.92\,\text{mm} \quad \text{(OK)}$$

Load transfer at column base

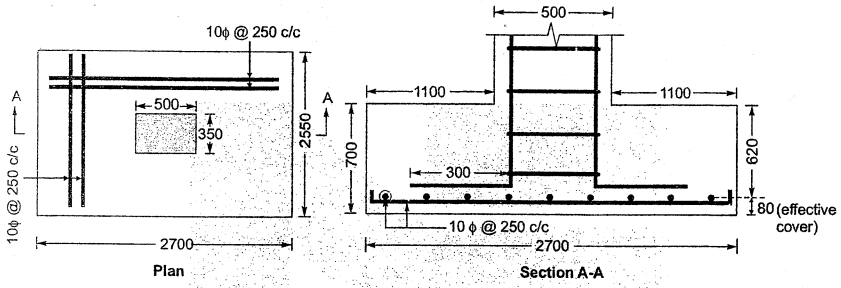
Since the column is subjected to axial load along with moments thus some column bars are in tension due to moment. Transfer of tensile force is not possible at the interface of column and footing and thus column bars are extended into the footing.

Development length for 20 Φ bars in M25 concrete = 805.92 mm

Development length available = 620 mm < 805.92 mm

Thus column bars must be given a 90° bend (anchorage value = 8 ϕ) and then extended upto 300 mm so that total development length provided = 620 + 8 (20) + 300 = 1080 mm > 805.92 mm (OK)

(Anchorage value of 90° bend)



Example 13.5 The size of a RC column is 300 x 600 mm. The column has to support a load of 1400 kN. Design the foundation for this column if safe bearing capacity of the soil is 150 kN/m². The width of the footing can not exceed 2.5 m. Use M 25 for concrete and Fe 500 steel. Use LSM.

Solution:

Size of foundation

Column load, $P = 1400$ kN

Let weight of the foundation, P_f = 10% of column load (P) = 0.1 x 1400 kN = 140 kN

∴ Total load, $P_t = 1400 + 140$ kN = 1540 kN

Area of footing required, $A = \frac{1540}{150} = 10.27\ \text{m}^2$

Note: Do not use factored load of 1.5 x 1540 kN, here since safe bearing capacity of soil itself takes into account the factor of safety.

Assume $B = 2.5$ m

$$\therefore \quad D = \frac{A}{B} = \frac{10.27}{2.5} = 4.11\,\text{m} = 4.2\,\text{m (say)}$$

∴ Area provided, $A = 4.2 \times 2.5\ \text{m}^2 = 10.5\ \text{m}^2 > 10.27\ \text{m}^2$ (OK)

Net soil pressure

$$w_0 = \frac{P}{A} = \frac{1400}{10.5} = 133.33\ \text{kN/m}^2$$

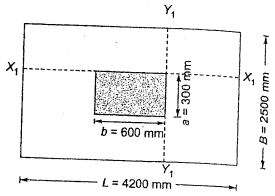
$< 150\ kN/m^2$ (= safe bearing capacity of the soil)

Factored soil pressure, $w_{u0} = 1.5 \times 133.33 = 200\ kN/m^2$ (OK)

Depth of foundation from bending moment criterion

Moment at section x_1-x_1, $M_{ux} = w_{u0}\left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$

$$= 200 \times \frac{(2.5-0.300)^2}{8} = 121\ kNm$$



Moment at section y_1-y_1, $M_{uy} = w_{u0} \times \left(\frac{L-a}{2}\right)\left(\frac{L-a}{4}\right)$

$$= 200 \times \frac{(4.2-0.6)^2}{8} = 324\ kNm$$

$$Q = 0.36 f_{ck} \times 0.46 \times (1 - 0.42 \times 0.46)$$
$$= 0.36 \times 25 \times 0.46\,(1 - 0.42 \times 0.46)$$
$$= 3.34$$

Depth of foundation, $d = \sqrt{\frac{M_{uy}}{QB}} = \sqrt{\frac{324 \times 10^6}{3.34 \times 1000}} = 311.46\ mm$

Take $d = 320$ mm

Check for one way shear

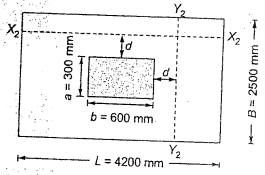
Critical section for one way shear will be at a distance of 'd' from the face of the column

Maximum shear force at section y_2-y_2

$$V_{uy} = w_{u0} \times 1\,m \times \left[\frac{L-a}{2} - d\right]$$

$$= 200 \times 1\left[\frac{4.2-0.6}{2} - 0.32\right]$$

$$= 296\ kN$$



Nominal shear stress

$$\tau_{vu} = \frac{V_{uy}}{Bd} = \frac{296 \times 10^3}{1000 \times 320} = 0.925\ N/mm^2$$

$k = 1.0$ for d > 300 mm

$k\tau_c = 1 \times 0.29\ N/mm^2 < \tau_{vu}\ (= 0.925\ N/mm^2)$ Failed

Revised depth of the footing

$$k\tau_c = 0.29 = \frac{V_{uy}}{Bd}$$

$$\therefore \quad d = \frac{V_{uy}}{B \times 0.29} = \frac{296 \times 10^3}{1000 \times 0.29} = 1020.69\ mm$$

Average of 320 and 1020 mm $= \frac{320+1020}{2} = 670\ mm \approx 700\ mm$

Check for $d = 700$ mm

$$V_{uy} = 200 \times \left[\frac{4.2-0.6}{2} - 0.7\right] = 220\ kN$$

$$\tau_v = \frac{V_{uy}}{Bd} = \frac{220 \times 10^3}{1000 \times 700} = 0.31\ N/mm^2$$

Try $d = 750$ mm

$$V_{uy} = 200 \times \left[\frac{4.2-0.6}{2} - 0.75\right] = 210\ kN$$

$$\tau_v = \frac{V_{uy}}{Bd} = \frac{210 \times 10^3}{1000 \times 750} = 0.28\ N/mm^2 < 0.29\ N/mm^2$$ (OK)

$\therefore$ Adopt effective depth, $d = 750$ mm

Check for punching/two way shear

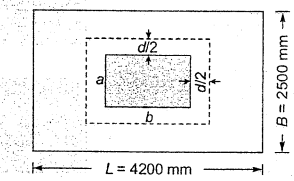
Net punching force $= P_u - w_{u0}(a+d)(b+d)$

$= 1.5 \times 1400 - 200 \times (0.6 + 0.75)(0.3 + 0.75)$

$= 1816.5\ kN$

Punching shear stress $= \tau_{vp\,(dev)} = \frac{\text{Net punching force}}{\text{Perimeter} \times \text{depth}}$

$$= \frac{1816.5 \times 10^3}{2[(a+d)(b+d)] \times d}$$



$$= \frac{1816.5 \times 10^3}{2[(600+750)(300+750)] \times 750} = 0.505\ N/mm^2$$

$\tau_{vp} = k_s 0.25\sqrt{f_{ck}}$

where $k_s = \left(0.5 + \frac{300}{600}\right) \not> 1 = 1$

$\tau_{vp\,(per)} = 1 \times 0.25 \times \sqrt{25} = 1.25\ N/mm^2 > 0.505\ N/mm^2$ (OK)

Steel Reinforcement:

Reinforcement required along long direction

$M_{uy} = 324$ kNm

$$R_y = \frac{M_{uy}}{bd^2} = \frac{324 \times 10^6}{1000 \times 750^2} = 0.576$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{sty}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - \frac{4.598 R_y}{f_{ck}}}\right]$$

$$= \frac{25}{2 \times 500}\left[1 - \sqrt{1 - 4.598 \times \frac{0.576}{25}}\right]$$

$\therefore \quad p_t = 0.136\%$

$$\therefore \quad A_{sty} = \frac{0.136}{100} \times 1000 \times 750 = 1020 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of } 20\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{1020} = 308 \text{ mm c/c}$$

Provide 20ϕ bars @ 250 m c/c along long direction

$$\therefore \quad A_{sty\,provided} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{250} = 1256.6 \text{ mm}^2 > 1020 \text{ mm}^2 \quad \text{(OK)}$$

Reinforcement required along short direction

$$M_{ux} = 121 \text{ kNm}$$

$$R_x = \frac{M_{ux}}{bd^2} = \frac{121 \times 10^6}{1000 \times (750-20)^2} = 0.22706$$

$$\therefore \quad p_t = \frac{25}{2 \times 500}\left[1 - \sqrt{1 - 4.598 \times \frac{0.22706}{25}}\right] 100$$

$$= 0.05\% < 0.12\% \text{ (minimum reinforcement)}$$

Provided minimum reinforcement of 0.12%

$$\therefore \quad A_{stx} = \frac{0.12}{100} \times 1000 \times (750-20) = 876 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of } 20\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{876} = 358.6 \text{ mm c/c but} \not> 300 \text{ mm c/c}$$

∴ Provide 20ϕ bars @ 250 mmc/c

$$\therefore \quad A_{stx\,provided} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{250}$$

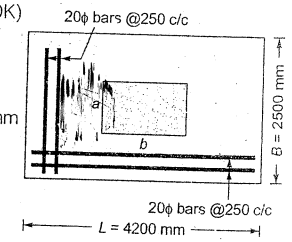
$$= 1256.6 \text{ mm}^2 > 876 \text{ mm}^2 \quad \text{(OK)}$$

$$\text{Development length required } (L_d) = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87(500)\phi}{4(1.6 \times 1.4)}$$

$$= 48.55\phi = 48.55 \times 20 = 917 \text{ mm}$$

$$\text{Available length} = \left(\frac{2500-300}{2}\right) - 75$$

$$= 1025 \text{ mm} > 971 \text{ mm} \quad \text{(OK)}$$



Example 13.6 Design the foundation of a brick masonry wall which is one brick thick and transfers a load of 110 kN/m length of the wall. The allowable bearing capacity of the soil below the footing is 90 kN/m². Take unit weight of earth as 16 kN/m³ and use M 25 concrete and Fe 415 steel.

Solution:

Let the footing is founded at a depth of 1 m below the natural ground level (NGL).

Axial load = 110 kN/m length of the wall

Assuming weight of the footing and soil backfill as 10% of axial load, then width of the footing required per meter length of the footing along the wall = $1.1 \times \frac{110}{90} = 1.3444$ m

Adopt width of the wall footing (B) = 2 m

Factored axial load = 1.5 × 110 kN/m = 165 kN/m

$$\text{Net upward soil pressure} = \frac{165}{(2 \times 1)} = 82.5 \text{ kN/m}^2$$

Now since the wall is one brick thick, so thickness of the wall = 230 mm (Neglecting the thickness of wall plaster etc.)

Critical bending moment in masonry wall:

In masonry wall, the critical section for bending moment is located half way between the wall center line and face/edge of the wall.

$$\text{Factored moment } (M_u) = 82.5 \times \frac{\left(\frac{2}{2} - \frac{0.15}{2}\right)^2}{2} = \frac{82.5 \times 0.925^2}{2} = 35.29 \text{ kNm}$$

Equating the factored moment to moment of resistance of the section for Fe 415 steel,

M_u = Moment of resistance (MOR)

$35.29 \times 10^6 = 0.138 f_{ck} bd^2$

$35.29 \times 10^6 = 0.138 \times 25 \times 1000 \times d^2$

$d = 101.138$ mm

Adopt an overall depth of the footing (D) as 200 mm

Taking the effective cover as 50 mm, effective depth of footing (d) = 200 – 50 mm = 150 mm

$$R = \frac{M_u}{bd^2} = \frac{35.29 \times 10^6}{1000 \times 150^2} = 1.5684 \text{ N/mm}^2$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right]$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2 \times 415}\left[1 - \sqrt{1 - 4.598\frac{1.5684}{25}}\right] = 0.004713$$

$p_t = 0.4713\%$

$A_{st} = 706.95 \text{ mm}^2$

Provide 12 mm diameter bars @ 150 mm c/c.

Thus, $A_{st,\,provided} = 753.98 \text{ mm}^2 > 706.95 \text{ mm}^2$

Development length:

The required development length is given by:

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87 \times 415 \times 12}{4 \times 1.4 \times 1.6} = 483.55 \text{ mm}$$

Length available = 1000 – 230/2 mm = 885 mm > 483.55 mm (OK)

Shrinkage reinforcement:

Provide longitudinal shrinkage reinforcement @ 0.12% = 0.0012 x 2000 x 200 mm² = 480 mm²

Provide 8-10Φ bars, $A_{st\ dist.} = 8\times\frac{\pi}{4}\times10^2 = 628.32\ mm^2 > 480\ mm^2$ (OK)

Shear check:

Now since the wall footing bends only in one direction normal to the wall, thus only one way shear is critical for wall footings.

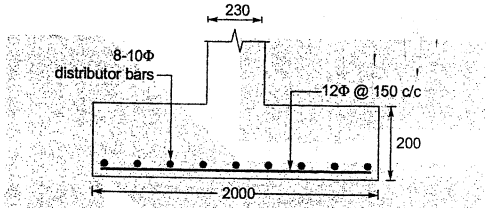
The critical section for one-way shear is located at a distance 'd' from the face of the wall.

Shear force (V_u) = 82.5 × (0.885 – 0.15) kN/m = 60.6375 kN/m=60.64 kN/m

Nominal shear stress (τ_v) = 60.64 × 1000/(1000 × 150) = 0.4043 N/mm²

$p_{t,\ provided}$ = 753.98 × 100/1000 × 150 = 0.503%

Design shear strength of concrete for p_t = 0.503% and M25 concrete (τ_c) = 0.49096 N/mm² (Table 19 of IS 456: 2000) > τ_v(0.4043 N/mm²) (O.K.)



Reinforcement Detailing

Example 13.7 Find the thickness of uniform footing slab for a square column of size 400 x 400 mm carrying an axial load of 1100 kN. The safe bearing capacity of the soil is 150 kN/sq.m. Use M20 and Fe415.

Solution:

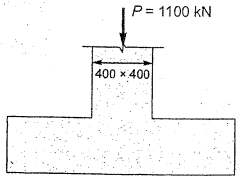
Safe bearing capacity of soil (spc) = 150 kN/m² (M20 concrete and Fe415 steel)

Axial load (P) = 1100 kN

Weight of footing = 10% of 1100 kN = 110 kN

∴ Total load = 1210 kN

∴ Footing area required = $\frac{1210}{150} = 8.07\ m^2$



Using square footing, size of the footing

$= \sqrt{8.07} = 2.84 m \simeq 2.9\ m$ (say)

∴ Provide a footing of size = 2.9 m × 2.9 m

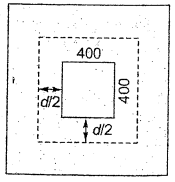
∴ Footing area provided = 2.9 × 2.9 = 8.41 m²

∴ Net soil pressure = $\frac{1100}{8.41} = 130.8\ kN/m^2$

∴ Factored soil pressure = 1.5 × 130.8 = 196.2 kN/m²

Projection of footing beyond column face along $x = O_y$

$= \frac{2900-400}{2} = 1250\ mm$



∴ $M_{yy} = 196.2\times1.25\times\frac{1.25}{2} = 153.28\ kNm$

Projection of footing beyond column face along $y = O_y$

$= \frac{2900-400}{2} = 1250\ mm$

∴ $M_{xx} = 196.2\times\frac{1.25^2}{2} = 153.28\ kNm$

$M_{u\,lim} = 0.138\ f_{ck}\ bd^2$

⇒ $153.28\times10^6 = 138\,(20)\,(1000)\,d^2$

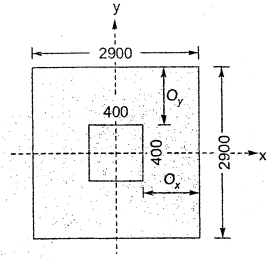
⇒ $d = 235.7\ mm$

Let clear cover = 75 mm and using 20 mm ϕ bars



$D = 235.7 + 75 + \frac{20}{2}$

$= 320.7\ mm = 350\ mm$ (say)



∴ Effective depth (d) = $350 - 75 - \frac{20}{2} = 265\ mm$

One way shear check:

Shear force along x-x = shear force along y-y

$= 196.2\times2.9\times\left(\frac{2.9-0.4}{2} - 0.265\right) = 560.45\ kN = V_u$ (say)

∴ $\tau_{xx} = \tau_{yy} = \frac{560.45\times1000}{2900\times265} = 0.729\ N/mm^2$

Design shear stress of concrete for minimum tensile reinforcement (0.15%)

For M20 concrete = $\tau_c = 0.28\ N/mm^2$

∴ Depth required = $\frac{V_u}{B\tau_c} = \frac{560.45\times1000}{2900\times0.28} = 690.2\ mm > 265\ mm$ (Not OK)

Let $d = 700\ mm$

$D = 700 + 75 + \frac{20}{2} = 785\ mm$

∴ $\tau_v = \frac{560.45\times1000}{2900\times700} = 0.276\ N/mm^2 < 0.28\ N/mm^2$ (OK)

∴ $d = 700$ m is OK.

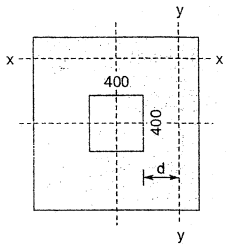
Two way shear check

Punching shear force = $1.5\times1100 - 196.2\,(0.4+d)^2$

$= 1650 - 196.2\,(0.4+d)^2$

Punching shear stress = $\frac{[1650-196.2(0.4+d)^2]1000}{4(0.4+0.7)0.7}$

$= 0.4656\ N/mm^2$



Permissible shear stress $= k_s\tau_c$ where, $k_s = 0.5+\frac{b}{a} \not> 1 = 0.5+\frac{2.92.9}{\quad} \not> 1 = 1.5 \not> 1$

$\therefore$ $k_s = 1$

$= 1\times\tau_c$

$= 0.25\sqrt{f_{ck}}$

$= 0.25\sqrt{20} = 1.118\ \text{N/mm}^2$

$> 0.4656\ \text{N/mm}^2$ (OK)

Thus, $d = 700$ mm is sufficient.

Reinforcement

$M_u = 153.28\ \text{kNm}$

$R_u = \frac{M_u}{Bd^2} = \frac{153.28\times10^6}{2900\times700^2} = 0.1079\ \text{N/mm}^2$

$\therefore$ $\frac{p_t}{100} = \frac{A_{st}}{Bd} = \frac{20}{2(415)}\left[1-\sqrt{1-4.598\times\frac{0.1079}{20}}\right]$

$\therefore$ $p_t = 0.03\%$ which is very less

$\frac{A_{st\,min}}{Bd} \geq \frac{0.85}{f_y}$

$\therefore$ $p_{t\,min} \geq \frac{85}{f_y} = \frac{8.5}{415} = 0.2048\%$

$\Rightarrow$ $A_{st\,min} = \frac{0.2048}{100}\times2900\times700 = 4157.44\ \text{mm}^2$

$\therefore$ No. of 20 mm ϕ bars $= \frac{41157.44}{\frac{\pi}{4}\times20^2} = 13.2-14\,(\text{say})$

$\therefore$ Spacing of bars $= \frac{2900-75-75-14\times20}{13}$

$= 190$ mm c/c

$< 3d\,(= 3\times700 = 2100\ \text{mm})$ (OK)

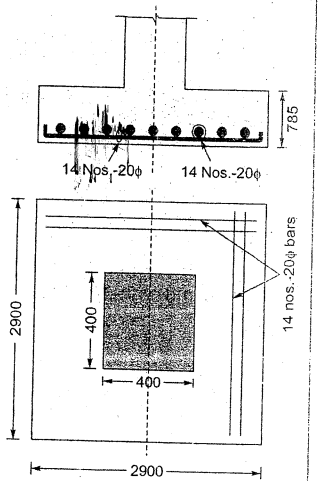
Development length

$L_d = \frac{0.87\times415\phi}{4\tau_{bd}(1.6)} = \frac{0.87\times415\phi}{4\times1.6\times1.2}$

$= 47\,\phi = 47\times20 = 940$ mm

Length available $= \left(\frac{2900-400}{2}\right)-75$

$= 1175\ \text{mm} > 940\ \text{mm}$ (OK)



13.12 Design of Combined Footings

A footing which supports more than a single column or wall is called as combined footing. This combined footing is called as continuous strip footing when columns are aligned in a straight line and it is called as raft or mat foundation when columns are aligned in two directions. Combined footing is required when columns are closely spaced and when exterior column lies on the property line of the structure.

13.13 Soil Pressure Distribution in Combined Footing

Assessment of exact determination of soil pressure below the footing is very difficult. In fact it depends on the rigidity of the footing and the soil properties. However for convenience, a straight line pressure distribution is assumed below the footing. This assumption is quite satisfactory for the design of rigid footings. But for flexible footings, this assumption is not valid and the problem is very complex involving soil-structure interactions aspects.

13.14 Geometric Design of Combined Footings

The geometry of the combined footing should be so selected that the centroid of the footing area should coincide with the resultant of column loads. This results in uniform soil pressure distribution which prevents the tilting of the footing. Depending on the relative magnitude of column loads, the footing may be rectangular or trapezoidal in plan.

13.15 Design Aspects of Two Column Combined Footing

1. **Fixing the plan dimensions:** The plan dimension of the footing should be such that the resulting pressure due to service loads is less than the safe bearing capacity of the soil and the centroid of the footing plan should coincide with the line of action of resultant column loads.
2. **Load transfer from column to footing:** Factored net soil pressure is calculated from the factored loads by dividing the factored load by the factored net soil pressure. This pressure is assumed to be uniformly distributed.

The base slab of the combined footing is subjected to the two way bending moments, two and one way shear. For simplicity, the footing is assumed to be a uniformly loaded longitudinal beam (of width B, length L and factored load $q_u \cdot B$ per unit length) supported on two columns strips.

The thickness of the footing is governed by shear considerations (generally two way or punching shear). Just like in uniformly loaded beams, in footings also, the critical section for one-way shear is located at a distance 'd' (effective depth) from the column face and for two way (punching) shear, the critical section is at a distance of $d/2$ from the column face along the periphery of the column. Flexural reinforcement is designed for the positive moments at the face of the column and negative moments occurring in between the columns. Reinforcement is placed at the bottom for the former case of bending moment and at the top for the later case of bending moment. Nominal transverse reinforcement is also provided to tie the main reinforcement.

13.16 Combined Footing of Beam-Slab

For large combined footings, providing a uniform depth for entire area of the footing results in un- economical design. For such cases, beam-slab combined footing is resorted to. In this, the footing consists of a base slab stiffened by a longitudinal beam of adequate depth interconnecting the two columns footings.

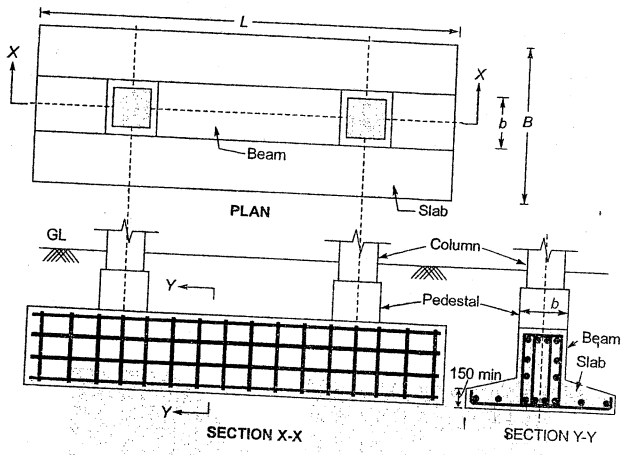


Fig.13.18 *Beam slab combined footing*

Base slab acts like a one way slab supported by the beam and bends in the transverse direction under the uniform soil pressure which acts from the below. Loads transferred by the slab are resisted by the longitudinal beam. The size of the beam generally depends on one way shear at a section located at a distance 'd' from the column face. The width of the footing beam is generally kept equal to the column width. High shear in the longitudinal beam is resisted by the heavy shear reinforcement provided in the form of multi-legged stirrups.

Thickness of the base slab is checked for one way shear at a distance 'd' (of the slab) from the face of the beam. Flexural reinforcement in the footing slab is designed for the cantilever moment coming in the footing slab at the face of the beam.

Example 13.8 **Design a reinforced concrete combined footing for two columns C_1 and C_2 of size 450 mm × 450 mm and 550 mm × 550 mm respectively. The column C_1 is reinforced with 4-25 mm diameter bars and column C_2 with 8-25 mm diameter bars (of Fe 415 in both the columns) and carries an axial load of 850 kN and 1550 kN respectively. The centre to centre distance between the two columns is 5 m. Permissible soil pressure below the base of the footing at 1.5 m depth below the ground is 270 kN/m². Use M 25 grade of concrete and Fe 415 steel.**

Solution:

Preliminary size of the combined footing:

Let weight of the footing and soil backfill is about 10% of the total column load.

Thus area of footing required $A_{reqd} = (850+1550)\times\frac{1.1}{270} = 9.78\ m^2$

For a uniform soil pressure distribution, the line of action of column loads must pass through the centroid of the footing. Let centroid is at a distance of x_c from the centre line of column C_1.

Determining the value of x_c:

Factored column load of column C_1 = P_{u1} = 1.5 × 850 kN = 1275 kN

Factored column load of column C_2 = P_{u2} = 1.5 × 1550 kN = 2325 kN

Total factored column load = $P_{u1} + P_{u2}$ = 1275 + 2325 kN = 3600 kN

Centre to centre spacing between the columns (s) = 5 m

$$x_c = \frac{P_{u2}s}{(P_{u1}+P_{u2})} = \frac{2325\times5000}{3600}$$

$$= 3229.167\ mm > \frac{s}{2}\left(=\frac{5000}{2}mm = 2500\ mm\right)$$

Thus adopt a rectangular footing of length $(L) = 2\left(x_c + \frac{450}{2}\right) = 2(3229.167+225) = 6908.334$ mm

= 6.908 m

Provide length of the footing (L) = 7 m.

Thus width of the footing required (B) = $\frac{A_{reqd.}}{L} = \frac{9.78}{7} = 1.397$ m

Provide width of the footing (B) = 1.5 m

Determination of Stresses in the Longitudinal Direction of the Footing

If the footing is assumed to be a wide beam of width 1500 mm spanning along the longitudinal direction then uniformly distributed load from beneath the base of the footing is given by:

$$q_u B = \frac{(P_{u1}+P_{u2})}{L} = \frac{3600}{7}$$

= 514.28757 kN/m

= 514.3 kNm

The **critical section for shear** is located at a distance 'd' from the inner face of the column C_2 and has the value:

V_{u1} = 2325 − 514.3 × (1.775 + 0.275 + d) kN

= (1270.685 − 0.5143d) kN

Maximum positive bending moment:

M_u^+ = 514.3 × 1.5²/2 = 578.6 kNm

Maximum negative bending moment:

Maximum positive bending moment occurs where shear forces is zero i.e. at a distance x_1 from the outer edge of column C_1 where x_1 is given by:

514.3 x_1 − 1275 = 0

or x_1 = 2.4791 m = 2479.1 mm

M_u^- = 514.3 × 2.4791²/2 − 1275 × (2.4791 − 0.225)

= −1293.5 kNm = −1294 kNm

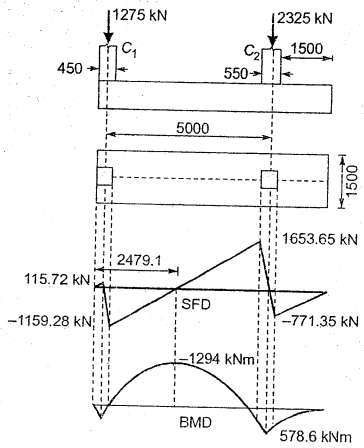


Fig.13.19 *Bending Moment Diagram and Shear Force Diagram*

Depth of the footing based on one way shear consideration:

Let percentage of tension reinforcement (p_t) = 0.5%

For 0.5% p_t and M 25 concrete, τ_c = 0.49 N/mm² (Table 19 of IS: 456-2000)

and for M 25, τ_{cmax} = 3.1 N/mm²

Design shear strength of concrete (V_{uc}) = 0.49 × 1500 × d = 735 d N

Now $V_{u1} \le V_{uc}$

or $(1270.685 - 0.5143d) \times 1000\ \text{N} \le 735d$

or $d \ge 1017.12$ mm

Depth of the footing based on two way shear consideration:

Critical section for two way shear is located at a distance of 'd/2' from the face of columns.

Factored soil pressure (q_u) = 514.3/1.5 kN/m² = 342.87 kN/m²

Taking d = 1018 mm (>1017.12 mm),

V_{u2} (for col. C_1) = 1275 – 342.87 × (0.45 + 1.018) × (0.45 + 1.018/2)

= 792.3 kN

V_{u2} (for col. C_2) = 2325 – 342.87 × (0.55 + 1.018)² = 1482.01 kN

Maximum permissible two way shear stress (τ_{c2}) = $k_s(0.25\sqrt{f_{ck}})$

where k_s = 1 for square columns

= 1 × (0.25√25) = 1.25 N/mm²

Two way shear strength of concrete (v_{c2}):

V_{c2} (for col. C_1) = 1.25 × (450 + 1018/2 + 450 + 1018/2 + 450 + 1018 + 450 + 1018) ×1018 N

= 1.25 × 4854 × 1018 N = 6176.715 kN > 792.3 kN

V_{c2} (for col. C_2) = 1.25 × (550 + 1018) × 4 × 1018 N

= 7981.112 kN > 1482.01 kN

Thus the depth of footing is sufficiently safe for two way shear also.

Taking clear cover to reinforcement in footing as 75 mm and using 25 mm diameter bars, overall depth of the footing (D) is:

D = 1017.12 + 75 + 25/2 mm = 1104.62 mm

Adopt an overall depth of footing as 1200 mm

Thus effective depth of the footing (d) = 1200 – 75 – 25/2 mm = 1112.5 mm > 1017.12 mm (OK)

Base pressure check:

Taking unit weight of concrete as 25 kN/m³ and that of backfill soil as 18 kN/m³, gross pressure under service loads is given by:

$$q = \frac{850+1550}{7\times1.5} + 25 \times 1.2 + 18 \times (1.5 - 1.2)\ \text{kN/m}^2$$

= 228.57 + 30 + 5.4 kN/m²

= 263.97 kN/m² < 270 kN/m² (OK)

Design of flexural reinforcement:

Maximum negative bending moment (M_u^-) = 1294 kNm

$$R = \frac{M_u^-}{Bd^2} = \frac{1294\times10^6}{(1500\times1112.5^2)} = 0.697\ \text{N/mm}^2$$

$$\frac{p_t}{100} = \frac{A_{st}}{Bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right]$$

$$\frac{p_t}{100} = \frac{A_{st}}{1500x1112.5} = \frac{25}{2x415}\left[1-\sqrt{1-4.598\left(\frac{0.697}{25}\right)}\right]$$

p_t = 0.1997% < 0.5%

0.5% steel was assumed and is required for one way shear.

Thus provide p_t = 0.5% so that A_{streqd} = 8343.75 mm²

$A_{st,\,min}$ = 0.12% of Bd = 0.0012 × 1500 × 1112.5 mm² = 2002.5 mm2 < 8343.75 mm² (OK)

Provide 18 nos. 25 mm diameter bars.

Spacing of bars = (1500 – 2 x 75 – 18 x 25)/17 = 52.94 mm

Thus provide 18 nos. 25 mm diameter bars at a spacing of 50 mm c/c.

Development length:

$$L_d = \frac{0.87f_y\phi}{4\tau_{bd}} = \frac{0.87x415\phi}{4x1.6x1.4} = 40.296\phi = 40.296\times25 = 1007.4\ \text{mm}$$

Sufficient length is available on both the sides of maximum bending moment section.

Maximum positive bending moment (M_u^+) = 578.6 kNm

$R = M_u/Bd^2$ = 578.6 × 10⁶/(1500 × 1112.5²) = 0.3117 N/mm²

$$\frac{p_t}{100} = \frac{A_{st}}{Bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right]$$

$$\frac{p_t}{100} = \frac{A_{st}}{1500x1112.5} = \frac{25}{2x415}\left[1-\sqrt{1-4.598\left(\frac{0.3117}{25}\right)}\right]$$

p_t = 0.0879% < 0.12%

Provide minimum reinforcement of 0.12% = 0.0012 × 1500 × 1112.5 mm² = 2002.5 mm²

Provide 8-25 mm diameter bars with spacing (s)

= (1500 – 2 × 75 – 8 × 25)/7 = 164.29 mm c/c

Development length (L_d) = 40.296ϕ = 40.296 × 25 mm = 1007.4 mm

Design of column strips as transverse beams:

Transverse beam under column C_1

Factored load per unit beam length = $\frac{1275}{1.5}$ = 850 kN/m

Projection of beam beyond column face = $\frac{150-450}{2}$ = 525 mm

Maximum moment at the face of column = $M_u = 850\times\frac{0.525^2}{2}$ = 117.14 kNm

Using 20 mm dia. bars placed over 25 mm dia. bars,

Effective beam depth (d) = $1200 - 75 - 25 - \frac{20}{2}$ = 1090 mm

Width of the beam (b) = Column width + 0.75 d

= 450 + 0.75 × 1090 = 1267.5 mm

∴ $R = \frac{M_u}{bd^2} = \frac{117.14\times10^6}{1267.5\times1090^2} = 0.078\ \text{N/mm}^2$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)\left[1-\sqrt{1-4.598\times\frac{0.078}{25}}\right]} = 2.1693\times10^{-4}$$

$\therefore \quad p_t = 0.021683\% < 0.12\%$ (min. reinforcement)

$\therefore$ Provide $p_t = 0.12\%$

$$\therefore \quad A_{st} = \frac{0.12}{100}\times1267.5\times1090 = 1657.89 \text{ mm}^2$$

$$\therefore \quad \text{No. of 20 mm dia. bars required} = \frac{1657.89}{\frac{\pi}{4}\times20^2} = 5.3 = 6(\text{say})$$

$$\therefore \quad \text{Spacing of bars} = \frac{1267.5-75\times2-6\times20}{5} = 199.5 \text{ mm c/c}$$

This spacing is comparatively large

Using 16 mm dia. bars, no. of bars required

$$= \frac{1657.89}{\frac{\pi}{4}\times16^2} = 8.2 = 10(\text{say})$$

$$\therefore \quad \text{Spacing} = \frac{1267.5-75\times2-16\times10}{9} = 106.4 \text{ mm c/c}$$

$\therefore$ Provide 10-16ϕ bars

$$\text{Development length required} = \frac{0.87\times415\phi}{4\times1.6\times1.4} = 40.29\phi\times16$$

$$= 644.64 > \text{length available } (=525-75=450 \text{ mm})$$

$$\text{Using 12}\phi \text{ bars, no. of bars required} = \frac{1657.89}{\frac{\pi}{4}\times12^2} = 14.7 = 15(\text{say})$$

$$\therefore \quad \text{Spacing} = \frac{1267.5-75\times2-15\times12}{14} = 66.96 \text{ mm c/c}$$

Development length required $= 40.29\phi = 40.29\times12 = 483.48 \text{ mm} < 450 \text{ mm}$ (OK)

The critical section for one way shear located at a distance d (= 1090 mm) lines outside the footing and thus no need to check for one way shear.

Transverse beam under column C_2

$$\text{Factored load per unit length} = \frac{2325}{1.5} = 1550 \text{ kN/m}$$

$$\text{Projection beyond column face} = \frac{1500-550}{2} = 475 \text{ mm}$$

$$\text{Maximum moment at column face} = M_u = 1550\times\frac{0.475^2}{2} = 174.86 \text{ kNm}$$

$$\text{Beam width} = 550 + 1.5\times1090 = 2185 \text{ mm}$$

$$\therefore \quad R = \frac{174.86\times10^6}{2185\times1090^2} = 0.06735 \text{ N/mm}^2$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1-\sqrt{1-4.598\times\frac{0.06735}{25}}\right]$$

$$= 1.8713\times10^{-4}$$

$\therefore \quad p_t = 0.018713\% < 0.12\% \ (= p_{t\,mm})$

$$\therefore \quad A_{st\,min.} = \frac{0.12}{100}\times2185\times1090 = 2857.98 \text{ mm}^2$$

$$\therefore \quad \text{No. of 12}\phi \text{ dia. bars required} = \frac{2857.98}{\frac{\pi}{4}\times12^2} = 25.3 \text{ nos. } 26(\text{say})$$

$$\text{Development length required} = \frac{0.87f_y\phi}{4\times1.6\tau_{bd}} = \frac{0.87\times415\phi}{4\times1.6\times1.4} = 40.29\phi$$

$$= 40.29\times12 = 483.48 \text{ mm} \quad \text{which is not available}$$

Increase width of the footing from 1500 mm to 1800 mm

$$\therefore \quad \text{Projection beyond column face} = \frac{1800-550}{2} = 625 \text{ mm}$$

$$= 625 \text{ mm} > 483.48 \text{ mm } (= L_d \text{ required})$$

Transfer of force at column base

Column C_1

Permissible bearing stress at

(i) Column face $= 0.45\,f_{ck} = 0.45\times25 = 11.25 \text{ N/mm}^2$

(ii) Footing face $= 0.45f_{ck}\sqrt{\frac{A_1}{A_2}}$ where $A_1 = 450^2 \text{ mm}^2$, $A_2 = 450^2 \text{ mm}^2$

$$\therefore \quad \sqrt{\frac{A_1}{A_2}} = \frac{450}{450} = 1 \qquad \text{But } \sqrt{\frac{A_1}{A_2}} \not> 2$$

$\therefore$ Permissible bearing stress at footing face

$$= 0.45f_{ck}\times1 = 11.25 \text{ N/mm}^2$$

$\therefore$ Limiting bearing resistance at column-footing interface

$$= 11.25\times450^2 = 2278.125 \text{ kN} > 1275 \text{ kN} \quad \text{(OK)}$$

Thus no reinforcement at interface is required but usually column bars are taken upto the bottom of footing.

Column C_2

Permissible bearing stress at

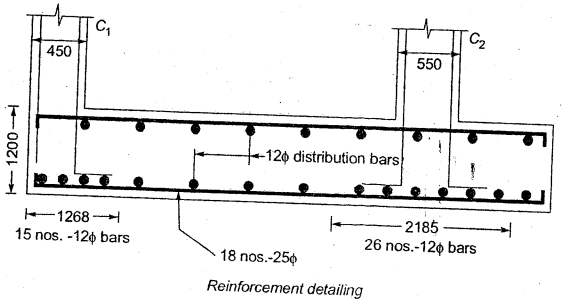
(i) Column face $= 0.45\,f_{ck} = 0.45\times25 = 11.25 \text{ N/mm}^2$

(ii) Footing interface $= 0.45f_{ck}\sqrt{\frac{A_1}{A_2}} = 0.45\times25\times2 = 22.5 \text{ N/mm}^2$

where $\sqrt{\frac{A_1}{A_2}} = \frac{1800}{550} = 3.27$ but $\not> 2$

$\therefore$ Limiting bearing resistance at column footing interface

$$= 11.25\times550^2 = 3403 \text{ kN} > 2325 \text{ kN} \quad \text{(OK)}$$



Reinforcement detailing

Objective Brain Teasers

Q.1 The thickness of the footing at the edge shall not be less than:
(a) 100 mm (b) 150 mm
(c) 200 mm (d) 300 mm

Q.2 The design soil pressure acting upwards is:
(a) More than the ultimate bearing capacity of the soil
(b) Less than the ultimate bearing capacity of the soil
(c) Dependent on the type of soil
(d) Cannot be said

Q.3 In the limit state design of footings, the permissible bearing stress at the interface of column and footing is _____ $0.45f_{ck}$.
(a) Less than
(b) More than
(c) Equal to
(d) Cannot be determined

Q.4 The check for development length in footing is critical when:
(a) Footing depth is large
(b) Footing depth is less
(c) Soil bearing capacity is high
(d) Soil bearing capacity is low

Q.5 The depth of an isolated footing is generally governed by:
(a) Bending moment consideration
(b) Shear consideration
(c) Development length consideration
(d) All of the above

Q.6 The *safe bearing capacity* of soil as reported in a typical *soil report*:
(a) Includes the factor of safety
(b) Excludes the factor of safety
(c) Is same as *net ultimate bearing capacity of soil*
(d) Is determined wholly by empirical expressions without undergoing field tests

Q.7 Foundation design of a power plant generator involves the **additional** consideration of:
(a) Static loading
(b) Dynamic loading
(c) Deformation characteristics of the generator
(d) Stiffness characteristics of the generator

Q.8 In a two column combined footing, the resultant of column axial loads passes through the geometric centroid of the footing. The resultant soil pressure in this case will be:
(a) Uniform
(b) Trapezoidal
(c) Parabolic
(d) Of any shape depending on the soil.

Q.9 The minimum clear cover requirement as per IS recommendations in case of footings is:
(a) 20 mm (b) 25 mm
(c) 75 mm (d) 50 mm

Q.10 Raft foundation is usually provided where:
(a) Total footing area of individual footings exceeds 60% of total plan area.
(b) Total footing area of individual footings exceeds 30% of total plan area.
(c) Soil strata is rocky.
(d) Differential settlements is not a big issue.

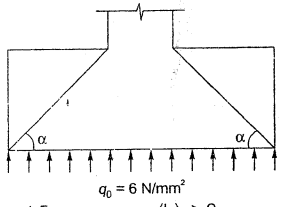
Q.11 A RC square footing of side length 2000 mm and 200 mm effective depth supports a 300 mm × 300 mm column. The line of action of load passes through the centroidal axis of footing and column. The magnitude of the load is 320 kN. The nominal transverse one way shear stress in the footing is
(a) 0.32 MPa (b) 3.2 MPa
(c) 0.26 MPa (d) 5.5 MPa

Q.12 The minimum edge thickness of footing should be more than
(a) 300 mm (b) 200 mm
(c) 250 mm (d) 150 mm

Q.13 If foundations of all the columns are designed on the basis of total dead and live loads then
(a) settlement of exterior columns will be more than interior columns
(b) there will be a uniform settlement
(c) there will not be any settlement at all
(d) settlement of interior columns will be more then exterior columns

Q.14 Punching shear is an important consideration in case of
(a) footings (b) beams
(c) columns (d) retaining wall

Q.15 A concrete pedestal is made of M25 concrete as shown below. The value of tanα for this case is



(a) > 4.5 (b) ≥ 3
(c) < 4.5 (d) ≤ 2

Q.16 A thin layer of PCC is provided below the footings because
(a) it lowers down the amount of total settlement
(b) it avoids point reactions from the soil
(c) it acts as a cushion for the footing
(d) All of the above

Q.17 Design of foundations for power plant turbo generators, is guided by the following parameter
(a) stiffness (b) strength
(c) flexibility (d) frequency

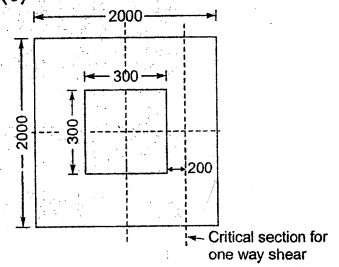
Answers

1. (b)	2. (a)	3. (b)	4. (b)	5. (b)
6. (a)	7. (b)	8. (a)	9. (d)	10. (a)
11. (c)	12. (d)	13. (a)	14. (a)	15. (a)
16. (b)	17. (d)			

Hints:

3. **(b)**
At interface, permissible bearing stress
$= 0.45f_{ck}\sqrt{\left(\frac{A_1}{A_2}\right)}$ which is greater than $0.45f_{ck}$.

11. **(c)**



$$p = \frac{P}{A} = \frac{320000}{2000 \times 2000} = 0.08 \text{ N/mm}^2$$
$$V_u = (0.08 \times 2000)(1000 - 200 - 150)$$
$$= 104000 \text{ N}$$
$$\tau_v = \frac{104 \times 1000}{2000 \times 200} = 0.26 \text{ N/mm}^2$$

15. **(a)**
$$\tan\alpha > 0.9\left[\frac{100q_0}{f_{ck}} + 1\right]^{1/2}$$
$$= 0.9\left[\frac{100 \times 6}{25} \times 1\right]^{1/2} = 4.5$$

Conventional Practice Questions

Q.1 Design an isolated footing for a column 350 mm × 450 mm, having 20 bars of 16 mm diameter carrying an axial load of 1600 kN and a moment of 150 kNm. Use M20 and Fe415. The safe bearing capacity of the soil is 170 kN/m^2 at 1 m below the ground level.

Q.2 Design a combined footing for two columns of size 400 × 400 mm and 350 × 450 mm. The columns carry axial loads of 800 kN and 1200 kN respectively and c/c spacing between the columns is 3.7 m. The 400 × 400 mm is flushed with the property line. The safe bearing capacity of the soil is 150 kN/m^2 at 1.5 m below the ground level. Use M25 and Fe500.

Q.3 Design an isolated footing for a rectangular column of size 300× 550 mm which is reinforced with 16 nos. 20 mm diameter bars of Fe415 and carries an axial load of 2000 kN. Take safe bearing capacity of the soil as 250 kN/m^2 at 1 m below the ground level. Use M30 and Fe415.

Q.4 Design a sloped footing for a square column of size 350 × 350 mm carrying an axial load of 1100 kN. The column is reinforced with 16 nos. 20 mm diameter bars of Fe415. The safe bearing capacity of the soil is 150 kN/m^2.

Q.5 Design a plain concrete footing for a column of size 350 × 350 mm carrying an axial load of 300 kN. The safe bearing capacity of the soil is 250 kN/m^2 at 1 m depth below the ground level. Use M25 and Fe415. Use M30

Q.6 Design a combined trapezoidal footing for two columns of size 300 × 400 mm and 450 × 550 mm carrying axial loads of 800 kN and 1100 kN respectively. The safe bearing capacity of the soil is 110 kN/m^2. Use M25 and Fe500.

Q.7 Design an isolated circular footing for a circular column of 400 mm diameter carrying an axial load of 1050 kN. Use M30 and Fe415. Take safe bearing capacity of the soil as 80 kN/m^2.

Q.8 Design an isolated stepped footing for a rectangular column of size 350 × 550 mm carrying an axial load of 1250 kN and moments of 120 kNm and 100 kNm in the major and minor axes respectively. Take safe bearing capacity of the soil as 150 kN/m^2. Use M30 and Fe415

CHAPTER 14

Prestressed Concrete

14.1 Introduction

According to **ACI Committee of Pre-stressed Concrete,** it is defined as "*Pre-stressed concrete is the one in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the external loading are counter balanced to a desired degree.*" Pre-stressed concrete got developed along with reinforced concrete. Basically in pre-stressed concrete, internal stresses are developed in a predetermined manner such that the stresses induced due to external loading gets nullified with these internal stresses of concrete. This is generally achieved by tensioning the reinforcing steel. Some earlier examples of pre-stressing include the force fitting of metal bands for construction of wooden barrels and the shrink fitting of metal tyres on wooden wheels. The principle of prestressing of concrete lies in the fact that stresses are induced in a concrete member in a planned manner so that when external loads are imposed then stresses due to external loading gets modified by stresses induced due to prestressing.

Do You Know ?

What is the Need of prestressing a Concrete Member?

Concrete is quite strong in compression and can take compressive stresses of upto 50 N/mm^2 or more for standard 150 mm cubes. However, concrete is very weak in tension and it is very difficult to have concrete of tensile strength greater than 2.5 N/mm^2. Thus in usual reinforced concrete, it is reinforced with steel bars. The bars are provided where tensile strength of concrete has to be increased. Due to the assumption of perfect bond between steel and concrete, cracks get developed in concrete as the steel yields i.e. undergoes strain. Thus under usual conditions of working loads, concrete develops minute cracks present in the tension zone of concrete.

The prestressing of concrete reduces these defects. A prestressed concrete member is a usual concrete member but tendons are provided to supply the necessary prestressing force. For getting the maximum advantage of prestressing, it is beneficial to have high strength concrete along with high tensile steel tendons.

14.2 Need of High Strength Concrete in Prestressing

High strength concrete for prestressing is required due to following reasons:

1. Due to large prestressing force to be applied by tendons, large bearing stresses get developed in concrete at the ends by anchoring devices.

2. Bursting stresses at the ends cannot be effectively resisted by low strength concrete.
3. Stress transfer from tendons to concrete takes place due to bond between concrete and steel. High bond strength can be achieved by the use of high strength concrete.
4. With the use of high strength concrete, shrinkage cracks are low.
5. High strength concrete will have high modulus of elasticity of concrete ($= 5000\sqrt{f_{ck}}$) and hence elastic and crack strains are very low resulting in smaller loss of prestressing force.
6. Use of high strength concrete reduces the size of concrete members thereby resulting in reduction of self weight of the member.

14.3 Need of High Tensile Steel in Prestressing

High tensile steel is used for the following reasons:

With the use of low tensile steel (e.g. mild steel), due to creep and shrinkage of concrete, loss of prestress is large. Thus, the net tensile stress which is offering the required prestressing force will be very small.

Generally losses due to creep and shrinkage is in the range of 0.0008 (= 0.08%)

$\therefore$ Strain lost = 0.0008

$\Rightarrow$ Stress lost = $0.0008 \times E_s = 0.0008 \times 2 \times 10^5 = 160$ N/mm^2

Total loss may be in the range of about 200 to 300 N/mm^2.

But losses $\not>$ 10 – 20%

So initial prestress in steel should be in the range of 1000 to 2000 N/mm^2.

14.4 Relative Comparison of Prestressed and Reinforced Concrete Beam

Table 14.1: *Comparison of RC beam with PSC beam*

Reinforced Concrete Beam	Prestressed Beam
Only the concrete in compression side is useful.	The whole concrete section is useful since whole section is in compression i.e., uncracked section.
Reinforced concrete beams are generally larger in size and require additional shear reinforcement to resist shear forces.	Prestressed beams are smaller in size and due to prestressed tendons, a considerable amount of shear reinforcement is avoided.
Use of high strength concrete is not a prime requirement.	High strength concrete is required for prestressed concrete members.
Reinforced concrete beams are quite heavy and large in size.	Prestressed beams are light in weight and smaller in size. These are suitable for long spans carrying heavy loads.
It does not require auxillary units much for construction.	Auxillay units are required like anchoring devices, equipments for prestressing etc.

14.5 Terminologies

Given below are some of the terms that are frequently used in pre-stressed concrete.

14.5.1 Tendon

It is a stretched element in a concrete member to pre-stress to the concrete. In general, high tensile steel wires are used as tendons.

14.5.2 Anchorage

Once the tendon has been stretched to the desired degree, anchorages are provided so that the tendons can impart compressive force to concrete and they also maintain the required pre-stress in tendons. Commonly adopted anchorages are **Freyssinet, Magnel Blaton, Gifford-Udall, Leonhardt-Baur, Dywidag, LeeMcCall** and Roebling system.

14.5.3 Pre-tensioning

It is a method of pre-stressing in which tendons are stretched/tensioned before the placement of concrete. In this method, pre-stress is imparted to concrete through **bond** between steel and concrete.

14.5.4 Post-tensioning

In this method of pre-stressing, tension is applied to tendons after the concrete has been set and hardened. Here the pre-stress is imparted to concrete through **bearing**.

14.5.5 Bonded pre-stressed concrete

Here the pre-stress is imparted to concrete through bond between the steel and surrounding concrete. Pre-tensioned members are classed into this group of concrete.

14.5.6 Non-bonded pre-stressed concrete

Here the pre-stress is imparted to concrete NOT by bond between the steel and the surrounding concrete but by bearing. Here the tendons are placed in ducts formed in concrete or they may be placed outside the concrete section. **Post-tensioned members** are classed into this group of concrete.

14.5.7 Full pre-stressing

Here the tensile stresses in concrete due to the applied loads are completely nullified by the existing pre-stress in the members.

14.5.8 Partial/limited pre-stressing

Here only a limited amount of tensile stresses are allowed in concrete under the working load conditions. Here in addition to tension steel, a good quantity of un-tensioned reinforcement is also used to limit the cracks developed at service loads.

14.5.9 Axial pre-stressing

Members in which the entire cross-section of concrete has a uniform compressive pre-stress. Here the centroid of the tendons coincide with that of the concrete section.

14.5.10 Eccentric pre-stressing

Here the tendons are eccentric to the centroid of the section thereby resulting in a triangular or trapezoidal compressive stress distribution.

14.5.11 Concordant pre-stressing

In this method of pre-stressing, the cable follows a concordant profile. In indeterminate structures, concordant pre-stressing does not cause any change in the support reaction.

14.5.12 Relaxation in steel

The decrease in stress in steel at a constant strain is called as relaxation in steel.

14.5.13 Creep coefficient

It is the ratio of creep strain to the elastic strain.

14.5.14 Degree of pre-stressing

It is the measure of magnitude of pre-stressing force that is being related to the resultant stress occurring in structural member at working load.

14.515 Debonding

It is the prevention of bond between the steel tendons and the surrounding concrete.

14.5.16 Transmission length

It is the length of bond anchorage of pre-stressing wire from the end of pre-tensioned member to the point of full steel stress.

14.5.17 Transfer

It represents the stage corresponding to the transfer of pre-stress to concrete. For pre-tensioned members, transfer takes place at the release of pre-stress from the bulk heads; and for post-tensioned members, it takes place after the completion of the process of tensioning.

14.5.18 Circular pre-stressing

It refers to the process of pre-stressing in circular members like pipes, circular tanks etc.

14.5.19 Proof stress

It refers to the tensile stress in steel which produces a residual strain of 0.002 or 0.2% of the original gauge length upon unloading.

14.5.20 Cap cable

It is the short curved cable arranged at the interior supports of a continuous member. Here the anchors are in compression zone while the curved portion is in tension zone.

14.5.21 Uniaxial, bi-axial and tri-axial pre-stressing

These refer to the situations where concrete is pre-stressed (a) in only one direction; (b) in two mutually perpendicular directions and (c) in three mutually perpendicular directions, respectively.

14.6 Advantages of Prestressed Concrete

Pre-stressed concrete offers more advantages than the reinforced concrete which are as under:

1. In fully pre-stressed concrete members which are free from tensile stresses under working loads, there the cross-section is utilized more efficiently as compared to reinforced concrete which gets cracked after the cracking moment at service loads.
2. The permanent dead load can be nullified by increasing the eccentricity of pre-stressing tendon in a pre-stressed concrete member thereby reducing the self-weight of the concrete member.

3. Pre-stressed concrete members offer improved resistance against shear forces due to the compressive pre-stressing force. Use of curved tendons especially in long span members helps to decrease the shearing forces at the supports.
4. For the same depth of concrete member, a pre-stressed concrete member is stiffer than the reinforced concrete member under working loads. However, once the cracks appear in pre-stressed member, then the behavior of pre-stressed concrete member is very similar to the reinforced concrete member.
5. The use of high strength concrete and high strength steel in pre-stressed concrete member results in smaller sections as compared to reinforced concrete.
6. Pre-stressing the concrete members improve the ability of concrete member to absorb impact loads more efficiently. Moreover the ability to resist repeated working loads has found to be very good in pre-stressed concrete member as compared to reinforced concrete member.
7. In long span structures, pre-stressed concrete is more economical than the reinforced concrete owing to smaller pre-stressed concrete sections.
8. Pre-stressed concrete members offer better resilience due to the capacity of completely recovering from the effects of overloading.
9. Fatigue strength of pre-stressed concrete is better than other materials due to the fact that there is small stress variation in steel which is recommended for dynamically loaded structures.
10. Owing to the fact that concrete in the tension zone is also useful, there is considerable savings in concrete (about 15-30 %) as compared to reinforced concrete.

Do You Know ?

Some remarkable Prestressed Concrete Structures in India.

Below mentioned are some of the structures in India built on pre-stressed concrete:

1. **Ganga Bridge, Patna:** It is the longest pre-stressed concrete bridge in the world with a total length of 5575 m consisting of continuous spans of 121.65 m long pre-stressed concrete girders of variable depth.
2. **Ball tank, Trombay, Mumbai:** It is a pre-stressed concrete tank of 4×10^6 liters capacity being constructed for the Department of Atomic Energy.
3. **Gomti Aqueduct, U.P.:** It is the longest and the biggest aqueduct of India consisting of 9.9 m deep pre-stressed concrete girders with weight of each girder being 550 tons (or 5500 kN) over a span of 31.8 m.
4. **Boeing Hanger, Santa Cruz Airport:** The roof of this hanger consists of barrel shells supported on pre-stressed concrete edge beams of span 45.73 m.
5. **Zuari Bridge, Goa:** This 807 m long bridge consists of pre-stressed concrete cantilever box girders with main spans of four in number of 122 m each; two end spans of 69.5 m and a via-duct with five spans of 36 m each.
6. **Pre-stressed concrete bridge on river Brahmaputra, Jogighopra, Assam:** It has a span of 286 m between the two towers with side spans of 114 m, consisting of single cell pre-stressed concrete box girder.

14.7 Design of High Strength Concrete Mixes

Pre-stressing requires the use of high strength concrete mix. High strength concrete mix can be designed by any of the following ways:

1. Empirical method given by Erntroy and Shacklock.
2. Mix design procedure as given by the American Concrete Institute for no slump concrete.
3. British method based on the works of Franklin, Erntroy and Teychenne.
4. IS Code method.

14.8 High Tensile Steel

14.8.1 Types of high tensile steel

In pre-stressed concrete, we require high strength steel wires for pre-stressing. High strength steel generally consists of wires, bars or strands. The higher tensile strength is generally achieved by slightly increasing the carbon content in steel as compared to mild steel. High tensile steel generally contains 0.6 to 0.85% carbon, 0.7 to 1% manganese, 0.05% sulphur and phosphorus with traces of silicon.

The process of cold drawing through dies decreases the durability of wires and thus is subsequently tempered to improve their properties. Tempering/ageing/stress relieving is done by heat treatment of wires at 150 to 420°C which results in enhanced tensile strength. Cold drawn stress relieved wires are available generally in the diameters of 2.5, 3, 4, 5, 7 and 8 mm and they should coform to IS 1785 (Part-I): 1983. Because of superior bond characteristics, hard drawn steel wires are preferred for pre-tensioned elements and they should conform to IS 6003: 1983. Strands generally comprises of 2 to 5 mm steel wires with number of wires ranging from two, three or seven. Helical form of wires in strands improves the bond strength.

The ultimate tensile strength of bars generally do not vary much with diameter since the high strength of bars is achieved by alloying and NOT by cold working which is done for wires.

14.8.2 Strength requirement of high tensile steel

The ultimate tensile strength of plain hard drawn wires varies with the diameter. The tensile strength of wire decreases with increase in the wire diameter.

The Indian Standard Code prescribes a minimum percentage elongation of 2.5% for wires and 10% for bars. For strands, the percentage elongation measured on a gauge length of 600 mm should not be less than 3.5% just prior to the fracture of wire. IS 1343 specifies the moduli of elasticity of high tensile wires as 210 kN/mm^2, high tensile bars as 200 kN/mm^2 and for strands as 195 kN/mm^2.

Table14.2: *Tensile strength and elongation characteristics of cold drawn stress relieved wires (IS 1785 (Part-i):1983)*

Nominal Diameter (mm)	Min. Tensile Strength (N/mm^2)	Elongation (%)
2.5	2010	2.5
3.0	1865	2.5
4.0	1715	3.0
5.0	1570	4.0
7.0	1470	4.0
8.0	1375	4.0

14.8.3 Stress relaxation in steel

When a high tensile steel wire is stretched and thereafter maintained at the constant strain, then the initial pre-stressing force does not remain constant but decreases with time. This is called as stress relaxation which is the decrease in the stress in steel at constant strain. In pre-stressed members, the high tensile wires between the anchorages are more or less in the state of constant strain but the actual state of stress relaxation will be less than that indicated by test of wire at constant length since there will be shortening of the member due to other reasons. The various codal provisions on stress relaxation are based on the results of a 1000 hour relaxation test on wires. The Indian Standard Code prescribes the 1000 hour relaxation test with no relaxation exceeding 5% of the initial stress.

14.8.4 Stress Corrosion

Stress corrosion results in sudden brittle failure and is thus more perilous. The combined action of corrosion and the static tensile stress results in stress corrosion. Heat treated wires are more prone to stress corrosion than the cold drawn wires. Also, if the ducts of the post tensioned members are not grouted, then there is a possibility of stress corrosion leading to sudden failure of the structure. Other common types of corrosion encountered in pre-stressed concrete are pitting corrosion and chloride corrosion.

14.9 Cover Requirements in Prestressed Concrete Members

It is important to note that not only the thickness of cover but also the density of concrete in the cover is crucial to provide protection to steel. IS 1343 specifies a minimum clear cover of 20 mm for protected pre-tensioned members and 30 mm (or the cable diameter, whichever is higher) for protected post-tensioned members. If the pre-stressed concrete members are exposed to aggressive environments then this cover requirement is increased by 10 mm.

14.10 Protection of Prestressing Steel

In order to prevent the pre-stressing steel from corrosion, un-bonded tendons should be coated by non-reactive materials like epoxy or zinc/zinc aluminium. Provision of non-corroding sheathing material like high density polyethylene (HDPE) is advantageous. The space between sheathing and the duct should be filled with corrosion resisting materials like grease, wax or the petroleum jelly.

14.11 Prestressing System

The various methods available to impart pre-compression in concrete are classified as follows:

1. Generation of compressive force between the structural element and its abutments using the flat jacks.
2. Imparting hoop compression in round/cylindrically shaped structures by winding the wire circumferentially.
3. By the use of longitudinally tensioned steel embedded in concrete.
4. By the use of deflected structural steel sections embedded in concrete until the hardening of concrete.

The most widely used method of pre-stressing is the longitudinally tensioning of steel by the tensioning devices. Development of expansive cements has now made possible the pre-stressing in concrete due to chemical pre-stressing.

14.12 Tensioning Devices

The various types of devices for tensioning are classified into the following four groups viz.:

1. Mechanical devices: These types of devices generally consists of weights with or without lever transmission, geared transmission along with pulley blocks, screw jacks with or without gears and the wire winding machines. These devices are generally used in factories where pre-stressed structural concrete components are produced in a large number.
2. Hydraulic devices: These are the simplest means of producing the pre-stressing force. There are several patented hydraulic jacks due to the works of Freyssinet, Magnel, Gifford Udall and Baur-Leonhardt for the range of 5 to 100 tons.

3. **Electrical/thermal devices:** These devices were used in USSR in 1950s for imparting tension to steel wires and deformed bars. The steel wires are electrically heated and anchored before placing the concrete in moulds. This method is also called as **thermo-electric pre-stressing.**
4. **Chemical devices:** Here the pre-stressing is imparted due to the expanding cements by controlled variation of curing conditions. Due to prevention of expansion of cement on setting, it induces tensile forces in tendons and compressive forces in concrete.

14.13 Pre-tensioning and Post-tensioning Systems

14.13.1 Pre-tensioning systems

In pre-tensioning system, the tendons are first prestressed/tensioned between the rigid anchor blocks. These blocks are casted on ground.

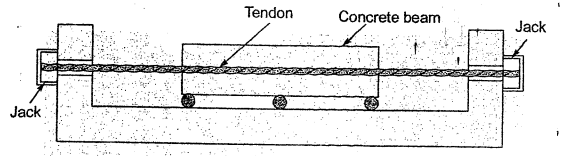


Fig.14.1 *Beam with straight tendon*

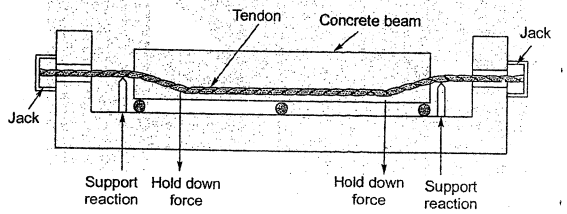


Fig.14.2 *Beam with variable tendon eccentricity*

High strength concrete is used for this purpose when concrete attains the sufficient strength, the pursue from the jacks is released. Due to elasticity of tendons, they tend to come to its original state thereby inducing compressive stress in concrete. Thus the stress gets transformed from tendon to concrete through bond.

For the production of pre-tensioned elements on a large scale, HOYER system is adopted. In the HOYER system of pre-tensioning, the tendons are stretched between the two bulk heads several hundred meters apart so that a number of pre-tensioned units can be casted on the same group of pre-tensioned tendons. The tension is applied through hydraulic jacks.

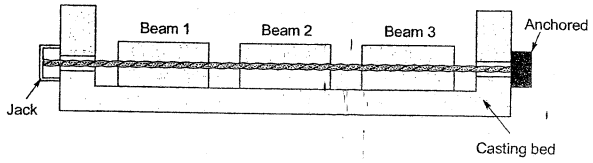


Fig.14.3 *Hoyer's system of pre-tensioning*

The transfer of prestress to concrete is achieved through bond and while releasing the tension in the wires, all wires are simultaneously released. The bond between the concrete and the tendon can be improved by having surface indentations on protusions by helical crimping of wires. Strands have better bond characteristics than plain wires of same cross-sectional area.

14.13.2 Post-tensioning systems

In this system, the concrete units casted first and having a provision of ducts incorporation a grooves to house the tendons. When concrete attains the required strength, the high tensile wires are tensioned by means of jack. The force is transferred to concrete by end anchorages and when the prestressing tendon is curved then this force transfer is achieved through radial pressures between the duct and the cable. The space between the tendon and the duct is grouted after the tensioning process. Most of the patented and commercially available pre-tensioning systems are based on following principles:

(i) Wedge action which produces a friction grip on the wire.
(ii) Direct bearing from rivet or bolt heads formed at the end of the wires.
(iii) Looping the wires around the concrete.

Post-tensioning anchorages

At present, there are about 64 patented post tensioning systems. The post tensioning systems based on wedge action are FREYSSINET, GIFFORD-UDALL, MAGNEL-BLATON etc.

Freyssinet System consists of a cylinder with a cone in tension through which high tensile wires pass Fig. (14.4). The major advantage of freyssinet system is that a large number of wires or strands can be tensioned simultaneously.

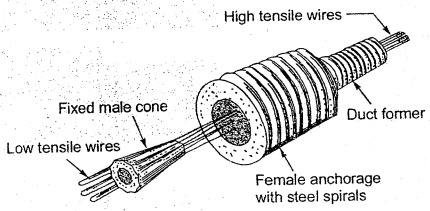


Fig.14.4 *Freyssinet Anchorage*

Gifford-Udall system was developed in the United Kingdom and consists of a steel split cone and cylindrical female cone anchorages to house the high tensile wires bearing against the steel plates. Here in this system, each wire is tensioned separately and anchored by pushing a sleeve wedge into the cylindrical grip resting against a bearing plate Fig (14.5).

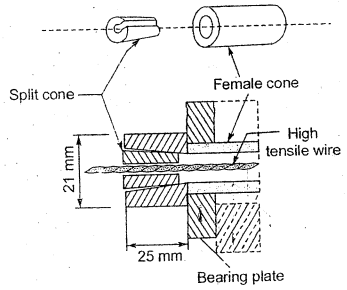


Fig.14.5 *Gifford-Udall System*

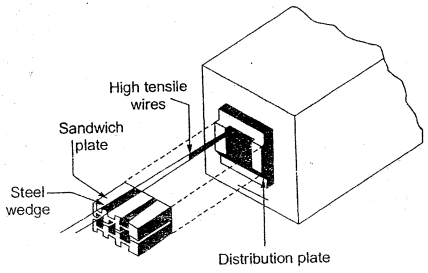
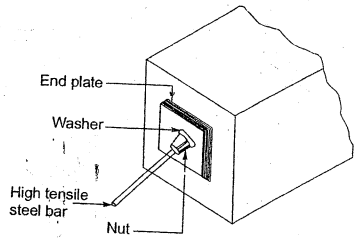


Fig.14.6 *Magnel-Blaton System*

Magnel Blaton system of post tensioning employs flat wedges, metallic sandwich plates, distribution plate for anchoring the wires. Each sandwich plate can house upto four pair of wires. The number of wires vary from 2 to 24 Fig. (14.6).

Lee-McCall System: This system uses bars having threads at end as tendons. Anchoring is done by screwing a nut and washer tightly against the end plates. In this system forces are transmitted by bearing action at end plates. Only one bar at a time can be tensioned. Disadvantage of this method is that curved tendons cannot be used.



Lee-McCall System

14.13.3 Thermo-electrical and chemical prestressing

Thermo Electric Prestressing: The prestressing is achieved by heating the tendons by passing an electric current through the high tensile wires. The wires are heated to a temperature of 300-400°C for about 3-5 minutes. Due to this, wires undergo an elongation of about 0.4-0.5%. These wires on cooling get shorter but are prevented in doing so by anchors fixed at the two ends.

This process induces a prestress of about 500-600 N/mm². Concrete is placed in the moulds only after the temperature of wires goes below 90°C.

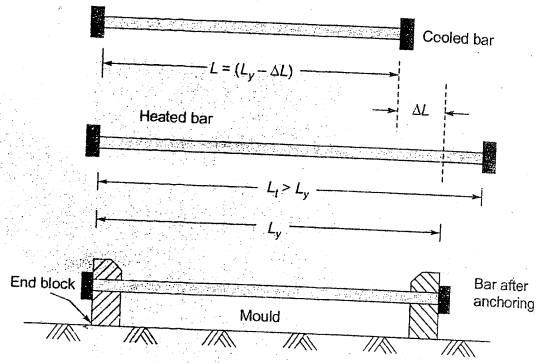


Fig.14.7 *Electro-Thermal Prestressing*

Chemical Prestressing: This is made possible by the development of expanding cements which consists of about 75% Portland cement 15% high alumina cement and 10% Gypsum. This results in the formation of calcium sulpho-aluminate. The linear expansion of this cement is about 3-4%. By varying the curing conditions, degree of expansion can be controlled.

Now the expansion of cement is restrained by high tensile steel wires thereby developing a compressive stress is concrete and tensile stress in wires. A tensile stress of about 850 N/mm² has been achieved in steel by the expansion of concrete. Structural elements suitable for chemical prestressing are shells, slabs, walls, pipes, composite columns etc.

14.14 Assumptions in the Analysis and Design of Prestressed Concrete Members

The following assumptions are made while analysing and designing a prestressed concrete member:

1. Plane sections remains plane after the bending.
2. It is assumed that Hooke's law is applicable within the limits of elastic deformation.
3. Stress in the reinforcement remain constant along the length of reinforcement i.e.. stress variation along the length of reinforcement is ignored. Variation of stress takes place in concrete only.
4. Change of stress in reinforcement due to external loads is also ignored.

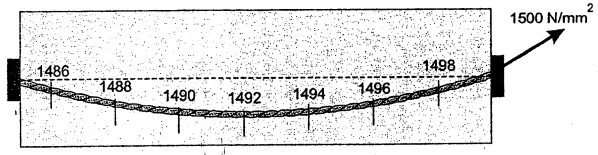


Fig.14.8 *Jacking from one end*

14.15 Analysis of Prestress

14.15.1 Concentric tendon

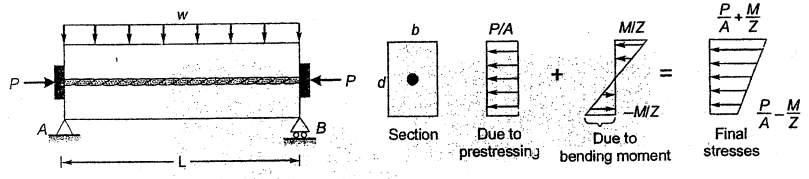


Fig.14.9 *Concentric Prestressing*

Due to concentric tendon, eccentricity is zero and thus a uniform pre-stress induced in concrete is (P/A) which is compressive in nature along the whole depth of the beam section. Consider a simply supported beam *AB* of span *L* which is prestressed by a tendon passing through the centroidal axis of beam.

Let P = Prestressing force provided by the tendon.
A = Cross-sectional area of the beam.
M = BM at a section due to dead and external loads.

Direct Stresses

Compressive stress induced in concrete = $f_c = \frac{P}{A}$

Bending/Flexural stresses due to bending moment $M = f_b = \pm\frac{M}{Z}\left(=\pm\frac{M}{I}\cdot y\right)$

where Z = section modulus of beam section.

∴ Extreme fibre stresses are obtained by superimposing the stresses due to prestress and flexural stresses.

Final Stress

∴ Extreme fibre stresses = $f_c \pm f_b = \frac{P}{A} \pm \frac{M}{Z}$

i.e. extreme fibre stress at top = $\frac{P}{A}+\frac{M}{Z}$

and extreme fibre stress at bottom = $\frac{P}{A}-\frac{M}{Z}$

Thus the action of external loads induce tensile stresses in the lower portion of neutral axis which is more effectively nullified by providing eccentric tendons.

14.15.2 Eccentric tendon

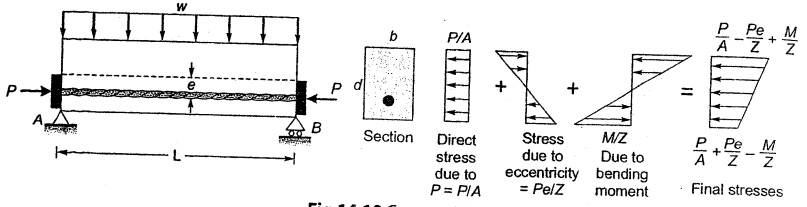


Fig.14.10 *Concentric Prestressing*

As shown in the figure above, the beam is subjected to an eccentric pre-stressing force P acting at an eccentricity of '*e*' from the neutral axis. The stresses induced at the top and bottom fibers of the beam are:

$$f_{top} = \left(\frac{P}{A} - \frac{Pe}{Z}\right); \quad f_{bottom} = \left(\frac{P}{A} + \frac{Pe}{Z}\right)$$

Due to eccentric prestressing force *P*, additional bending/flexural stresses will get induced in the beam. Thus,

1. Direct stress due to axial load $(P) = f_a = \frac{P}{A}$
2. Flexural stresses due to eccentricity of prestressing force $(P) = f_e = \mp\frac{Pe}{Z} = \mp\frac{Pe}{I}\cdot y$

   Here *Pe* = Moment induced in beam due to eccentric prestressing force.
3. Flexural stresses due to self weight and external loading i.e. due to moment *M* at any section.

$$f_b = \pm\frac{M}{Z} = \pm\frac{M}{I}.Y$$

14.15.3 Final Stress/Resultant Stresses

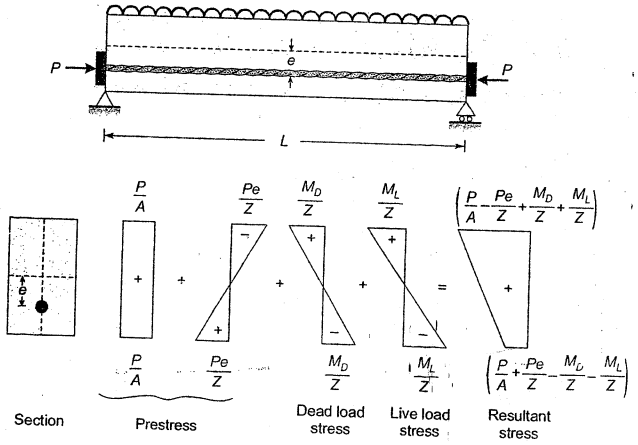


Fig.14.11 *Stress Distribution due to Eccentric Prestressing, Dead and Live Loads*

As shown in the Fig. 14.11, the beam is subjected to a uniformly distributed live load and dead load of w_L and w_D respectively. The beam is pre-stressed by a straight tendon at an eccentricity '*e*' from the neutral axis. The resultant stress at any section of the beam is obtained by super-imposing the effects of pre-stressing, dead and live loads. If M_L and M_D are respectively the live load moment and dead load moment for a simply supported beam of span '*l*' then,

$$M_L = \frac{w_L L^2}{8}; \quad M_D = \frac{w_D L^2}{8}$$

The resultant stresses at the top and bottom fibers are:

$$f_{top} = \left(\frac{P}{A} - \frac{Pe}{Z}\right) + \frac{M_D}{Z} + \frac{M_L}{Z} = \left(\frac{P}{A} - \frac{Pe}{I}y_t\right) + \frac{M_D}{I}y_t + \frac{M_L}{I}y_t$$

$$f_{bottom} = \left(\frac{P}{A} + \frac{Pe}{Z}\right) - \frac{M_D}{Z} - \frac{M_L}{Z} = \left(\frac{P}{A} + \frac{Pe}{I}y_b\right) - \frac{M_D}{I}y_b - \frac{M_L}{I}y_b$$

where, y_t and y_b are the distances of extreme top and bottom fibers respectively from the neutral axis of the beam.

Remember



- Comparing the final stresses of the above two cases, it is clear that tendon placed at an eccentricity offers additional compressive stress along with axial prestressing stress which improves the load carrying capacity of the beam.
- In pre-stressed concrete members, usually the cross-sectional area of high tensile steel wires is very small as compared to the gross area of concrete and thus stress computations are done based on gross concrete area. Use of equivalent concrete section is indispensable for interpreting and analyzing the test results but this does not significantly alter the stresses computed from the gross concrete area.

Example 14.1 A PSC beam of size 200 mm × 350 mm is prestressed by 5 high tensile steel wires of 6 mm diameter. The wires are stressed to 1350 N/mm² and located at an eccentricity of 75 mm. Find the percentage difference in stresses developed at the soffit of beam by considering nominal concrete and equivalent concrete.

Solution:

Applied prestressing force $(P) = 5 \times \frac{\pi}{4} \times 6^2 \times 1350\ \text{N} = 190.85\ \text{kN}$

For nominal concrete section,

Cross-sectional area, $(A) = 200 \times 350 = 70000\ \text{mm}^2$

Moment of inertia $(I) = 200 \times \frac{350^3}{12} = 714.583 \times 10^6\ \text{mm}^4$

∴ Axial compressive stress at the soffit of the section

$$= \frac{P}{A} = \frac{190.85 \times 10^3}{70000} = 2.73\ \text{N/mm}^2$$

∴ Final stress at the soffit of section $= \left(\frac{P}{A} + \frac{Pe}{I}y\right) = \left(2.73 + \frac{190.85 \times 10^3 \times 75}{714.583 \times 10^6} \times 175\right)$

$$= 6.24\ \text{N/mm}^2$$

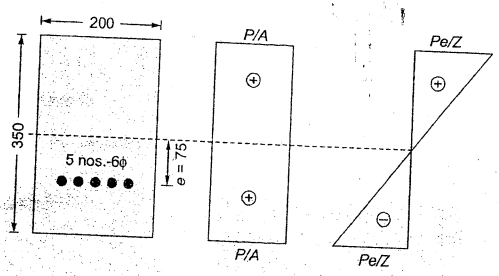
For equivalent concrete section,

Let modulur ratio (m) = 6

$\therefore$ Equivalent concrete area (A_e) = $70000 + (6-1)\times 5 \times \frac{\pi}{4}\times 6^2 = 70706.86\ mm^2$

Centroid of section from soffit = $100\ mm \left(= \frac{350}{2} - 75\ mm\right)$

Location of Neutral Axis



$\therefore$ Stress at soffit of section = $\frac{190.85\times 10^3}{70706.86} + \frac{190.85\times 10^3 \times 75}{718.52\times 10^6}\times 174.25$

= 2.7 + 3.47 = 6.17 N/mm²

$\therefore$ Percentage difference in stresses at soffit

= $\frac{6.24-6.17}{6.24}\times 100 = 1.12\%$

Example 14.2 An unsymmetrical I-section is required to support an imposed load of 1.8 kN/m over a span of 7.8 m. Top flange is 300 mm wide and 60 mm thick, bottom flange is 100 mm wide and 60 mm thick, web thickness is 80 mm with overall beam depth as 400 mm. An effective prestressing force of 150 kN is there at 50 mm from soffit of beam at mid-span. What are the stress at the centre of span for:

(i) Prestress + self weight?

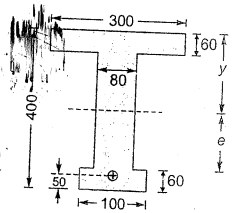
(ii) Prestress + self weight + imposed load??

Solution:

Prestress force (P) = 150 kN

Area of concrete = (300 × 60) + (100 × 60) + (80 × 280)

= 46400 mm²



Let y = Distance of centroid from top

$\therefore$ $y = \frac{300\times 60\times 30 + 280\times 80\times 200 + 100\times 60\times 370}{46400}$

= 156 mm

$\therefore$ $e = (400 - 50) - y = 350 - 156 = 194$ mm

$I = 300\times\frac{60^3}{12} + 300\times 60\times 126^2 + 80\times\frac{280^3}{12} + 80\times 280\times 44^2$

$+ 100\times\frac{60^3}{12} + 100\times 60\times 214^2$

$= 5.4\times 10^6 + 285.768\times 10^6 + 146.35\times 10^6 + 43.37\times 10^6 + 1.8\times 10^6 + 274.776\times 10^6$

$= 757.464\times 10^6\ mm^4$

$\therefore$ $z_t = \frac{I}{y} = \frac{757.464\times 10^6}{156} = 4.8555\times 10^6$ mm

$z_b = \left(\frac{I}{400-y}\right) = \frac{757.464\times 10^6}{400-156} = 3.104\times 10^6\ mm^3$

Self weight of beam = $2.5\times 46400\times 10^{-6} = 1.16$ kN/m

$\therefore$ Moment due to self weight of beam (M_1) = $1.16\times\frac{7.8^2}{8} = 8.82$ kNm

Moment due to imposed load (M_2) = $1.8\times\frac{7.8^2}{8} = 12.69$ kNm

Stress type	Top fibre	Bottom fibre
Prestress	$\frac{P}{A} = \frac{150\times 1000}{46400} = +3.23\ N/mm^2$	$\frac{P}{A} = \frac{150\times 1000}{46400} = +3.23\ N/mm^2$
	$\frac{P.e}{z_b} = \frac{150\times 1000\times 194}{4.8555\times 10^6} = -5.99\ N/mm^2$	$\frac{P.e}{z_b} = \frac{150\times 1000\times 194}{3.104\times 10^6} = +9.375\ N/mm^2$
Stress due to self weight of beam	$\frac{M_1}{z_t} = \frac{8.82\times 10^6}{4.8555\times 10^6} = 41.82\ N/mm^2$	$\frac{M_1}{z_b} = \frac{8.82\times 10^6}{3.104\times 10^6} = -2.84\ N/mm^2$
Stress due to imposed load	$\frac{M_2}{z_t} = \frac{13.69\times 10^6}{4.8555\times 10^6} = +2.82\ N/mm^2$	$\frac{M_2}{z_b} = \frac{13.69\times 10^6}{3.104\times 10^6} = -4.41\ N/mm^2$

Thus, stress due to (prestress + self weight)

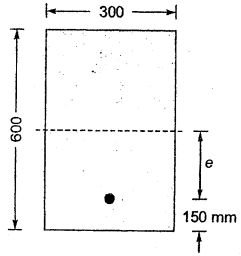
= −0.94 N/mm² (at top), 9.765 N/mm² (at bottom)

Stress due to (prestress + self weight + imposed load)

= 1.88 N/mm² (at top), 5.355 N/mm² (at bottom)

Example 14.3 A 300 mm × 600 mm rectangular concrete beam is prestressed by four 14 mm diameter bars of high tensile steel located at 150 mm from the beam soffit. The net stress in the bars is 750 MPa. What is the maximum BM that can be applied at the section with no tension in the beam soffit?

Solution:



Area of beam section = $A = 300\times 600 = 18\times 10^4\ mm^2$

Section modulus (z) = $\frac{300\times 600^2}{6} = 18\times 10^6\ mm^3$

Area of steel bars (A_s) = $\frac{300\times 600^2}{6} = 18\times 10^6\ mm^3$

Eccentricity (e) = $\frac{600}{2} - 150 = 150$ mm

$\therefore$ Prestressing force (P) = $750 \times 615.75\ \text{N} = 461.81\ \text{kN}$

$$\therefore \quad \frac{P}{A} = \frac{461.81 \times 1000}{18 \times 10^4} = 2.57\ \text{N/mm}^2$$

$$\frac{P.e}{z} = \frac{461.81 \times 1000 \times 150}{18 \times 10^6} = 3.85\ \text{N/mm}^2$$

$\therefore$ Total stress the beam soffit = $2.57 + 3.85 = 6.42\ \text{N/mm}^2$

Let M = maximum BM for zero stress at the beam soffit

$$\therefore \quad \frac{M}{z} = 6.42$$

$$\Rightarrow \quad M = 6.42 \times 18 \times 10^6 = 115.56\ \text{kNm}$$

14.16 Prestress Pressure Distribution in Beams

From the above BMD for a simply supported beam subjected to a uniformly distributed load (w) including the self weight of beam, it is evident that maximum flexural stress occurs at mid-span and these flexural stresses go on reducing as distance from the mid-span increases.

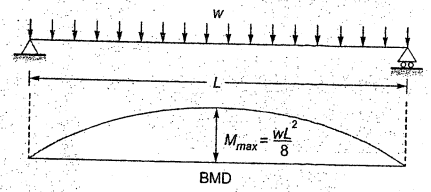


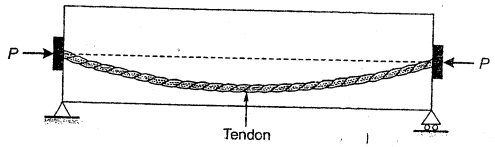
Fig.14.12 *BMD for a simply supported uniformly loaded beam*

In order to counter act the external flexural stresses as per the BMD above, it will be beneficial to have maximum prestressing stress at mid-span and this prestressing stress can be reduced on either side of the mid-span. This type of prestress distribution is called as non-uniform prestress distribution. Thus we have following types of prestress presure distribution viz.:

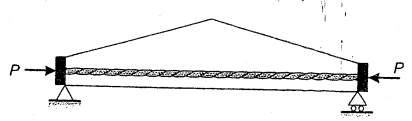
1. Uniform prestress pressure distribution
2. Non-uniform prestress pressure distribution.

Non-uniform prestress pressure distribution is achieved by following ways:

1. By varying the tendon/cable profile



2. By varying the beam cross-section



14.17 Effect of Loading on Stresses in Tendons

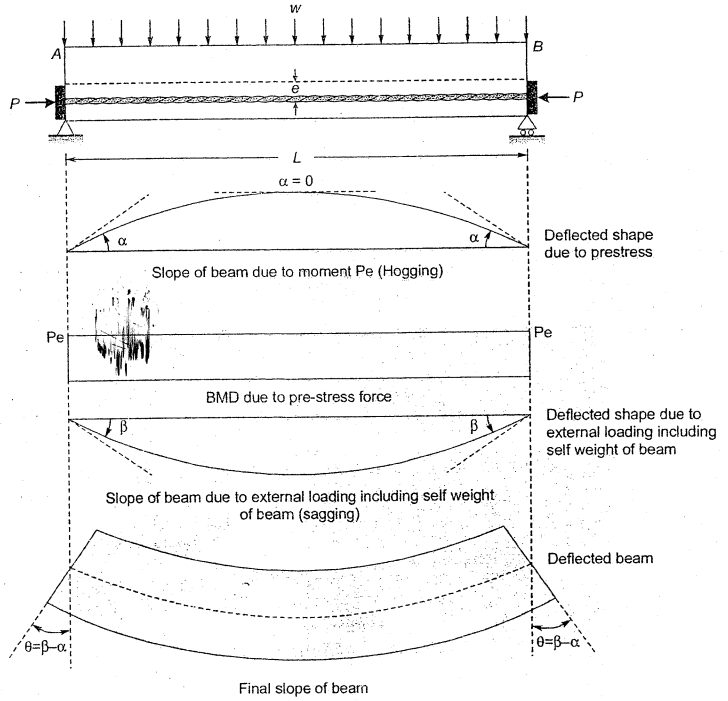


Fig.14.13 *Variation in tendon stress*

Consider a simply supported beam AB of span L prestressed by a force P acting at an eccentricity 'e' from the centroidal axis of beam.

Hogging moment due to prestress = Pe

$$\therefore \quad \text{Slope at the beam ends} = \alpha - 0 = \frac{\text{Area of BMD}}{EI}$$

$$\therefore \quad \alpha = \frac{PeL}{2EI}$$

Similarly slope at the beam ends due to uniformly distributed load (w) including the self weight of beam

$$\beta = \frac{wL^3}{24EI}$$

Case $\beta > \alpha$

If $\beta > \alpha$ then net slope will be in the downward direction

$$\therefore \text{Net slope at each end of beam} = \theta = \beta - \alpha = \frac{wL^3}{24E_cI} - \frac{P_eL}{2E_cI}$$

Due to slope in the downward direction there will be an increase in the length of tendon.

Total increase in length $= \delta l_1 = e\theta + e\theta = 2e\theta$

Increase in tendon strain $= \dfrac{2e\theta}{L}$

$\therefore$ Gain in stress in steel reinforcement $= \dfrac{\delta l_1}{L} \times E_S = \dfrac{2e\theta}{L} \cdot E_s = \dfrac{2e\theta E_s}{L}$

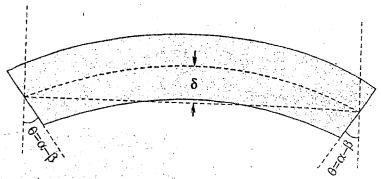
Case $\beta < \alpha$

There will be a net slope in upward direction

$$= \theta = \alpha - \beta = \frac{P_e L}{2E_c I} - \frac{wL^3}{24E_c I}$$

reduction in length $= 2e\theta = \delta l_2$

$\therefore$ Loss in stress $= \dfrac{\delta l_2}{L} \times E_S = \dfrac{2e\theta}{L} E_s = \dfrac{2e\theta E_s}{L}$



In general, any increase of loading on the pre-stressed member within the elastic range does not result in significant change in the tendon stress. Alternatively, it can be said that the stress in the tendon remains more or less constant provided the transverse loading is within the elastic limits.

14.18 Prestressed Beam with Parabolic Tendon Profile

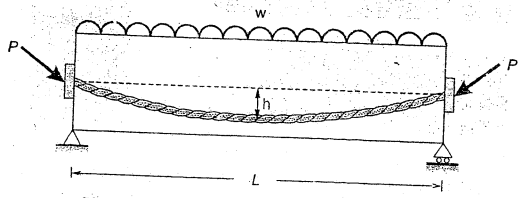
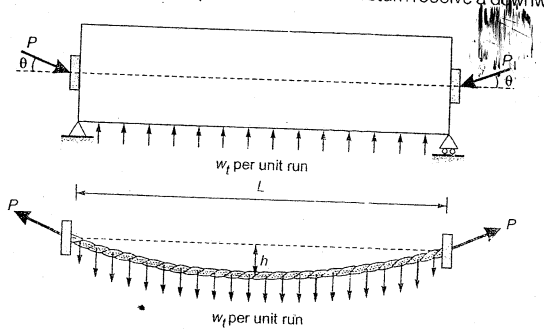


Fig.14.14 *Prestressed beam with parabolic tendon profile*

Consider a simply supported beam of span L with parabolic tendon profile as shown above. The stressed tendon will exert a uniform pressure upwards (w_t) in the beam and in return receive a downward uniform pressure.



From structural analysis of cables and suspension bridges, horizontal reaction at each end is given by:

$$H = \frac{w_t L^2}{8h}$$

Let the inclination of tendon at the ends in very small so that it can be conveniently assumed that,

$$H \simeq P$$

$$\therefore \quad P = \frac{w_t L^2}{8h} \quad \Rightarrow \quad w_t = \frac{8Ph}{L^2}$$

= Upward uniformly distributed pressure to a parabolic presented tendon carrying a prestressing force P.

Thus a part of downward acting load will be counteracted by this uniform upward pressure (w_T). This gives rise to the concept of LOAD BALANCING which is described in later sections.

14.19 C-line or Pressure Line

As seen in earlier sections, the resultant stress at any section of a pre-stressed concrete member is the combined effect of the pre-stressing force and the stresses induced due to external loads which results in the distribution of concrete stresses that can be resolved into a single force. The **locus of point of application of this resultant force in any structure is called as the pressure line or the thrust line.**

In pre-stressed concrete members, obviously the location of pressure line depends on the magnitude and direction of applied moments at the section combined with the magnitude and direction of the effect of stresses due to the pre-stressing force.

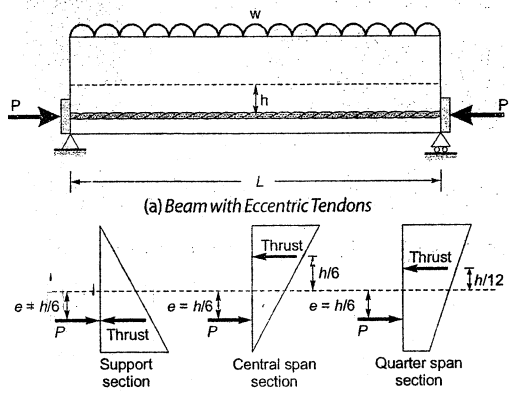


Fig. 14.15

As shown in the figure above, the load is of such magnitude that the bottom fibers at mid span section have zero stresses. At the support section, the stresses due to the applied load (and thus applied moment) are zero and thus the pressure line coincides with the centroid of steel located at an eccentricity of (h/6) from the neutral axis. It is quite evident that at the mid span section of the beam, the bottom stresses are zero and the pressure line has shifted towards the top by an amount of (h/3) from the initial position. At the quarter span section, the externally applied moment is small and so does the shift in pressure line is also small, being equal

to (h/4) from the initial position. Similarly, the pressure line location at other sections can also be determined. The locus of all such points gives the pressure line as shown in the **Fig. 14.16.**

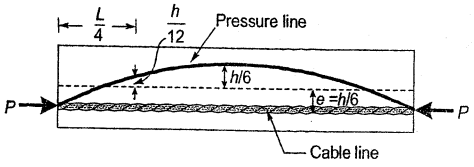


Fig.14.16 *Location of Pressure Line in the Prestressed Beam*

NOTE

A change in the external moments in the elastic range of pre-stressed concrete beam results in a shift of pressure line rather than an increase in the resultant force of the beam. This is in contrast to the reinforced concrete beam where increase in the externally applied moments results in an increase in the tensile stresses and compressive stresses. This increase in the stresses in reinforced concrete is due to the **constant lever arm.** In pre-stressed concrete sections, the load carrying mechanism is the **constant force with changing lever arm** and in reinforced concrete; it is the **constant lever arm with changing force.** But as a pre-stressed concrete member gets cracked then it behaves just like reinforced concrete section.

Apart from direct method of analysis of stresses at a section as described in earlier sections, the method based on **pressure line or the thrust line** can also be adopted for analysis of stresses. This method is generally called as **internal resisting couple method** or the **C-line method.** In this method, the pre-stressed concrete beam is analyzed just like plain concrete elastic beam with the usual principles of statics. The pre-stressing force is treated as external compressive force with a constant tensile force (T) in the tendons throughout the span.

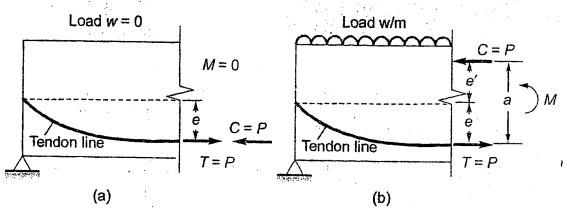


Fig.14.17 *Free-Body Diagram of Forces and Moments at a Section of Prestressed Concrete Beam*

As shown in the figure above, when gravity loads are zero then the *C* and *T* lines coincide because there is no moment anywhere at the section. Under the influence of transverse loads, the *C* line or the center of pressure line/thrust line is a varying distance '*a*' from the T line.

Let
M = Flexural moment at any section due to dead and live loads
e = Eccentricity of the pre-stressing tendon
$T = P$ = Pre-stressing force in the tendon

Thus from moment equilibrium,

$$M = Ca = Ta = Pa$$

or

$$a = \frac{M}{P}$$

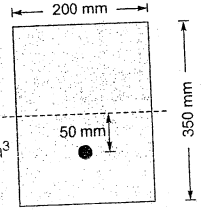
The shift of pressure line '*e*' is measured from the centroidal axis as:

$$e' = a - e = \left(\frac{M}{P}\right) - e$$

The resultant stresses at the top and bottom fibers of the section are:

$$f_{top} = \frac{P}{A} + \frac{Pe'}{Z}; \quad f_{bottom} = \frac{P}{A} - \frac{Pe'}{Z}$$

Example 14.4 A prestressed concrete beam of size 200 mm × 350 mm is used for an effective span of 5.5 m in order to support a uniformly distributed load of 5 kN/m (including self weight of the beam). The beam is prestressed by a straight cable carrying a force of 200 kN and located at an eccentricity of 50 mm. Determine the location of thrust line in the beam.



Solution:

Eccentricity, $e = 50$ mm

Gross cross-sectional area of beam, $A = 200 \times 350 = 70000\ mm^2$

Sectional modulus of the beam cross-section

$$Z = 200 \times \frac{350^2}{6} = 4.083 \times 10^6\ mm^3$$

Prestressing force, $P = 200$ kN

Direct stress due to prestressing force $= \frac{P}{A} = \frac{200 \times 1000}{70000} = 2.875\ N/mm^2$

Flexural stresses due to prestressing force $= \mp\frac{Pe}{Z} = \mp\frac{200 \times 10^3 \times 50}{4.083 \times 10^6} = \mp 2.45\ N/mm^2$

Maximum BM, $M = 5 \times \frac{5.5^2}{8} = 18.91$ kNm

Flexural stress due to $M = \pm\frac{18.91 \times 10^6}{4.083 \times 10^6} = \pm 4.63\ N/mm^2$

Thus resultant stress at mid-span section $=$
$2.857 - 2.45 + 4.63 = 5.037\ N/mm^2$ (at top)
$2.857 + 2.45 - 4.63 = 0.677\ N/mm^2$ (at bottom)

$\therefore$ Shift of pressure line from cable line $= \frac{M}{P} = \frac{18.91 \times 10^6}{200 \times 10^3} = 94.55$ mm

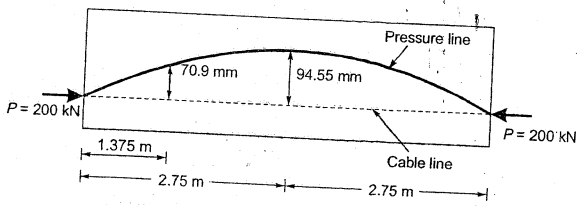
BM at quarter span section, $M_1 = R\frac{L}{4} - \frac{wL}{4}\frac{L}{8} = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3wL^2}{32}$

$= \frac{3}{32} \times 5 \times 5.5^2 = 14.18$ kNm

$\therefore$ Flexural stresses due to $M_a = \pm\frac{14.18 \times 10^6}{4.083 \times 10^6} = \pm 3.47\ N/mm^2$

Resultant stresses at quarter span = $\begin{cases} 2.857 - 2.45 + 3.47 = 3.877\ \text{N/mm}^2 \text{ (at top)} \\ 2.857 + 2.45 - 3.47 = 1.837\ \text{N/mm}^2 \text{ (at bottom)} \end{cases}$

$\therefore$ Shift of pressure line from cable line $= \dfrac{M_1}{P} = \dfrac{14.18 \times 10^6}{200 \times 10^3} = 70.9\ \text{m}$



Locus of pressure line

Example 14.5 A prestressed concrete beam of size 120 mm x 300 mm is prestressed concentrically by a prestressing force of 180 kN. The effective span of the beam is 6 m and it supports a uniformly distributed load of 5 kN/m including the self weight of beam. Find the locus of pressure line in the beam.

Solution:

Prestressing force, $P = 180\ \text{kN}$

$\because$ Beam is concentrically prestressed.

$\therefore$ Eccentricity, $e = 0$

Gross cross-sectional area of beam, $A = 120 \times 300 = 36000\ \text{mm}^2$

Sectional modulus, $Z = \dfrac{120 \times 300^2}{6} = 1.8 \times 10^6\ \text{mm}^3$

Mid-span flexural moment in beam, $M = 5 \times \dfrac{6^2}{8} = 22.5\ \text{kNm}$

Direct stress due to prestressing force $= \dfrac{P}{A} = \dfrac{180 \times 10^3}{36000} = 5\ \text{N/mm}^2$

Flexural stress due to moment $= \dfrac{M}{Z} = \dfrac{22.5 \times 10^6}{1.8 \times 10^6} = 12.5\ \text{N/mm}^2$

Stresses at the mid-span section

At top, stress = 5 + 12.5 = 17.5 N/mm² (compression)

At bottom, stress = 5 – 12.5 = –7.5 N/mm² (tensile)

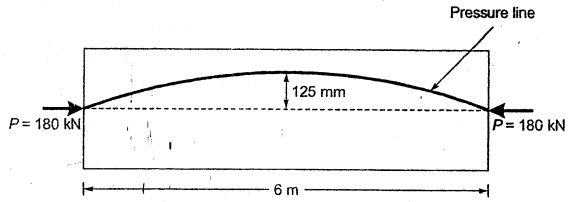
Thus shift of pressure line is given by,

$$\frac{180 \times 10^3}{36000} + \frac{180 \times 10^3 e}{1.8 \times 10^6} = 17.5$$

$\Rightarrow \quad 5 + 0.1e = 17.5$

$\Rightarrow \quad e = 125\ \text{mm}$

Alternatively, $e = \dfrac{M}{P} = \dfrac{22.5 \times 10^6}{180 \times 10^3} = 125\ \text{mm}$



Pressure line

Example 14.6 A Prestressed concrete beam is 120 mm x 300 mm and is used for an effective span of 6 m. It supports a uniformly distributed load of 5 kN/m including its own weight. The beam is prestressed by a prestressing force of 180 kN at an eccentricity of 30 mm. Find the locus of pressure line.

Solution:

Prestressing force, $P = 180\ \text{kN}$

Eccentricity, $e = 30\ \text{mm}$

Gross area of cross-section, $A = 120 \times 300 = 36000\ \text{mm}^2$

Sectional modulus, $Z = 120 \times \dfrac{300^2}{6} = 1.8 \times 10^6\ \text{mm}^3$

Direct stress $= \dfrac{P}{A} = \dfrac{180 \times 1000}{36000} = 5\ \text{N/mm}^2$

Flexural stress due to eccentricity $= \dfrac{Pe}{Z} = \dfrac{180 \times 1000 \times 30}{1.8 \times 10^6} = 3\ \text{N/mm}^2$

Mid span moment, $M = 5 \times \dfrac{6^2}{8} = 22.5\ \text{kNm}$

Flexural stress due to moment $= \dfrac{M}{Z} = \dfrac{22.5 \times 10^6}{1.8 \times 10^6} = 12.5\ \text{N/mm}^2$

Stresses at mid-span section

At top, stress = 5 – 3 + 12.5 = 14.5 N/mm² (compression)

At bottom, stress = 5 + 3 – 12.5 = –4.5 N/mm² (tension)

Shift of pressure line is given by,

$$\frac{180 \times 10^3}{36000} + \frac{180 \times 10^3 e}{1.8 \times 10^6} = 14.5$$

$\Rightarrow \quad 5 + 0.1e = 14.5$

$\Rightarrow \quad e = 95\ \text{mm}$

$\therefore$ Pressure line gets shifted from original pressure line by 95 + 30 = 125 mm

Alternatively:

This shift of pressure line can be computed as,

Shift $= \dfrac{M}{P} = \dfrac{22.5 \times 10^6}{180 \times 10^3} = 125\ \text{mm}$ (which is same as above)

At quarter span

Moment at quarter span, $M_1 = \left(\frac{wL}{2}\right)\frac{L}{4} - \frac{wL}{4}\frac{L}{8} = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3wL^2}{32}$

$= \frac{3\times5\times6^2}{32} = 16.875$ kNm

$\therefore$ Flexural stress due to moment, $M_1 = \frac{M_1}{Z} = \frac{16.875\times10^6}{1.8\times10^6} = 9.375$ N/mm^2

$\therefore$ At top, stress $= 5 - 3 + 9.375 = 11.375$ N/mm^2 (compression)

$\therefore$ At bottom, stress $= 5 + 3 - 9.375 = -1.375$ N/mm^2 (tension)

$\therefore$ Shift of pressure line is,

$$\frac{180\times10^3}{36000} + \frac{180\times10^3 e}{1.8\times10^6} = 11.375$$

$\Rightarrow e = 63.75$ mm

Thus pressure line gets shifted from original location by

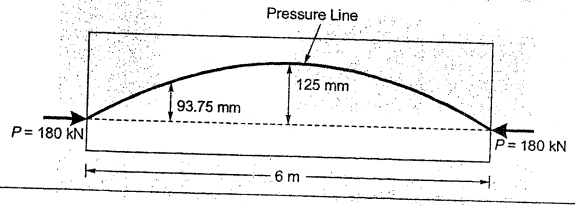
$63.75 + 30 = 93.75$ mm

Alternatively:

This shift of pressure line can be computed as,

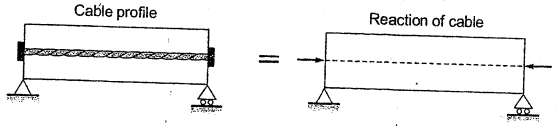
$$\text{Shift} = \frac{M_1}{P} = \frac{16.875\times10^6}{180\times10^3} = 93.75 \text{ mm}$$

(same as calculated above)

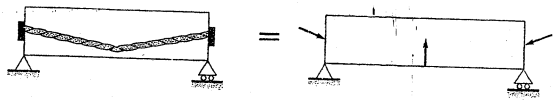


14.20 The Concept of Load Balancing

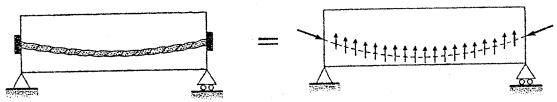
In a pre-stressed concrete member, it is always possible to have a suitable cable profile such that the transverse component of the cable force balances the external transverse loading.



(a) Straight tendon



(b) Bent tendon



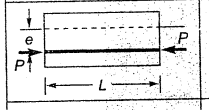
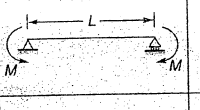
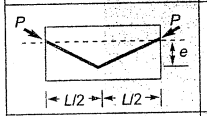
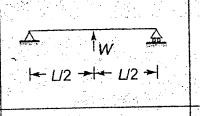
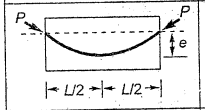
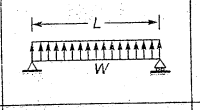
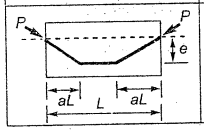
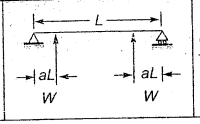
(c) Curved tendon

Fig.14.18 *Reactions of cable on beam*

The reaction of the cable on the concrete member depends on the shape of the cable profile. Straight cable profiles do not induce any reactions except at the supports but the curved cables result in uniformly distributed loads. Furthermore, sharp cable profile induce concentrated load on the concrete member.

In general, the requirement of load balancing will be satisfied if the cable profile in the pre-stressed member corresponds to the shape of the bending moment diagram due to the external loads. For a beam supporting two concentrated loads, the cable profile must have two bends i.e. its profile should be like a trapezium; similarly for a beam supporting uniformly distributed load, the cable profile should be parabolic.

Table 14.1 *Tendon Profiles and Equivalent Loads in Prestressed Concrete Beams*

Tendon profile	Equivalent moment or Load	Equivalent loading	Camber
	$M = Pe$		$\frac{ML^2}{8EI}$
	$W = \frac{4Pe}{L}$		$\frac{WL^2}{48EI}$
	$W = \frac{8Pe}{L^2}$		$\frac{5WL^2}{384EI}$
	$W = \frac{Pe}{aL}$		$\frac{a\left(3-4a^2\right)WL^2}{24EI}$

Example 14.7 A rectangular prestressed concrete beam of size 150 x 300 mm is to be used for an effective span of 9 m. The prestressing cable is having zero eccentricity at supports and eccentricity varies linearly upto 40 mm at the centre. The effective prestressing force is 550 kN. Find the concentrated load at the mid-span location is the load nullifies the flexural effect of prestressing force (excluding the self weight of beam)

Solution:

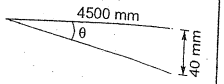
Gross cross-sectional area of beam, $A = 150 \times 300 = 45000$ mm^2

Sectional modulus, $Z = 150\times\frac{300^2}{6} = 225\times10^4$ mm^3

Prestressing force, $P = 550$ kN

Self weight of beam $= 0.15 \times 0.3 \times 25 = 1.125$ kN/m

$$\tan\theta = \frac{40}{4500}$$



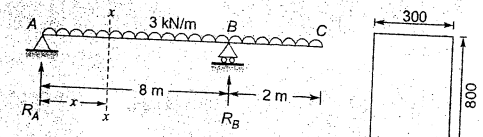
∴ Concentrated load, $Q = 2P\sin\theta \approx 2P\tan\theta$

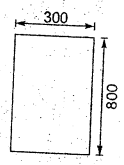
$$= 2\times 550\left(\frac{40}{4500}\right) = 9.78 \text{ kN}$$

Thus concentrated load required at mid-span section = 9.78 kN

Example 14.8 A single overhang prestressed concrete beam is simply supported over a span of 8 m with overhang of 2 m. The section of beam is 300 × 800 mm. The beam supports a uniformly distributed live load of 3 kN/m over the entire span. Find the cable profile of prestressing cable for an effective prestress of 450 kN which can counteract the effect of dead and live load.

Solution:





Prestressing force, $P = 450$ kN

Self weight of beam $= 0.3 \times 0.8 \times 25 = 6$ kN/m

Live load on beam $= 3$ kN/m

∴ Total load $= 6 + 3 = 9$ kN/m

$R_A + R_B = 9 \times 10 = 90$ kN

and taking moment about A is zero

$R_B(8) - 9 \times 10 \times 5 = 0$

⇒ $R_B = 56.25$ kN

∴ $R_A = 90 - R_B = 33.75$ kN

Moment at section x-x, distant x from A $= R_A x - 9x.\frac{x}{2}$

$= 33.75x - 4.5x^2$

For maximum moment, $\frac{dM_x}{dx} = 0$

⇒ $33.75 - 9x = 0$

⇒ $x = 3.75$ m

∴ $M_{max} = 33.75(3.75) - 4.5(3.75)^2 = 63.28$ kNm

⇒ $e = \frac{M_{max}}{P} = = 140.62$ mm

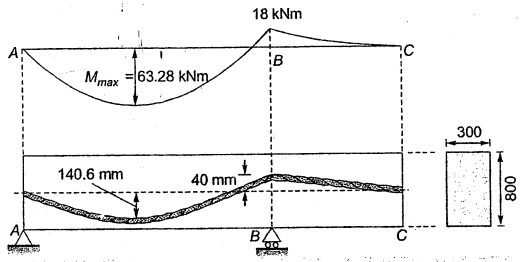
When $M_x = 0$

⇒ $33.75x - 4.5x^2 = 0$

⇒ $x = 0, 7.5$m (from support A)

Eccentricity of cable at support $B = \frac{M_B}{P} = \frac{18\times 10^6}{450\times 1000} = 40$ mm

At support A, moment is zero and thus eccentricity is zero at A.



Cable profile

Example 14.9 A prestressed concrete beam of effective span 5.5 m is 150 mm wide and 300 mm deep. The tendon is located at a constant eccentricity of 45 mm having a prestressing stress of 850 N/mm². What is the percentage increase of stress in tendons if the beam supports a line load of 4.5 kN/m. Take cross-sectional area of tendons as 90 mm², modulus of elasticity of concrete as 36 kN/mm² and that of steel as 210 kN/mm².

Solution:

Prestressing force, $P = 850 \times 90 = 76.5$ kN

Moment of inertia, $I = 150\times\frac{300^3}{12} = 337.5\times 10^6$ mm⁴

Rotation due to prestress, $\theta_1 = \frac{PeL}{2E_cI} = \frac{(76.5\times 1000)45(5500)}{2\times 36000\times 337.5\times 10^6}$

$= 0.0007792$ radians (Hogging)

Self weight of beam $= 0.15 \times 0.3 \times 25 = 1.125$ kN/m

Live load $= 4.5$ kNm

∴ Total load $= 5.625$ kN/m $= 5.625$ N/mm

Rotation due to loads, $\theta_2 = \frac{wL^3}{24E_cI} = \frac{5.625(5500)^3}{24\times 36\times 1000\times 337.5\times 10^6}$

$= 0.00321$ radians (Sagging)

∴ Net rotation $= 0.00321 - 0.0007792$

$= 0.0024308$ radians (Sagging)

∴ Cable elongation $= 2e\theta$

$= 2 \times 45 \times 0.0024308 = 0.218772$ mm

∴ Increase in stress $= \left(\frac{2e\theta}{L}\right)E_s = \left(\frac{0.218772}{5500}\right)210\times 10^3 = 8.353$ N/mm²

Initial prestress = 850 N/mm²

∴ Percentage increase in stress = $\frac{8.353}{850} \times 100 = 0.983\%$

14.21 Stresses in Beam at Different Stages of Loading

Stresses get induced in beam at the following stages of loading viz.

1. At transfer
2. At final stage of loading

14.21.1 Stress at transfer stage

It gets induced just after the transfer of prestressing force in beam. Some important points must be considered in this regard:

1. Total prestressing force (initial value) is considered
2. No loss of prestress is considered
3. Dead loads may be considered. If the beam is still kept on ground or still supported on supports then no dead load should be considered.
4. No live load is considered

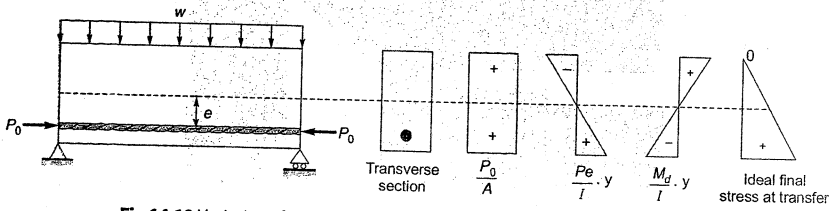


Fig.14.19 *Variation of stresses in the transverse section of an eccentrically prestressed beam*

$$\text{Stress at top} = \frac{P_0}{A} - \frac{Pe}{I} \cdot y + \frac{M_d}{I} \cdot y$$

$$\text{Stress of bottom} = \frac{P_0}{A} + \frac{Pe}{A} \cdot y - \frac{M_d}{I} \cdot y$$

These two forces i.e. the *P*-force and the *C*-force are equal and opposite. Now external moment is zero. No level difference between the *C*-force and P-force i.e. *P*-force and *C*-force acts at the same level. Thus the *p*-line and *C*-line coincide in the absence of external loading.

Now let the beam is subjected to an external moment (*M*). The c-line will shift upwards to counteract the external moment. Let this shift of *c*-line from *p*-line is *a*,

$$\therefore \quad a = \frac{M}{P}$$

$$\therefore \quad \text{Final stress in concrete} = \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}$$

Ideal stress diagram at transfer should have zero stress at top and maximum stress at bottom at shown above.

14.21.2 Stress at final stage of loading

It occurs after the application of line loads.

1. Loss of prestress is considered

   Prestressing force = kP_0

   where k = loss factor = $\left(1 - \frac{p_L}{100}\right)$ where p_L is in percentage.

2. All loads i.e. dead loads and live loads are considered.

$$\text{Final stress} = \frac{kP_0}{A} \mp \frac{kP_0 e}{I} \cdot y \pm \frac{M_d}{I} \cdot y \pm \frac{M_l}{I} \cdot y$$

Ideal stress diagram at final stage of loading should have zero stress at bottom and maximum stress at the top.

14.22 Prestress Losses

The initial pre-stress in concrete does not remain constant with time but in fact decreases with time at transfer stage due to many possible reasons. This gradual decrease in pre-stress is called as **loss of pre-stress.** It is to be noted that different types of pre-stress losses are encountered in the two types of pre-stressed concrete viz. pre-tensioned and post-tensioned concrete.

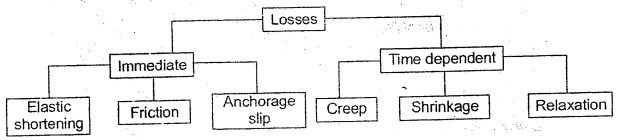


Fig.14.20 *Losses in prestressed concrete*

Table 14.4: *Types of pre-stress losses in pre-stressed concrete*

Pre-tensioned concrete	Post-tensioned concrete
Loss due to elastic deformation of concrete.	If wires are tensioned simultaneously, then no loss due to elastic deformation of concrete but however if the wires are tensioned successively, then loss due to elastic deformation occurs.
Loss due to stress relaxation in steel.	Loss due to stress relaxation in steel.
Loss due to creep and shrinkage of concrete.	Loss due to creep and shrinkage of concrete.
No loss due anchorage slip and friction.	Loss due to anchorage slip and friction.

Apart from the above losses, there are other types of losses also like loss due to temperature, particularly in steam curing of pre-tensioned concrete members.

14.22.1 Loss due to elastic deformation of concrete

This type of pre-stress loss in pre-stressed concrete members depends on the modular ratio and average stress in concrete at the level of centroid of steel tendons/wires.

Let f_c = Pre-stress in concrete at the level of centroid of tendons/wires

E_s = Modulus of elasticity of steel

E_c = Modulus of elasticity of concrete

Then modular ratio $(m) = \frac{E_s}{E_c}$

So, strain in concrete at the level of steel = $\frac{f_c}{E_c}$

Corresponding to the above strain, stress in steel = $\left(\frac{f_c}{E_c}\right)E_s = mf_c$

$\therefore$ Loss of stress in steel = mf_c.

Example 14.10 A pre-stressioned concrete beam of size 150 x 300 mm is prestressed by straight wires with an initial prestressing force of 150 kN. The eccentricity of the wire is 60 mm. What is the percentage loss of stress in steel due to elastic deformation of concrete? Take E_s = 210 kN/mm², E_c = 35 kN/m², area of wires as 180 mm².

Solution:

Initial prestressing force (P) = 150 kN

Gross cross-sectional area of beam (A) = $150 \times 300 = 45000$ mm²

Moment of inertia (I) = $150 \times \frac{300^3}{12} = 337.5 \times 10^6$ mm⁴

Initial stress in steel wires = $\frac{150 \times 1000}{180} = 833.33$ N/mm²

Stress in concrete at the level of prestress wire = $\frac{P}{A} + \frac{Pe}{I}e = \frac{450 \times 1000}{45000} + \frac{(150 \times 1000)60}{337.5 \times 10^6} \times 60$

$= 3.33 + 1.6 = 4.93$ N/mm²

Modular ratio (α) = $\frac{E_s}{E_c} = \frac{210}{35} = 6$

$\therefore$ Loss of stress due to elastic deformation of concrete = $6 \times 4.93 = 29.58$ N/mm²

$\therefore$ Percentage loss of stress in steel = $\frac{29.58}{833.33} \times 100 = 3.55\%$

14.22.2 Loss due to shrinkage of concrete

It is well known fact that concrete shrinks with age. This shrinkage of concrete in a pre-stressed concrete member results in the shortening of prestressed wires thereby resulting in the loss of pre-stress. Obviously, the shrinkage of concrete is influenced by the type of cement used and the type of aggregates (both coarse and fine aggregates) along with the water-cement ratio and also the method of curing employed. Thus use of high strength concrete is preferred with low water cement ratio which results in low shrinkages and thus the resulting loss due to concrete shrinkage reduces.

For **pre-tensioned members,** shrinkage of concrete is prevented by moist curing of members till the time of stress transfer. Thus, the total residual shrinkage strain is large in pre-tensioned members after the stress

transfer in comparison to post tensioned members, where a part of shrinkage strain has already occurred at the stage of stress transfer.

For **pre-tensioned members,** the total residual shrinkage strain is 3×10^{-4}.

For **post-tensioned members,** total residual shrinkage strain is given by,

$$\frac{2 \times 10^{-4}}{\log_{10}(t+2)}$$

where t = age of concrete at the time of transfer in days.

This value can be enhanced by 50% in dry atmospheric conditions but must not exceed 3×10^{-4}.

Example 14.11 A concrete beam is prestressed by a cable with a prestressing force of 450 kN. Area of cable wires is 150 mm². Find the percentage loss of stress due to shrinkage of concrete assuming the beam to be (a) Pre-tensioned (b) Post-tensioned. Take E_s = 210 kN/mm² and age of concrete at transfer = 8 days.

Solution:

Initial stress in wires = $\frac{450 \times 1000}{150} = 3000$ N/mm²

(a) Pre-tensioned

For pre-tensioned beam total residual shrinkage strain = 3×10^{-4}

$\therefore$ Loss of stress = $E_s \times 3 \times 10^{-4} = 210 \times 10^3 \times 3 \times 10^{-4} = 63$ N/mm²

$\therefore$ Percentage loss of stress = $\frac{63}{3000} \times 100 = 2.1\%$

(b) Post-tensioned beam

For post-tensioned beam, total residual shrinkage strain

$$= \frac{2 \times 10^{-4}}{\log(t+2)} = \frac{2 \times 10^{-4}}{\log(8+2)} = 2 \times 10^{-4}$$

$\therefore$ Loss of stress = $210 \times 10^3 \times 2 \times 10^{-4} = 42$ N/mm²

$\therefore$ Percentage loss of stress = $\frac{42}{3000} \times 100 = 1.4\%$

14.22.3 Loss due to creep of concrete

It is well known fact that sustained loading on concrete results in creep of concrete. Due to this reason only, the sustained pre-stress in a pre-stressed concrete member results in creep of concrete. The loss of stress in steel due to creep of concrete can be judged if the ultimate creep coefficient is known. Thus we have two methods, viz.:

1. **Creep coefficient method:** As described earlier, we define creep coefficient as:

$$\text{Creep coefficient } (\phi) = \frac{\text{Creep strain } (\varepsilon_c)}{\text{Elastic strain } (\varepsilon_e)}$$

Thus, $\varepsilon_c = \varepsilon_e \cdot \phi = \phi\left(\frac{f_c}{E_c}\right)$

Thus loss of stress in steel = $\varepsilon_c E_c \phi E_s = \phi E_s \left(\frac{f_c}{E_c}\right) = m\phi f_c$

The magnitude of creep coefficient (ϕ) varies with humidity of the atmosphere, quality of concrete and the duration of applied loading along with the age of concrete at the time of loading. Generally the creep coefficient varies from 1.5 for wet situations to 4.0 for dry situations with relative humidity of about 35%.

2. **Ultimate creep strain method:** If ε_{cc} is the ultimate creep strain for a sustained stress of magnitude unity, then the loss of pre-stress due to creep in concrete is equal to $\varepsilon_{cc} f_c E_s$ where f_c is the compressive stress in concrete at the level of centroid of steel and E_s is the modulus of elasticity of steel.

Example 14.12 A prestressed rectangular concrete beam of size 150 × 450 mm is prestressed by 5 wires of 6 mm diameter located at an eccentricity of 45 mm. The initial prestress in the wires is 1350 N/mm². What is the loss of stress in steel due to creep of concrete? Take E_s = 210 kN/mm², E_c = 35 kN/mm², ultimate creep strain = 41 × 10^{-6} mm/mm per N/mm² , creep coefficient ϕ = 1.6.

Solution:

Gross cross-sectional area of beam, $A = 150 \times 450 = 67500 \text{ mm}^2$

Moment of inertia, $I = 150 \times \frac{450^3}{12} = 11.3906 \times 10^8 \text{ mm}^4$

Prestressing force, $P = 1350 \times 5 \times \frac{\pi}{4} \times 6^2 = 190.85 \text{ kN}$

Modular ratio, $m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

Creep coefficient, $\phi = 1.6$

Stress in concrete at the level of steel $= \frac{190.85 \times 10^3}{67500} + \frac{190.85 \times 10^3 \times 45 \times 45}{11.3906 \times 10^8}$

$= 2.83 + 0.34 = 3.17 \text{ N/mm}^2$

Loss of stress from ultimate creep strain method

$= (41 \times 10^{-6}) \times 3.17 \times 210 \times 1000 = 27.29 \text{ N/mm}^2$

Loss of stress from creep coefficient method

$= 1.6 \times 3.17 \times 6 = 30.432 \text{ N/mm}^2$

Example 14.13 A post-tensioned prestressed concrete beam of size 150 × 450 mm prestressed using a parabolic cable with zero eccentricity at supports. The eccentricity at the centre of span is 50 mm. Initial prestress in the cable is 1300 N/mm² and area of cable is 175 mm². If ultimate creep strain is 3 × 10^{-5} mm/mm per N/mm² and E_s = 210 kN/m², find the loss of stress in steel due to creep.

Solution:

Gross cross-sectional area of beam, $A = 150 \times 450 = 67500 \text{ mm}^2$

Moment of inertia of beam, $I = 150 \times \frac{450^3}{12} = 11.3906 \times 10^8 \text{ mm}^4$

$P = 1300 \times 175 = 227.5 \text{ kN}$

Stress in concrete at the level of steel,

at support $= \frac{227.5 \times 1000}{67500} = 3.37 \text{ N/mm}^2$

at mid-span $= \frac{227.5 \times 1000}{67500} + \frac{227.5 \times 1000 \times 50 \times 50}{11.3906 \times 10^8}$

$= 3.37 + 0.5 = 3.87 \text{ N/mm}^2$

∴ Average stress at the level of steel $= 3.37 + \frac{2}{3} \times (3.87 - 3.37)$ 3.703 N/mm²

∴ Loss of stress due to creep $= 3 \times 10^{-5} \times 3.703 \times 210 \times 1000 = 23.33 \text{ N/mm}^2$

14.22.4 Loss due to stress relaxation in steel

In order to account for loss due to relaxation in steel, most of the codes all over the world describe the loss of stress due to stress relaxation as a percentage of initial stress in steel. In line with this only, the IS Code recommends a value ranging from 0 to 90 N/mm² for stress in wires varying from 0.5f_{pu} to 0.8f_{pu}.

Loss of stress due to stress relaxation in steel as per IS 1343 at 1000 hrs. at 27°C is as given in the table.

In order to reduce this type of pre-stress loss, a temporary over stressing by about 5 to 10% for a period of 2 min is usually recommended for drawn wires.

Table 14.5: IS 1343 recommendations for relaxation loss at 1000 hrs. at 27°C

Initial Stress	Relaxation loss (N/mm²)
0.5f_{pu}	0
0.6f_{pu}	35
0.7f_{pu}	70
0.8f_{pu}	90

14.22.5 Loss of stress due to friction

This type of loss occurs in post-tensioned members. In post-tensioned members, the tendons are passed through the ducts. These ducts are either straight or follow a curved profile as per the requirements. Thus on applying the pre-stressing force to these tendons, loss of stress occurs due to the friction between the tendons and the surrounding concrete. Friction loss is of two types viz.:

1. Loss of stress due to the effect of curvature which in fact depends on the alignment of the tendon.
2. Loss of stress due to wobble effect. It depends on the local variations in the alignment of the cable. The wobble or the wave effect occurs due to accidental or unavoidable misalignment in the cable profile because the ducts or the sheaths cannot be perfectly placed in order to follow exactly the cable profile throughout the length of the member.

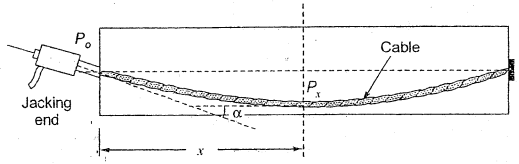


Fig.14.21 *Loss of Stress due to Friction*

The magnitude of the pre-stressing force (P_x) at a distance x from the tensioning end follows the following exponential function:

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

where, P_o = Pre-stressing force at the jacking end

μ = Friction coefficient between the cable and the duct

α = Angle (in radians) through which the tangent to the cable profile has turned between the two points under consideration

k = Coefficient of friction for wave effect

Typical values of coefficient of friction (μ):

0.55 for steel moving on smooth concrete

0.35 for steel moving on steel fixed to duct

0.25 for steel moving on steel fixed to concrete

0.25 for steel moving on lead

Value of friction coefficient for wave effect (k) is taken as 0.15 per 100 m for normal conditions.

This value can be taken as zero where clearance between the duct and the cable is large enough to nullify the wave effect.

The coefficient of friction (μ) can be considerably reduced by using various types of lubricants like grease, oils, graphite, paraffin wax etc. Experimentally, it is found that the graphite gives the lowest value of coefficient of friction. For small values of ($\mu\alpha + kx$), the above expression can be simplified as,

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

$$\Rightarrow \quad P_x = P_o(1 - \mu\alpha - kx)$$

$$\Rightarrow \quad \frac{P_o - P_x}{A} = (\mu\alpha + kx)\frac{P_o}{A}$$

$$\Rightarrow \quad \text{Prestress loss} = (\mu\alpha + kx) \times \text{Initial stress}$$

The above simplified expression shows that for a cable of single curvature, the pre-stress varies linearly with distance from the stretching end.

14.22.6 Loss due to anchorage slip

It is observed mostly in post-tensioning system wherein the cable is tensioned and the jack is released to transfer the pre-stress to concrete, the friction wedges that are used to grip the wires, get slipped for a small distance before the wires are anchored firmly between the edges. When anchor plates are used, it is quite essential to allow for a small settlement of plate into the end of the concrete member.

Let, Δ = Slip of the anchorage (in mm)

L = Length of the cable (in mm)

A = Cross-sectional area of the cable (in mm^2)

E_s = Modulus of elasticity of steel (in N/mm^2)

P = Pre-stressing force in the cable (in N)

Thus, $$\Delta = \frac{PL}{AE_s}$$

Thus loss of stress due to anchorage slip = $\left(\frac{P}{A}\right) = \frac{\Delta E_s}{L}$

The loss during anchoring is usually accounted for (at site) by over extending the tendon in pre-stressing process. But the momentary over-stress must not exceed 80-85% of ultimate tensile strength of wires.

Table 14.6: *Some typical values of anchorage slip*

Anchorage System	Anchorage Slip (Δ)
Freyssinet	
12.5 mm diameter strands	4 mm
12.8 mm diameter strands	6 mm
Magnel	8 mm
Dywidag System	1 mm

In long line pre-tensioning system, the slip at the anchorages is usually very small as compared to the length of the tensioned wire and thus is generally neglected. However, while pre-stressing a short member, due allowance should be made for loss of pre-stress due to anchorage slip which in fact forms a major portion of the total loss of pre-stress.

Example 14.14 A concrete beam is post-tensioned by a cable carrying a pre-stress of 900 N/mm^2. At the jacking end, the slip was observed to be 8 mm. Find the percentage loss of stress due to anchorage slip if length of beam is (a) 20 m and (b) 5 m. Take E_s = 210 kN/mm^2.

Solution:

Loss of stress due to anchorage slip = $E_s = \frac{\Delta}{L}$

(a) For 20 m beam, loss of stress = $210 \times 1000 \times \frac{8}{20000} = 84$ N/mm^2

$\therefore$ Percentage loss of stress = $\frac{84}{900} \times 100 = 9.33\%$

(b) For 5 m, beam, loss of stress = $210 \times 1000 \times \frac{8}{5000} = 336$ N/mm^2

$\therefore$ Percentage loss of stress = $\frac{336}{900} \times 100 = 37.33$ N/mm^2

Example 14.15 A prestressed concrete beam of size 250 mm x 400 mm has a span of 10 m. The beam is prestressed by steel wires of area 360 mm^2 provided at a uniform eccentricity of 50 mm with an initial prestress of 1150 N/mm^2. Find the percentage loss of stress in the wires of (a) the beam is pre-tensioned (b) the beam is post tensioned. Take E_s = 210 kN/mm^2, E_c = 35 kN/m^2, Ultimate creep stress = 45 x 10^{-6} mm/mm per N/mm^2 for pre-tensioned beam and 22 x 10^{-6} mm/mm per N/mm^2 for post tensioned beam, shrinkage of concrete = 3 x 10^{-4} for pre-tensioned beam = 2.15 x 10^{-4} for post tensioned beam, steel stress relaxation = 5% of initial stress, anchorage slip = 1.25 mm, friction coefficient of wave effect = k = 0.0013.

Solution:

Gross cross-sectional area of beam (A) = 250 x 400 = 1 x 10^5 mm^2

Moment of inertia of the beam section (I) = $\frac{250 \times 400^3}{12} = 13.33 \times 10^8$ mm^4

Initial prestressing force (P) = 1150 x 360 = 414 kN

$$m = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

Stress in concrete at the level of steel,

$$f_c = \frac{P}{A} + \frac{Pe}{I}e = \frac{414000}{10^5} + \frac{414000(50)^2}{13.33 \times 10^8}$$

$$= 4.14 + 0.7764 = 4.9164 \text{ N/mm}^2$$

(a) Loss of stress in pre-tensioned beam

(i) Loss of stress due to elastic shortening of concrete

$= mf_c = 6 \times 4.964 = 29.4984$ N/mm^2 = 29.5 N/mm^2

(ii) Loss of stress due to creep of concrete

$= 45 \times 10^{-6} \times 4.9164 \times 210 \times 10^3 = 46.5\ \text{N/mm}^2$

(iii) Loss of stress due to shrinkage of concrete

$= 3 \times 10^{-4} \times 210 \times 10^3 = 63\ \text{N/mm}^2$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5\ \text{N/mm}^2$

(v) Loss of stress due to anchorage slip = 0

(vi) Loss of stress due to friction = 0

Total loss of stress = $196.5\ \text{N/mm}^2$

$\therefore$ Percentage loss of stress $= \frac{196.5}{1150} \times 100 = 17.09\%$

(b) Loss of stress in post-tensioned beam

(i) Loss of stress due to elastic shortening of concrete = 0

(ii) Loss of stress due to creep of concrete $= 22 \times 10^{-6} \times 4.9164 \times 210 \times 10^3$

$= 22.7\ \text{N/mm}^2$

(iii) Loss of stress due to shrinkage of concrete $= 2.15 \times 10^{-4} \times 210 \times 10^3$

$= 45.15\ \text{N/mm}^2$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5\ \text{N/mm}^2$

(v) Loss of stress due to anchorage slip $= \frac{1.25}{10 \times 1000} \times 210 \times 1000 = 26.25\ \text{N/mm}^2$

(vi) Loss of stress due to friction $= 1150 \times 0.0013 \times 10 = 14.95\ \text{N/mm}^2$

Total loss of stress = $166.55\ \text{N/mm}^2$

$\therefore$ Percentage loss of stress $= \frac{166.55}{1150} \times 100 = 14.48\%$

14.22.7 Loss due to successive tensioning of curved cables

As explained in the section of load balancing, the tendon profile is selected in such a way that it counteract the superimposed transverse loads. Mostly in bridges, the tendons are curved having maximum eccentricity at the center of span. For such cases, the loss of pre-stress due to elastic deformation of concrete is computed by taking average stress in concrete at the level of centroid of steel.

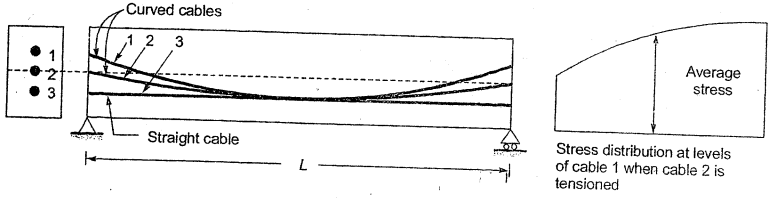


Fig.14.22 *Successive Tensioning of Curved Cables*

The above figure shows the three parabolic cables that are post-tensioned. The stress distribution in concrete at the level of cable no. 1 is also shown when cable no. 2 is tensioned. For calculating the loss of stress, we take into account the average stress.

14.22.8 Total loss of prestress

In normal design practice of pre-stressed concrete members, it is quite convenient to express the total loss of stress as a percentage of the initial stress and make suitable provisions in the design to account for these stress losses. Now the loss of pre-stress depends on various factors like the properties of concrete and steel, method of curing, the method of pre-stressing used and so on. Thus, it is quite difficult to have an exact idea of the total amount of loss of pre-stress.

Let, f_{pe} = Effective stress in tendons after loses

f_{pi} = Stress in tendons at transfer

η = Prestress loss reduction factor

$\therefore \quad \eta = \left(\frac{f_{pe}}{f_{pi}}\right)$

This η is generally taken as 0 .75 for pre-tensioned members and 0.80 for post-tensioned members.

Table 14.7: *Percentage loss of prestress*

Cause of prestress loss	Post-tensioned members	Pre-tensioned members
Shrinkage of concrete	6%	7%
Creep of concrete	5%	6%
Creep of steel	3%	2%
Elastic shortening of concrete	1%	3%
Total	15%	18%

Further different types of prestress loss can be summarized as:

Table 14.8: *Summary of prestress losses*

Types of loss	Equation
1. Curvature and friction effect	$(\mu\alpha + kx)P_0$
2. Slip at anchorages	$(\Delta/L)E_s$
3. Loss due to shrinkage	$\in_s E_s$
4. Loss due to creep of concrete	$m\phi f_c$
5. Loss due to elastic shortening of concrete	mf_c
6. Loss due to stress relaxation	2-5%

14.23 Cracking Moment

It is the moment at which visible cracks will appear in the section and this moment is called as cracking moment. Cracks appear when tensile stresses developed in the section are more than the permissible tensile strength of concrete. Let cracking moment = M_c

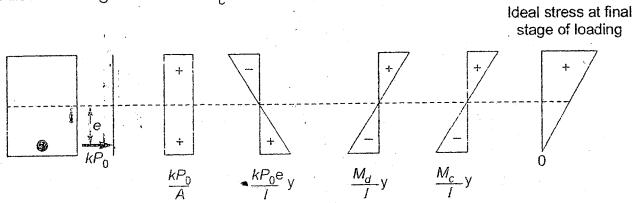


Fig.14.23 *Concept of cracking moment*

Final stress at bottom $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M_c}{Z} = f_t$

Load corresponding to cracking moment $= w_c$

$\therefore \quad M_c = \frac{w_c l^2}{8}$

Load factor against cracking $= \frac{\text{Cracking Load}}{\text{Working Load}}$

Here, Load = Dead load + Live load

Example 14.16 A prestressed concrete beam of 8 m span is prestressed by three cables each of area 150 mm^2 and initial prestress of 1100 N/mm^2. Cable is parabolic with eccentricities of 40 mm each at support and mid-span. Cable 2 is also parabolic but with zero eccentricity at supports and 40 mm at mid-span. Cable 3 is straight with uniform eccentricity of 40 mm. Calculate the loss of stress in each cable due to friction is cables are stretched from one end only. Take $k = 0.0015$ per m and $\mu = 0.35$.

Solution:

For parabolic cable profile, $y = \frac{4e}{L^2}x(L-x)$

$\therefore \quad \text{Slope} = \frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$

$\therefore$ Slope at support for cable 1

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{4eL}{L^2} = \frac{4e}{L} = \frac{4\times 40}{8000} = 0.025 \text{ radians}$$

$\therefore \quad \alpha = 2 \times 0.025 = 0.04$ radians

Slope at support for cable 2

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{4e}{L} = \frac{4\times 20}{8000} = 0.01 \text{ rad}$$

$\therefore \quad a = 2 \times 0.01 = 0.02$ radian

Initial prestressing force $(P_x) = 1100 \times 150 = 165$ kN

$\therefore \quad P_x = P_o e^{-(\mu\alpha + kx)} = P_o(1 - \mu\alpha - kx)$

$\therefore$ Loss of prestress $= P_o - P_x = P_o(\mu\alpha + kx)$

For cable 1, prestress loss $= P_o(0.35 \times 0.04 + 0.0015 \times 8)$

$= 0.026\, P_o = 0.026 \times 1100 = 28.6$ N/mm^2

For cable 2, prestress loss $= P_o(0.35 \times 0.02 + 0.0015 \times 8)$

$= 0.019 P_o = 0.019 \times 1100 = 20.9$ N/mm^2

For cable 3, prestress loss $= P_o(0.35 \times 0 + 0.0015 \times 8)$

$= 0.012 P_o = 0.012 \times 1100 = 13.2$ N/mm^2

14.24 Design of Prestressed Concrete Beam Members

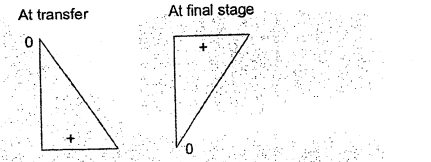
For design of prestressed concrete beam/slab, the following cases are considered:

Case I:

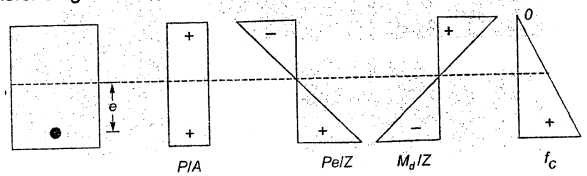
Given:
1. Permissible stress of concrete
   (i) in compression $= f_c$
   (ii) in tension = 0
2. Permissible stress in steel
3. Live load
4. Loss of prestress = 0

Find out:
1. Size of beam/slab requested
2. Prestressing force
3. Eccentricity

For design purposes, use following stress diagrams:



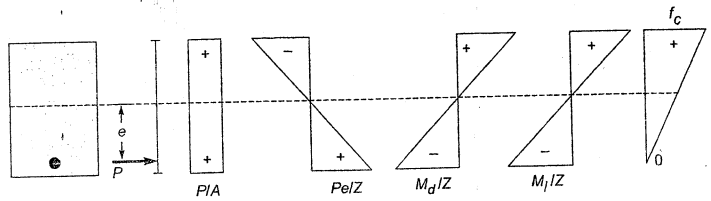
1. At transfer stage:



Stress at top $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} = 0$ ...(i)

Stress at bottom $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} = f_c$ ...(ii)

2. At final stage:



Stress at top $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} + \frac{M_l}{Z} = f_c$ ...(iii)

Stress at bottom $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} = 0$ (iv)

Design Formula

Let M_l = BM due to live load

The beam should be so designed that resultant stress will be as shown above i.e.

$$\frac{M_l}{Z} = f_c \quad \text{...(v)}$$

from (ii) $$\frac{P}{A}+\frac{Pe}{Z}-\frac{M_d}{Z} = f_c = \frac{M_l}{Z} \quad \text{...(from v)}$$

$$\Rightarrow \quad \frac{Pe}{Z} = \frac{M_d+M_l}{Z}-\frac{P}{A} \quad \text{...(vi)}$$

But from (i), $$\frac{P}{A}+\frac{M_d}{Z} = \frac{Pe}{Z} \quad \text{...(vii)}$$

Also from (v) $$\frac{M_l}{Z} = f_c$$

$$\Rightarrow \quad Z = \frac{M_l}{f_c} = \text{Section modulus required}$$

Select suitable value of beam width (B)

$$\therefore \quad \text{Section modulus, } Z = \frac{BD^2}{6}$$

$$\Rightarrow \quad D = \sqrt{\frac{6Z}{B}}$$

$$\therefore \quad \text{Cross-sectional area, } A = B \times D$$

Now either from (i) + (ii) or (iii) + (iv)

$$\frac{2P}{A} = f_c$$

$$\Rightarrow \quad P = \frac{Af_c}{2}$$

from (i), $$\frac{P}{A}+\frac{M_d}{Z} = \frac{Pe}{Z}$$

from (iv), $$\frac{-P}{A}+\frac{M_d}{Z}+\frac{M_l}{Z} = \frac{Pe}{Z}$$

Adding, $$\frac{2M_d}{Z}+\frac{M_l}{Z} = \frac{2Pe}{Z}$$

$$\Rightarrow \quad e = \frac{2M_d+M_l}{2P} = \text{Eccentricity of prestressing tendons}$$

Design Steps for design of prestressed concrete beam member

1. Calculate live load moment $(M_l) = \frac{w_l l^2}{8}$ (for simply supported beam)
2. Calculate section modulus, $Z = \frac{M_l}{f_c}$

3. Select any size, say B (width), $Z = \frac{BD^2}{6} \Rightarrow D = \sqrt{\frac{6Z}{B}}$
4. Area, $(A = BD)$

   Prestressing force $(P) = \frac{Af_c}{2}$
5. Calculate area of steel, $A_s = \frac{P}{f_s}$
6. Calculate dead load, $w_d = BD \times 1 \times \gamma_c (\gamma_c = 25\ kN/m^3)$
7. Calculate moment due to dead load, $M_d = \frac{w_d l^2}{8}$
8. Eccentricity, $e = \frac{2M_d+M_l}{2P}$

Case II:

Given:	Find out:
Permissible stress of concrete compression = f_c	Size of beam/slab requested
Permissible stress of concrete in tension = 0	Prestressing force
	Eccentricity

Loss of prestress = P_L%

$$k = \text{Loss factor} = \left(1-\frac{P_L}{100}\right)$$

Stress at transfer stage:

$$\text{Stress at top} = \frac{P}{A}-\frac{Pe}{Z}+\frac{M_d}{Z} = 0 \quad \text{....(i)}$$

$$\text{Stress at bottom} = \frac{P}{A}-\frac{Pe}{Z}-\frac{M_d}{Z} = f_c \quad \text{...(ii)}$$

Stress at final stage:

$$\text{Stress at top} = \frac{kP}{A}-\frac{kPe}{Z}+\frac{M_d}{Z}+\frac{M_l}{Z} = f_c \quad \text{....(iii)}$$

$$\text{Stress at bottom} = \frac{kP}{A}+\frac{kPe}{Z}-\frac{M_d}{Z}-\frac{M_l}{Z} = 0 \quad \text{...(iv)}$$

Design Formula

From equation (iii) – k. equation (i),

$$\frac{M_d}{Z} = \frac{M_l}{Z}-\frac{kM_d}{Z} = f_c$$

$$\Rightarrow \quad Z = \frac{(1-k)M_d+M_l}{f_c}$$

From equation (iii) + equation (iv)

$$\frac{2kP}{A} = f_c$$

$$\Rightarrow \quad P = \frac{f_c \cdot A}{2k} = \text{Prestressing force}$$

Multiplying equation (i) by k and rearranging,

$$\frac{kPe}{Z} = \frac{kP}{A} + \frac{kM_d}{Z}$$

From equation (iv)

$$\frac{kPe}{Z} = \frac{-kP}{A} + \frac{M_d}{Z} + \frac{M_l}{Z}$$

Adding the two above equations

$$\frac{2kPe}{Z} = \frac{(1+k)M_d + M_l}{kP}$$

$$e = \frac{(1+k)M_d + M_l}{2kP}$$

Design Steps

Assume size of beam/slab i.e. width (B) and depth (D).
Calculate

$$A = BD$$

$$\therefore \quad M_d = \frac{w_d l^2}{8} = \text{BM due to dead load}$$

$$M_l = \frac{w_l l^2}{8} = \text{BM due to live load}$$

Section modulus required.

$$Z = \frac{(1+k)M_d + M_l}{f_c}$$

Now $$Z = \frac{BD^2}{6}$$

$$\Rightarrow \quad D = \sqrt{\frac{6Z}{B}}$$

Decide B and D

$$\therefore \quad A = BD$$

Prestressing force

$$P = \frac{Af_c}{2k}$$

Eccentricity $$e = \frac{(1+k)M_d + M_l}{2kP}$$

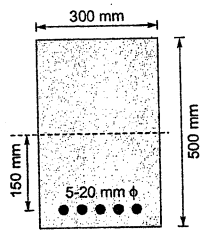
Calculate area of steel $$A_s = \frac{P}{f_c}$$

Example 14.17 A prestressed concrete beam of size 300 x 500 mm is provided with a straight cable of 20 wires of 5 mm of bars stressed at 1600 N/mm² prestress eccentricity = 150 mm. Beam is subjected an UDL of 30 kN/m over a span of 7 m. Calculate the stress in beam at (i) At ends. (ii) At mid span. (Neglect losses)

Solution:

Beam width, $B = 300\,\text{mm}$

Beam depth, $D = 500\,\text{mm}$



Area of tendons, $A_s = 20 \times \frac{\pi}{4} \times (5)^2 = 392.7\ \text{mm}^2$

Prestressing force, $P = A_s \times f_s = 392.7 \times 1600$

$= 628318.53\,\text{N} = 628.32\,\text{kN}$

Eccentricity, $e = 150\,\text{m}$

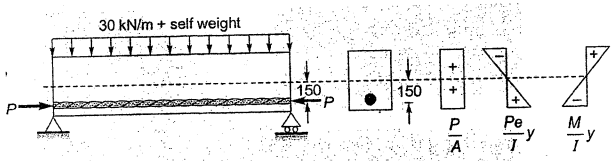
Loads

Live load = 30 kNm

Dead load = 0.3 x 0.5 x 1 x 25 = 3.75 kN/m

Total load = 33.75 kN/m

Bending moment at ends $M = 0$



Bending moment at mid span $= \frac{wl^2}{8} = \frac{33.75 \times 7^2}{8} = 206.72\ \text{kNm}$

Stresses in beam at ends:

Stress at top, $f_{top} = \frac{P}{A} - \frac{Pe}{Z} = \frac{628.32 \times 10^3}{300 \times 500} - \frac{628.32 \times 10^3 \times 150}{300 \times \frac{500^2}{6}} = -3.35\ \text{N/mm}^2$

Stress at bottom, $f_{bottom} = \frac{P}{A} + \frac{Pe}{Z}$

$= \frac{628.32 \times 10^3}{300 \times 500} + \frac{628.32 \times 10^3 \times 150}{300 \times \frac{500^2}{6}}$ (11.73 – 3.35 = 8.88)

$= 11.73\,\text{N/mm}^2$

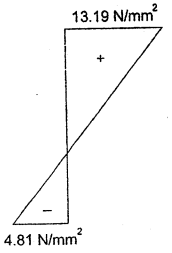
Stresses in beam at mid span:

Stress at top, $f_{top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z} = 4.19 - 7.54 + \frac{206.72 \times 10^6}{300 \times \frac{500^2}{6}}$

$= 4.19 - 7.54 + 16.54 = 13.19\,\text{N/mm}^2$

Stress at bottom, $f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$ (13.19 – 4.81 = 8.38)

$= 4.19 + 7.54 - 16.54 = -4.81\,\text{N/mm}^2$



Example 14.18 Determine the profile of a load balancing cable for a beam of size 300 x 800 mm subjected to a live load of 10 kN/m over a span of 8 m. If the beam is

(i) simply supported at ends.

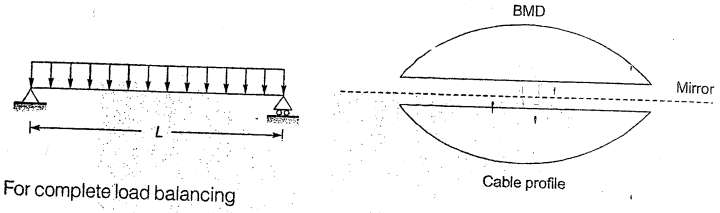
(ii) cantilever.

In both case prestressing force that can be applied = 1500 kN. Find out the shape of cable and show the position in beam. Also find the equation of cable profile in both cases.

Solution:

(i) If it is simply supported

(a) The shape of cable shall be mirror image of BM diagram as shown in figure.



(b) For complete load balancing

$$w = w_p = \frac{8Ph}{l^2}$$

Load, self weight = 0.3 x 0.8 x 25 x 1 = 6.0 kNm

Live load = 10 kNm

w = 16 kNm

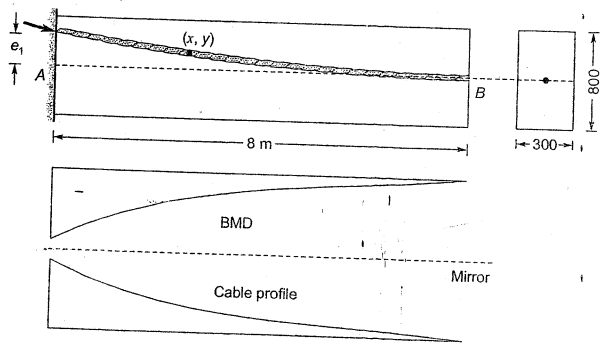
Eccentricity at centre, $h = \frac{w_p l^2}{8P} = \frac{16\times8^2}{8\times1500} = 0.08533\text{ m} = 85.33\text{ mm}$

Equation at cable (with origin at ends)

$$y = \frac{4h}{l^2}x(l-x) = \frac{4\times0.085333}{8^2}x(l-x) = \frac{x(8-x)}{187.5}m$$

(ii) Cantilever

Shape of the cable shall be as shown in figure



BM at $A = \frac{wl^2}{2}$

$$Pe_1 = \frac{wl^2}{2}$$

$$e_1 = \frac{wl^2}{2P} = \frac{16\times8^2}{2\times1500} = 0.34133\text{ m} = 341.33\text{ mm}$$

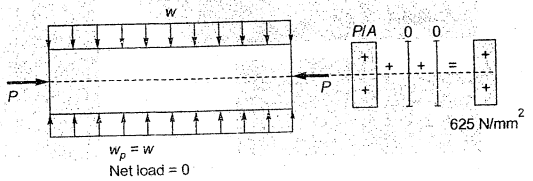
Consider any point (x, y) keeping origin at free end,

BM at point (x, y) $M = \frac{wx^2}{2}$

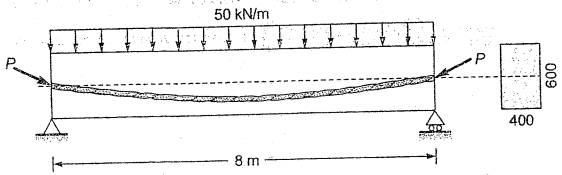
$$Py = \frac{wx^2}{2}$$

$$y = \frac{wx^2}{2P} = \frac{16x^2}{2\times1500} = \frac{x^2}{187.5} \quad ...(i)$$

Stresses in the beam in both cases $= \frac{P}{A} = \frac{1500\times10^3}{300\times800} = 625\text{ N/mm}^2$



Example 14.19 A PSC beam as shown in figure.



P = 1500 kN. The beam is subjected to a live load moment of 50 kN/m over a span = 8 m. Calculate stresses in the beam at quarter span and calculate value by three method.

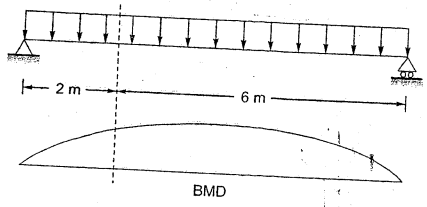
Solution:

1. Eccentricity at quarter span:

$$y = \frac{4h}{l^2}x(l-x)$$

At $x = 2$ $y = \frac{4\times0.2}{8^2}\times2(8-2) = 0.15\text{ m} = 150\text{ mm}$

2. Moment at quarter span:



Live load = 50 kN/m
Self weight = 0.40 × 0.60 × 1 × 25 = 6 kN/m
Total weight, w = 56 kN/m

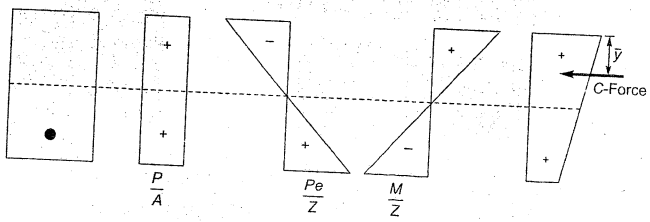
$$R_1 = R_2 = \frac{wl}{2} = \frac{56\times 8}{2} = 224\text{ kN}$$

$$M = R_1 x - \frac{wx^2}{2} = 224\times 2 - \frac{56\times 2^2}{2} = 336\text{ kNm}$$

3. Cross-sectional area, $A = 400 \times 600 = 24 \times 10^4\ \text{mm}^2$

Section modulus, $Z = \frac{BD^2}{6} = \frac{400\times 600^2}{6} = 24\times 10^6\ \text{mm}^3$

A. By stress concept method



$$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

$$\tan\theta = \frac{4\times 0.2}{8^2}\times(8-2\times 2) = \frac{1}{20}$$

$\theta = 2.86°$

$\cos\theta = 0.99875$

$P\cos\theta \approx P$



⇒ Stress at top

$$f_{top} = \frac{P}{A} - \frac{Pe_p}{(Z)} + \frac{M}{Z}$$

$$= \frac{1500\times 10^3}{400\times 600} - \frac{1500\times 10^3\times 150}{24\times 10^6} + \frac{336\times 10^6}{24\times 10^6}$$

$$= 6.25 - 9.375 + 14 = 10.875\ \text{N/mm}^2$$

Stress at bottom

$$f_{bottom} = \frac{P}{A} + \frac{Pe_p}{Z} - \frac{M}{Z} = 6.25 + 9.375 - 14 = 1.625\ \text{N/mm}^2$$

$$\text{C-force} = B\times D\left(\frac{f_1+f_2}{2}\right) = 400\times 600\left(\frac{10.875+1.625}{2}\right)\times\frac{1}{10^3}$$

$$= 1500\text{ kN}$$

$\bar{y}$ value

$$\bar{y} = \left(\frac{f_1+2f_2}{f_1+f_2}\right)\times\frac{D}{3} = \left(\frac{10.875+2\times 1.625}{10.875+1.625}\right)\times\frac{600}{3}$$

$$= 226\text{ mm}$$

Eccentricity of C-force

$$e_c = \frac{D}{2} - \bar{y} = 300 - 226 = 74\text{ mm}$$

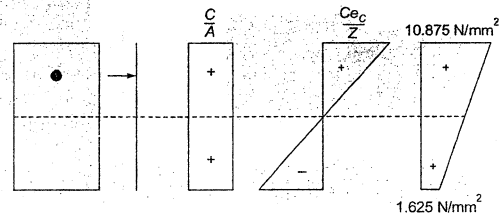
B. By strength concept method (based on F-force)

Shift of C-line w.r.t. P-line

$$x = \frac{M}{P} = \frac{336\times 10^6}{1500\times 10^3} = 224\text{ mm}$$

Eccentricity of C-line $e_c = x - e_p = 224 - 150 = 74\text{ mm}$

Stresses



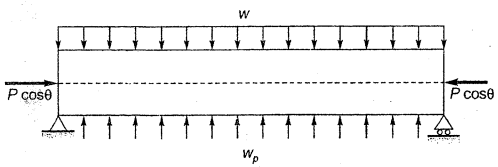
Stress at top, $f_{top} = \frac{C}{A} + \frac{C.e_c}{Z}$

$$= \frac{1500\times 10^3}{24\times 10^4} + \frac{1500\times 10^3\times 74}{24\times 10^6}$$

$$= 6.25 + 4.625 = 10.875\ \text{N/mm}^2$$

Stress at bottom, $f_{bottom} = 6.25 - 4.625 = 1.625\ \text{N/mm}^2$

C. By load balancing method



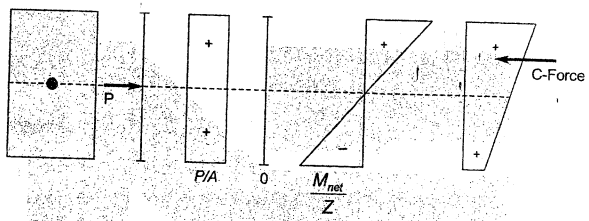
$$w_p = \frac{8Ph}{l^2} = \frac{8\times1500\times0.2}{8^2} = 37.5 \text{ kN/m}$$

$$\text{Net UDL} = w - w_p = 56 - 37.5 = 18.5 \text{ kN/m}$$

Reaction, $$R_1 = R_2 = \frac{18.5\times l}{2} = \frac{18.5\times8}{2} = 74 \text{ kN}$$

Flexural moment, $$M_{net} = R_1 x - \frac{wx^2}{2}$$

$\Rightarrow$ $$M_{net} = 74\times2 - \frac{18.5\times2^2}{2} = 111 \text{ kNm}$$



Stress at top

$$f_{top} = \frac{P}{A} + \frac{M_{net}}{Z} = \frac{1500\times10^3}{400\times600} + \frac{111\times10^6}{24\times10^6}$$
$$= 6.25 + 4.625 = 10.875 \text{ N/mm}^2$$

Stress at bottom

$$f_{bottom} = \frac{P}{A} - \frac{M_{net}}{Z} = 6.25 - 4.625 = 1.625 \text{ N/mm}^2$$

Example 14.20 For a prestress beam for pretensioned, reinforcement is provided as shown in figure.

Initial prestress = 1600 N/mm² in both wires. Calculate loss of prestress only due to elastic shortening is each wire. $E_s = 2\times10^5$ N/mm², $E_C = 0.32\times10^5$ N/mm².

Solution:

$$P_1 = 12\times\frac{\pi}{4}\times(5)^2\times1600\times\frac{1}{10^3} = 377 \text{ kN}$$

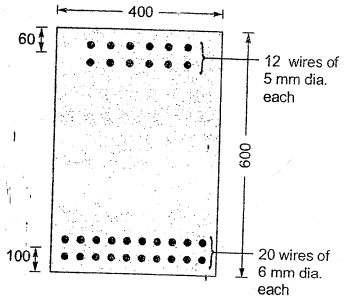
$$e_1 = 240 \text{ mm}$$

$$P_2 = 20\times\frac{\pi}{4}\times6^2\times1600\times\frac{1}{10^3} = 904.8 \text{ kN}$$

$$e_2 = 200 \text{ mm}$$

$$A = 400\times600 = 24\times10^4 \text{ mm}^2$$

$$I = \frac{400\times600^3}{12} = 72\times10^8 \text{ mm}^4$$



Stress in concrete at the location of top steel

$$f_{ct} = \frac{P_1}{A} + \frac{P_1e_1}{I}e_1 + \frac{P_2}{A} - \frac{P_2e_2}{I}e_1$$
$$= \frac{377\times10^3}{24\times10^4} + \frac{377\times10^3\times240^2}{72\times10^8} + \frac{904.8\times10^3}{24\times10^4} - \frac{904.8\times10^3\times200\times240}{72\times10^8}$$
$$= 1.57 + 3.02 + 3.77 - 6.03 = 2.33 \text{ N/mm}^2$$

$$\text{Loss of stress} = f_{ct}\times\frac{E_s}{E_c} = 2.33\times\frac{2.0\times10^5}{0.32\times10^5} = 14.56 \text{ N/mm}^2$$

Stress in concrete at the location of bottom steel

$$f_{cb} = \frac{P_1}{A} - \frac{P_1e_1}{I}e_2 + \frac{P_2}{A} + \frac{P_2e_2}{I}e_2$$
$$= \frac{377\times10^3}{24\times10^4} - \frac{377\times10^3\times240\times200}{72\times10^8} + \frac{904.8\times10^3}{24\times10^4} + \frac{904.8\times10^3\times200\times200}{72\times10^8}$$
$$= 1.57 - 2.51 + 3.77 + 5.02 = 7.85 \text{ N/mm}^2$$

$$\text{Prestress loss} = 7.855\frac{2\times10^5}{0.32\times10^5} = 49.06 \text{ N/mm}^2$$

Alternatively

$$P_1 = 377 \text{ kN}$$
$$P_2 = 904.8 \text{ kN}$$

Total force, $$P_R = P_1 + P_2 = 1281.8 \text{ kN}$$

$$\bar{y} = \frac{P_1y_1 + P_2y_2}{P_1 + P_2} = \frac{377\times60 + 904.8\times500}{1281.8} = 370.6 \text{ mm}$$

$$e_R = 370.6 - 300 = 70.6 \text{ mm}$$

$$f_{ct} = \frac{P_R}{A} - \frac{P_Re_R}{I}e_1 = \frac{1281.8\times10^3}{24\times10^4} - \frac{1281.8\times10^3\times70.6\times240}{72\times10^8}$$
$$= 5.34 - 3.01 = 2.33 \text{ N/mm}^2$$

$$\therefore \text{ Prestress loss} = 2.33\times\frac{2}{0.32} = 14.56 \text{ N/mm}^2$$

Stress in concrete at the location of bottom steel

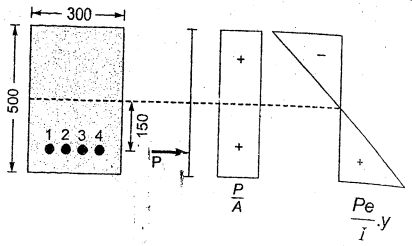
$$f_{cb} = \frac{P_R}{A} + \frac{P_Re_R}{I}e_2 = 5.34 + \frac{1281.8\times10^3\times70.60\times200}{72\times10^8} = 5.34 + 2.51 = 7.85 \text{ N/mm}^2$$

$$\text{Prestress loss} = 7.85\times\frac{2}{0.32} = 49.05 \text{ N/mm}^2$$

Example 14.21 A post-tensioned concrete beam of size 300 x 500 mm is prestressed by 4 tendons of 400 mm² each provided at an eccentricity of 150 mm. Cable are straight and initially prestress in each tendon is 1500 N/mm². Calculate loss of prestress only due to elastic shortening in wire. If

1. All wires tensioned at same time.
2. Each wires are tensioned one by one, m = 6

Solution:



1. **Case-1:** All wires tensioned at same time
   Loss due to elastic shortening = 0
2. **Case-2:** If wires are tensioned one after another

(a) In wire no. 4 (last reinforcement)

loss = 0

(b) In wire no. 3, Prestressed loss is due to P_4

Stress in concrete at the location of wire no. 3

$$f_c = \frac{P_4}{A} + \frac{P_4 e}{I} e$$

$$= \frac{400\times1500}{300\times500} + \frac{400\times1500\times150\times150}{\left(300\times\frac{500^3}{12}\right)}$$

$$= 4.0 + 4.32 = 8.32 \text{ N/mm}^2$$

Loss of stress $= mf_c$

$$= 6 \times 8.32 = 49.92 \text{ N/mm}^2$$

(c) In wire no. 2

Loss of stress in steel will be due to P_3 and P_4.

Stress in concrete at the location of wire no. 2

$$f_c = \frac{P_3 + P_4}{A} + \frac{(P_3 + P_4)e}{I} e$$

$$= 2 \times 8.32 = 16.64 \text{ N/mm}^2$$

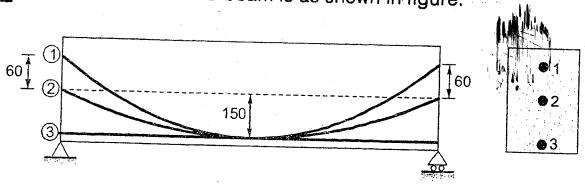
Loss of prestress

$$= 2 \times 49.92 = 99.84 \text{ N/mm}^2$$

(d) In wire no. 1

Prestress loss is due to $(P_2 + P_3 + P_4)$ $= 3 \times 49.92 = 149.76 \text{ N/mm}^2$

Example 14.22 A post-tensioned PSC beam is as shown in figure.



Size of beam = 800 × 550 mm

Area of each wire = 500 mm²

Initial prestress = 1500 N/mm²

$E_s = 2 \times 10^5$ N/mm²

$E_C = 0.35 \times 10^5$ N/mm²

Wire No. 1 tensioned first

Wire no. 3 tensioned last

Calculate loss of stress due to elastic shortening of concrete in each wire.

Solution:

In wire no. 3,

Loss of stress = 0

Wire no. 2

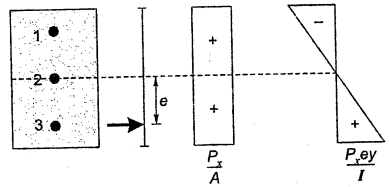
Loss of stresses in wire no. 2 is due to prestressing force in wire no. 3

Stress in concrete at the location of wire no. 2 due to prestressing force in wire no. 3

At ends:

$$f_{c1} = \frac{P_3}{A} = \frac{500\times10^3}{300\times550}$$

$$= 4.54 \text{ N/mm}^2$$



At mid span,

$$f_{c2} = \frac{P_3}{A} + \frac{P_3 e_3}{I} \times e_2$$

$$= 4.54 + \frac{500\times1500\times150\times150}{\frac{(300\times500)^3}{12}}$$

$$= 4.54 + 4.05 = 8.59 \text{ N/mm}^2$$

Average stress $f_c = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1}) = 4.54 + \frac{2}{3}(8.59 - 4.54) = 7.24 \text{ N/mm}^2$

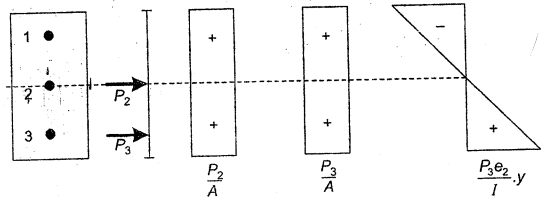
Loss of stress $= mf_c = \frac{2\times10^5}{0.35\times10^5} \times 7.24 = 41.37 \text{ N/mm}^2$

In wire no. 1

Loss of stress in wire no. 1 will be due to P_3 and P_2.

∴ Stress in concrete at the location of wire no. 1 is due to prestressing force in wire 2 and 3.

At ends

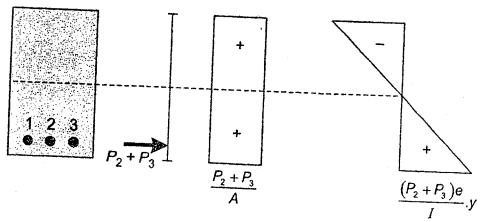


$$f_{c1} = \frac{P_2}{A} + \frac{P_3}{A} - \frac{P_3 e_3}{I} e_1$$

$$= \frac{500\times1500}{300\times550} + \frac{500\times1500}{300\times550} - \frac{500\times1500\times150\times60}{\frac{(300\times500)^3}{12}}$$

$$= 4.54 + 4.54 - 1.62 = 7.46 \text{ N/mm}^2$$

At mid span



$$f_{c2} = \frac{P_2+P_3}{A}+\frac{(P_2+P_3)}{I}e_1$$

$$= \frac{2\times500\times1500}{550\times300}+\frac{2\times500\times1500\times150^2}{\frac{300\times550^2}{12}} = 9.09 + 8.11 = 17.2\ \text{N/mm}^2$$

Average stress

$$f_c = f_{c1}+\frac{2}{3}(f_{c2}-f_{c1}) = 7.46+\frac{2}{3}(17.2-7.45) = 13.95\ \text{N/mm}^2$$

Loss of stress

$$= \frac{2\times10^5}{0.35\times10^5}\times13.95 = 79.73\ \text{N/mm}^2$$

Example 14.23 Design a beam simply supported over a span of 20 m subjected to a load of 80 kN/m (live load). The permissible stress in concrete in compression is 16 N/mm² and in tension it is zero. Loss of prestress is 15%, permissible stress in steel is 1500 N/mm². Design PSC beam and show the reinforcement. Use straight cable.

Solution:

Step-1: Assume size of beam

Width = 500 mm

Depth = 800 mm

Self weight $w_d = 0.5\times0.8\times1\times25 = 10\ \text{kN/m}$

$$M_d = \frac{10\times20^2}{8} = 500\ \text{kNm}$$

Live load moment $M_l = \frac{w_l\times l^2}{8} = \frac{30\times20^2}{8} = 1500\ \text{kNm}$

Step-2: Section modulus required

$$Z = \frac{(1-k)M_d+M_l}{f_c}$$

$$k = \left(1-\frac{P_L}{100}\right) = \left(1-\frac{15}{100}\right) = 0.85$$

$$Z = \frac{(1-0.85)500\times10^6+1500\times10^6}{16} = 98.44\times10^6\ \text{mm}^3$$

Depth required, $D = \sqrt{\frac{6Z}{B}} = \sqrt{\frac{6\times98.44\times10^6}{500}} = 1086.86\ \text{mm}$

Consider $D = 1100\ \text{mm}$

$B = 500\ \text{mm}$

$w_d = 0.5\times1.1\times1\times25 = 13.75\ \text{kN/m}$

$$M_d = \frac{13.75\times20^2}{8} = 687.5\ \text{kNm}$$

Section modulus (Z) required.

$$Z = \frac{(1-0.85)687.5\times10^6+1500\times10^6}{16} = 100.2\times10^6\ \text{mm}^3$$

$$D = \sqrt{\frac{6Z}{B}} = 1096\ \text{mm}$$

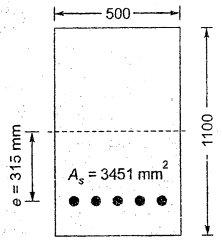
So, $D = 1100\ \text{mm}$, $B = 500\ \text{mm}$

Step-3: Cross-sectional area,

$$A = B\times D = 500\times1106 = 55\times10^4\ \text{mm}^2$$

Prestressing force,

$$P = \frac{A\times f_c}{2k} = \frac{55\times10^4\times16}{2\times0.85} = 5176.5\ \text{kN}$$



Step-4: Area of steel, $A_s = \frac{P}{f_s} = \frac{5176.5\times10^3}{1500} = 3451\ \text{mm}^2$

Step-5: Eccentricity, $e = \frac{(1+k)M_d+M_l}{2kP}$

$$= \frac{(1+0.85)\times687.5\times10^6+1500\times10^6}{2\times0.85\times5176.5\times10^3}$$

$$= 315\ \text{mm}$$

Example 14.24 A prestressed concrete rectangular beam of 250 mm × 600 mm is prestressed by four 16 mm dia. high tensile bars located at 225 mm from the soffit of beam. If the effective stress in the bars is 690 N/mm². What is the maximum moment that can be applied without inducing any tension in the beam?

Solution:

Area cross-section of beam $= 250\times60 = 15\times10^4\ \text{mm}^2$

Section modulus of the beam cross-section (z) $= \frac{250\times600^2}{6} = 15\times10^6\ \text{mm}^4$

Area of prestressing bars (A_s) $= 4\times\frac{\pi}{4}\times16^2 = 804.25\ \text{mm}^2$

Eccentricity (e) $= 300-225 = 75\ \text{mm}$

$\therefore$ Prestressing force (P) $= 690\times804.25\ \text{N} = 554.93\ \text{kN} \simeq 555\ \text{kN}$

Direct stress $= \frac{P}{A} = \frac{555\times10^3}{15\times10^4} = 3.7\ \text{N/mm}^2$

Flexural stress $= \mp2.775\ \text{N/mm}^2$

$\therefore$ Maximum stress at the bottom of beam $= \frac{P}{A}+\frac{Pe}{Z} = 3.7+2.775 = 6.475\ \text{N/mm}^2$

External loading will induce tensile stresses in the bottom of beam.

Let M = Moment induced due to external loading

∴ To have zero stress (no tension) any where in the beam cross-section,

$$\frac{M}{Z} = 6.475$$

$$\Rightarrow \quad M = 6.475\,Z = 6.475 \times 15 \times 10^6 \text{ Nmm} = 97.125 \text{ kNm}$$

= Max. moment that can be induced due to external loading

Example 14.25 A prestressed concrete beam of size 225 mm × 325 mm is used for an effective span of 5.5 m. It supports a super-imposed load of 4.2 kN/m. At the mid-span section of the beam find.

(a) the concentric prestressing force necessary to have zero fibre stress at the bottom of beam when the beam is fully loaded.

(b) the eccentric prestressing force located at 115 mm from the beam bottom which will totally counteract the stresses due to external loading.

Solution:

Cross-section of beam cross-section (A) = $225 \times 325 \text{ mm}^2 = 73125 \text{ mm}^2$

Section modulus of the beam cross-section (Z) = $225 \times \frac{325^2}{6} = 3.961 \times 10^6 \text{ mm}^3$

Self weight of beam = $0.225 \times 0.325 \times 25 \text{ kN/m} = 1.828 \text{ kN/m}$

∴ Moment due to $DL = M_D = 1.828 \times \frac{5.5^2}{8} = 6.91 \text{ kNm}$

Moment due to line load = $M_L = 4.2 \times \frac{5.5^2}{8} = 15.88 \text{ kNm}$

∴ Tensile stress at the bottom of beam due to dead and live load = f_{bt}

$$= \frac{M_D}{Z} + \frac{M_L}{Z} = \frac{(6.91 + 15.88)10^6}{3.961 \times 10^6} = 5.75 \text{ N/mm}^2$$

(a) Let P_1 = concentric prestressing force for zero stresses at the beam bottom

$$= \frac{P_1}{A} = 5.75 \text{ N/mm}^2$$

$$\Rightarrow \quad P_1 = 5.75 \times 73125 \text{ N} = 420.5 \text{ kN}$$

(b) Let P_2 = eccentric prestressing force for zero stresses at the beam bottom

Eccentricity (e) = $\frac{325}{2} - 115 = 47.5 \text{ mm}$

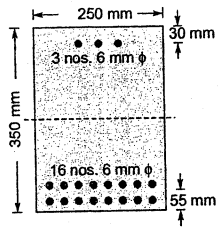
$$\therefore \quad \frac{P_2}{A} + \frac{P_2 e}{Z} = 5.75 \text{ N/mm}^2$$

$$\Rightarrow \quad P_2\left[\frac{1}{73125} + \frac{47.5}{3.961 \times 10^6}\right] = 5.75$$

$$\Rightarrow \quad P_2 = 224 \text{ kN}$$

Example 14.26 A prestressed concrete beam of size 250 × 350 mm is prestressed by 16 wires of 6 mm diameter located at a distance of 55 mm from the bottom of beam, and 3 wires of 6 mm diameter at a distance of 30 mm from the top. If prestress in the steel is 850 MPa, find the stresses at the extreme fibres of mid-span section if the beam spans over a distance of 6 m. The imposed live load on the beam is 5 kN/m.

Solution:



Gross area of beam cross-section (A) = $250 \times 350 = 87500 \text{ mm}^2$

Section modulus of the beam cross-section (Z) = $250 \times \frac{350^2}{6}$

= $5.104 \times 10^6 \text{ mm}^3$

Moment of inertia of the beam cross-section (I) = $250 \times \frac{350^3}{12}$

= $893.33 \times 10^6 \text{ mm}^4$

Self weight of beam = $0.25 \times 0.35 \times 25 = 2.1875 \text{ kN/m}$

Moment due to self weight of beam = $2.1875 \times \frac{6^2}{8} = 9.84 \text{ kNm}$

Moment due to line load on beam = $5 \times \frac{6^2}{8} = 22.5 \text{ kNm}$

Distance at point of application of prestressing force from the bottom of beam

$$= \frac{(16 \times 55) + (3 \times 320)}{19} = 96.842 \text{ mm}$$

∴ Eccentricity, $e = \frac{350}{2} - 96.842 = 78.158 \text{ mm}$

Pre-stressing force (P) = $(850 \text{ N/mm}^2) \times (19 \text{ wires}) \times \left(\frac{\pi}{4} \times 6^2\right) = 456.63 \text{ kN}$

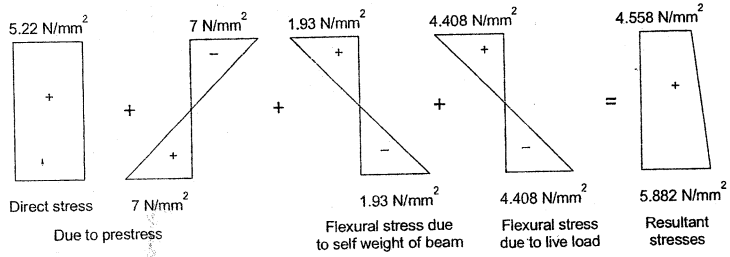
Direct stress = $\frac{P}{A} = \frac{456.63 \times 1000}{87500} = 5.22 \text{ N/mm}^2$

Flexural stress = $\mp \frac{456.63 \times 1000 \times 78.158}{5.104 \times 10^6} \mp 6.99 \text{ N/mm}^2$

Flexural stress due to prestressing = $\mp 6.99 \text{ N/mm}^2 \simeq \mp 7 \text{ N/mm}^2$

Flexural stresses due to beams self weight = $\pm \frac{9.84 \times 10^6}{5.104 \times 10^6} = \pm 1.93 \text{ N/mm}^2$

Flexural stresses due to line load = $\pm \frac{22.5 \times 10^6}{5.10^4 \times 10^6} = \pm 4.408 \text{ N/mm}^2$



Example 14.27 Design a prestressed concrete beam of rectangular section for a span of 12 m to support a total UDL of 280 kN excluding the self weight of beam. The permissible stresses for concrete and steel are 15 N/mm² and 1100 N/mm² respectively.

Solution:

Assume zero loss of prestress i.e. $k = 1$

Flexural moment due to superimposed load $(M_l) = 280 \times \frac{12}{8} = 420$ kNm

$\therefore$ Section modulus required $(Z_{reqd.}) = \frac{M_l}{f_c} = \frac{420 \times 10^6}{15} = 28 \times 10^6$ mm³

Let depth of beam $= \frac{1}{20}$ for span $= \frac{12000}{20} = 600$ mm

and let width of beam $= b$

$\therefore \quad \frac{b(600)^2}{6} = 28 \times 10^6$

$\Rightarrow \quad b = 466.67$ mm $\simeq 470$ mm (say)

$\therefore$ Beam size is 470 mm × 600 mm

Prestressing force, $P = \frac{f_c}{2} A = \frac{15}{2}(470 \times 600) = 2115$ kN

$\therefore$ Area of tendons required, $A_s = \frac{2115 \times 10^3}{1100} = 1922.7$ mm²

If 8 mm dia. bars are used.

Then number of bars required $= \frac{1922.7}{\frac{\pi}{4}(8)^2} = 38.25 \simeq 40$ bars (say)

$\therefore$ Provide 40 bars of 8 mm diameter.

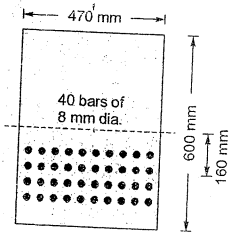
Self weight of beam = 0.47 × 0.6 × 25 = 7.05 kN/m

$\therefore$ Dead load moment $(M_d) = 7.05 \times \frac{12^2}{8} = 126.9$ kNm

$\therefore$ Eccentricity, $e = \frac{2M_d + M_l}{2P} = \frac{2(126.9) + 420}{2 \times 2115}$

$= 0.15929$ m $= 159.29$ mm

$\simeq 160$ mm (say)



Objective Brain Teasers

Q.1 While going for a pre-tensioning system
(a) the member is prestressed by external anchors
(b) member is casted prior to tensioning of high tensile tendons/wires.
(c) the high tensile wires are tensioned prior to placing of concrete
(d) None of the above

Q.2 The advantage of concentric tendons over eccentric tendons are that
(a) it induces uniform compressive stress over the entire section of beam
(b) it induces uniform tensile stress over the entire section of beam
(c) in induces linearly varying stress
(d) it induces parabolically varying stress

Q.3 Loss of prestress due to concrete shrinkage depends on
(a) shrinkage strain
(b) modulus of elasticity of concrete
(c) modular ratio
(d) concrete stress at the level of steel

Q.4 Which of the following cable profile effectively counter balances the uniformly distributed load on the beam?
(a) eccentric cable (b) parabolic cable
(c) concentric cable (d) All of the above

Q.5 High tensile bars with thread at ends are employed in which of the following prestressing system?
(a) Gifford-Udall system
(b) Lee-McCall system
(c) Freyssinet system
(d) Mangel-Blaton system

Q.6 In the system of pre-stressing by post-tensioning
(a) the tensioning wires are stretched prior to placement of concrete
(b) the wire are tensioned against the hardened concrete
(c) concrete is compressed after setting
(d) None of the above

Q.7 Loss of prestress due to stress relaxation in steel depends on
(a) initial prestress in steel
(b) coefficient of friction in between the steel and concrete
(c) amount of shrinkage in concrete
(d) amount of elongation in steel

Q.8 Which of the following prestressed beam has the highest moment of resistance?
(a) post-tensioned with unbounded wires
(b) pre-tensioned beam
(c) post-tensioned with bonded wires
(d) data insufficient

Q.9 The advantages of providing concrete beam with sloping a curved cables is that
(a) it increases the moment of resistance of the beam
(b) it decreases the shrinkage cracks
(c) it increases the torsional strength of beam
(d) it increases the shear strength of the beam

Q.10 The tensile stress in steel at the end face of a pre-tensioned beam is
(a) the minimum (b) the maximum
(c) zero (d) can't say

Q.11 Transmission length as per IS 1343 is
(a) 100ϕ (b) 30ϕ
(c) 60ϕ (d) 45ϕ
where ϕ = dia. of strand.

Q.12 Short term deflection of prestressed concrete beam is estimated by
(a) Mohr's theory (b) Rankine's theory
(c) Euler's theory (d) Hooke's law

Q.13 Stress transfer in pre-tensioned beam occurs dueto
(a) shear resistance
(b) flexural resistance
(c) bearing resistance
(d) bond between steel and concrete

Answers

1. (c)	2. (a)	3 (a)	4. (b)	5. (b)
6. (b)	7. (a)	8. (b)	9. (d)	10. (c)
11. (b)	12. (a)	13. (d)		

Conventional Practice Questions

Q.1 A pre-tensioned beam of size 250 mm x 300 mm is pre-stressed by twelve wires each of 7 mm diameter. All the wires are stressed initially to 1200 N/mm² with their centroids located at 100 mm from the soffit of beam. Compute the final percentage loss in stress due to elastic deformation, creep, shrinkage and relaxation. Take relaxation of steel stress as 90 N/mm², E_s as 210 kN/mm², E_c as 35 kN/mm², creep coefficient as 1.6 and residual shrinkage strain as 0.0003.

Ans. [22%]

Q.2 A double over hang concrete beam has a mid-span of 10 m with over hangs of 2.5 m each. Find the profile of the pre-stressing cable for an effective pre-stressing force of 250 kN which can nullify the load effect of 8 kN/m uniformly distributed on the beam.

Ans. [$e_{support}$ = 100 mm, $e_{mid\text{-}span}$ = 300 mm]

Q.3 A rectangular concrete beam of 350 mm x 700 mm size and length 10 m supports a uniformly distributed load of 20 kN/m excluding the self-weight of beam. If a 200 mm eccentricity is to be maintained at the mid span portion of the beam, find the pre-stressing force and cable profile to support dead and live loads.

Ans. [Prestressing force = 1632 kN]

Q.4 A post tensioned cable of a beam with 24 parallel cables of total area 800 mm^2 is tensioned with two wires at a time. The cable is a circular curve with zero eccentricity at the supports and 150 mm at mid span. The span and size of the beam respectively are 10 m and 200 mm x 450 mm. The wires are stressed to a value of f_1 to overcome friction and then released to a value of f_2 such that immediately after anchoring, the initial pre stress is 840 N/mm^2. Find f_1 and f_2 and final design stress in steel after all losses. Take coefficient of friction for curvature as 0.6, friction coefficient for wave effect as 0.003 per m, deformation and anchorage slip as 1.25 mm, E_s as 210 kN/mm^2, concrete shrinkage as 0.0002, and stress relaxation of steel as 3%.

Ans. [f_1 = 954 N/mm^2, f_2 = 866 N/mm^2, 668 N/mm^2]

CHAPTER 15

Earthquake Resistant Design of Structures

CODE REFERENCE

IS Number	Title
1893 : 2002 (Part I)	Criteria for earthquake resistant design of structures. (General provision and buildings)
456 : 2000	For plain and reinforced concrete
13920 : 1993	Ductile detailing of reinforced concrete structures subjected to seismic forces
4326 : 1993	Earthquake resistant design and construction of buildings.
13935 : 2009	Seismic evaluation, repair and strengthening of masonry buildings.

A. Seismology

- Seismology is the study of the generation, propagation and measurement of seismic waves through the surface of earth and the sources that produce them.
- The study of seismic waves through the earth surface gives the best understanding of the internal structure of earth.

15.1 Earthquake

- Earthquake is the vibration of earth's surface caused by waves coming from a source of disturbance inside the earth.

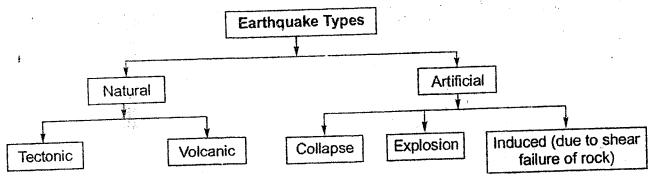


Fig. 15.1

- Tectonic earthquake occurs when the earth crust breaks due to geological forces on rocks and adjoining plates.
- Most of earthquakes are tectonic in nature.
- Volcanic earthquake results from tectonic forces in conjunction with volcanic activity.
- Collapse earthquake occurs in underground caverns and mines due to explosion of rock on the surface.
- Explosion earthquake is due to detonation of nuclear or chemical device.
- Induced seismicity occurs in many ways. Injecting or extracting fluid from the earth such as oil and gas extraction and geothermal energy development has been found to cause seismic events.

Do You Know?

- Sometimes earthquake is classified as interplate and intraplate earthquake. Interplate earthquake occurs at the boundary of two tectonic plates (majority of earthquakes are of this type).
  Examples: North America, Himalayas and Japan.
- An intraplate earthquake occurs in the interior of a tectonic plate. These are relatively rare compared to interplate.
  Example: Bhuj (Gujarat) earthquake, 2001 and Indian ocean earthquake, 2012.

15.2 Internal Structure of Earth

- The shape of earth is oblate spheroid having equatorial and polar diameter of 12740 km and 12700 km respectively.
- The interior of earth mainly consists of three zones namely crust, mantle and core.

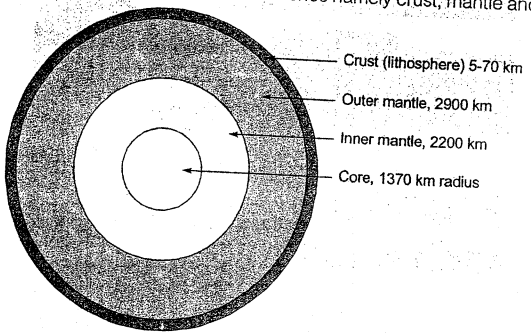


Fig. 15.2 Cross-section of interior of earth

Do You Know?

- Crust beneath continents is about 40 km comprising granite rocks overlying the basaltic rocks. Whereas oceanic crust thickness is 5 km comprising of basaltic rocks.
- Mantle consists of upper mantle and lower mantle. Upper mantle comprises of olivine and pyroxene and lower mantle comprises of magnesium, iron oxide and quartz.
- The mantle's average temperature is about 2200°C and it is in the form of viscous semi molten state.
- Core consists of inner and outer core. The inner solid core comprising very dense nickel-iron material under very high pressure.

15.3 Fault

- Fault is a fracture or zone of fractures along which relative movement of blocks had happened in past.

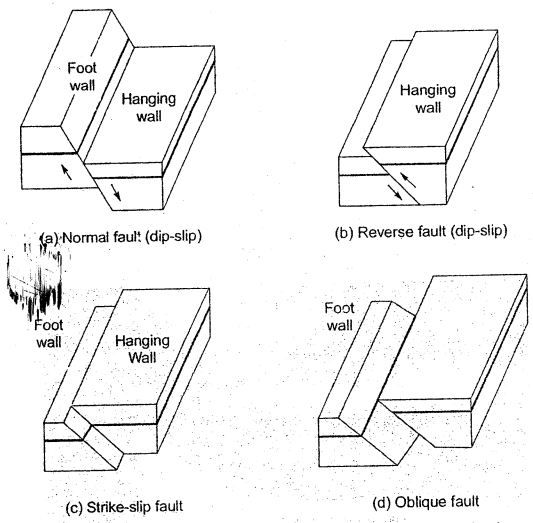


Fig. 15.3 Types of faults (Arrow shows direction of relative displacement)

- Normal faults form when the hanging wall drops down, relative to foot wall.
- Reverse faults form when the hanging wall moves up, relative to ward.
- Strike-slip faults (transcurrent) have walls that move sideways.
- Oblique-slip faults have walls movements diagonally across the fault plane.

Note: Forces that create normal and reverse faults are extensional and compressional respectively.

15.4 Elastic Rebound Theory

- Strain energy gets built-up along the boundaries of plates when plates try to move relative to each other. Thus when increasing stress reaches the ultimate strength of rock, the rock fractures and accumulated strain energy is released.
- In strong and brittle material sudden rupture occurs. Sudden fracture releases the stored energy in form of elastic stress waves resulting vibratory motion that is earthquake.

15.5 Earthquake Terminologies

- Focus or hypocenter is the region on the faults where rupture initiates i.e., the place or origin of earthquake.
- Epicentre is the point on the earth surface, vertically above the focus, the destruction caused by earthquake is maximum near epicentre, decreases with increasing distance from this point.
- Distance between focus to epicenter is focal length.

- The point's on earth's surface diametrically opposite to the epicentre is called anti-centre.

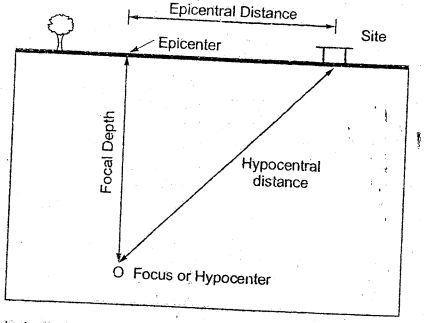


Fig. 15.4 Various distance measurements associated with earthquake

NOTE

- Most of damaging earthquakes are shallow focus earthquake.
- Earthquake are classified into shallow focus, intermediate focus and deep focus earthquake based on its focal depth.

15.6 Measurement of Earthquake

15.6.1 Earthquake Intensity

- The earthquake intensity at a place is a measure of the strength of shaking and extent of damage caused during the earthquake.
- Popular methods of measuring earthquake intensity are Modified-Mercalli intensity (MMI) scale and Mendvedev-Spoonheuer-Karnik (MSK) scale.
- An imaginary line joining the points of same intensity is known as isoseismal.

Note: Intensity at a place also depends upon soil type, type of construction, magnitude and duration of earthquake, depth of focus and distance from the epicenter.

15.6.2 Earthquake Magnitude

- Magnitude of earthquake is a measure of energy released by an earthquake.
- It is measured by Richter magnitude scale.

Table 15.1: Increase in energy release for various range of increase in value of magnitude

Increase in Magnitude	Increase in Energy Release
0.2	2 Times
0.447	5 Times
0.67	10 Times
1	31.6 Times
2	1000 Times

15.7 Seismic Zone Map, 2002

- India is classified into four seismic zones that is zone II, zone III, zone IV and zone V.

About **59 percent** of the land area of India is liable to seismic hazard damage

Zone	Intensity
Zone V	**Very High Risk Zone** Area liable to shaking Intensity IX (and above)
Zone IV	**Very Risk Zone** Intensity VIII
Zone III	**Moderate Risk Zone** Intensity VII
Zone II	**Low Risk Zone** VI (and lower)

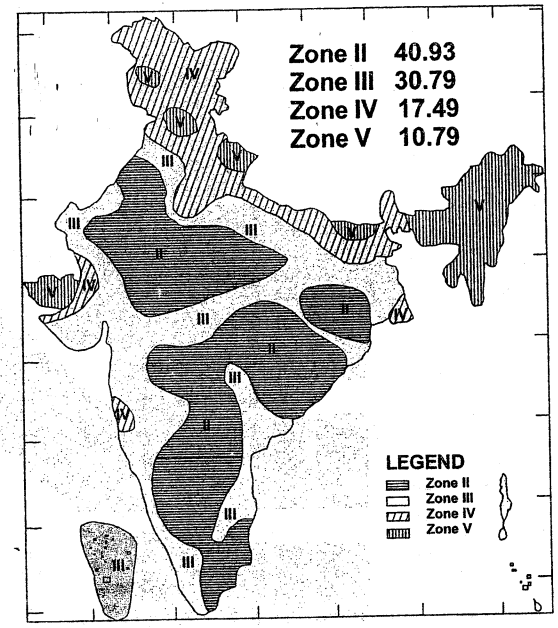


Fig. 15.5 Seismic zonation and intensity map of India

B. Ground Motion Characteristics

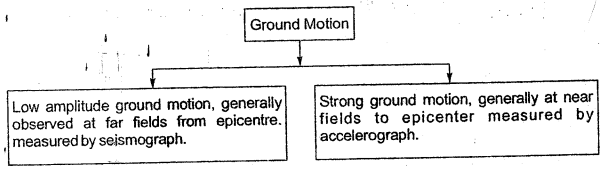


Fig. 15.6

Note: When frequency of instrument is very high compared to that of the vibration to be measured, the instrument picks up acceleration of the motion hence accelerometer have high natural frequency.

15.8 Strong Motion Characteristics

- Earthquake measurement is done by measuring ground displacement by seismograph instrument and recorded data is known as seismogram.

- Ground acceleration can be measured by accelerograph and recorded data is known as accelerogram. This is usually preferred for strong ground motion.
- The ground velocity and displacement can be obtained by direct integration of an accelerogram.

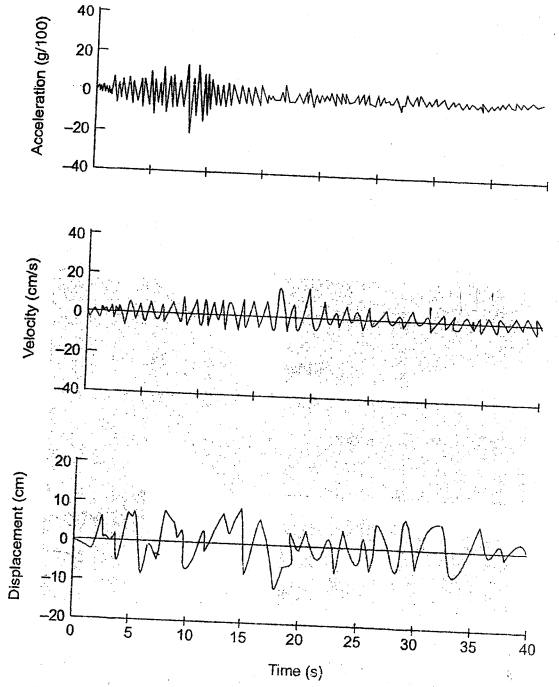


Fig. 15.7 Example of time based earthquake strong motion record

15.9 Strong Motion Seismology Terminology

15.9.1 Amplitude Parameter

- These are peak values of ground acceleration, velocity or displacement, which gives an idea of the severity of shaking of the ground, predominant period of vibration etc.

  Peak Velocity: Peak velocity is the largest absolute value of velocity time history. It is more sensitive to the intermediate frequency components of motion.

  Peak displacement: Peak displacement reflect the amplitude of lower frequency components in ground motion.

Note: Estimation of these parameters is difficult as the errors in signal processing and numerical integration greatly affect the estimation of amplitude.

15.9.2 Peak Acceleration

- Peak horizontal acceleration is the largest absolute value of acceleration time history recorded at site during earthquake and it is primarily used to report the ground motion of structures and intensity of earthquake.

Note: Duration of excitation is also an important criteria to judge destruction potential of earthquake.

15.9.3 Design Acceleration Spectrum

- Design acceleration spectrum refers to an average smoothening plot of maximum acceleration as a function of frequency or time period of vibration for a specified damping ratio for earthquake excitations at the base of a single degree of freedom systems (SDFS).

15.9.4 Effective Peak Ground Acceleration (EPGA)

- It is 0.4 times the 5 percent damped average spectral acceleration between period 0.1 to 0.3s. This can be taken as zero period acceleration (ZPA), as per IS 1893 : 2002.

15.9.5 Zero Period Acceleration (ZPA)

- It is the value of acceleration response spectrum for zero natural period (period below 0.03 s) so that it can be assumed that there is no relative motion between structure and ground.

15.9.6 Design Basis Earthquake (DBE)

- It is the earthquake which is expected to occur atleast once during the design life of the structure.

15.9.7 Maximum considered Earthquake (MCE)

- The most severe earthquake considered by IS 1893 : 2002.

15.9.8 Response Spectrum

- The representation of the maximum response of idealized single degree freedom systems (SDFS) of different fundamental time periods and same damping, during earthquake ground motion.
- The maximum response is plotted against the undamped natural period and for various damping values and can be expressed in terms of spectral acceleration, spectral velocity or spectral displacement.
- Response spectrum are used to report interaction between ground acceleration and structural systems. It reflects frequency content, amplitude of ground motion and effect of subsequent behaviour of the strcuture.

15.9.9 Design Spectrum

- Seismic design of structure is generally carried out be equivalent static forces evaluated from the maximum acceleration response of structures during earthquake.
- Currently, there is no accurate method to predict the ground motion for future earthquake. Thus design response spectrum is prepared by incorporating the spectra for various earthquake whose datas are already available.
- Design spectrum is the specification for the required structure strength. It is the response spectrum illustrating the level of the seismic resistance, required for design.

C. Loads

- Structures are subjected to two types of loading that is static and dynamic.
- Considering dynamic loading in analysis makes the solution complicated and time consuming.

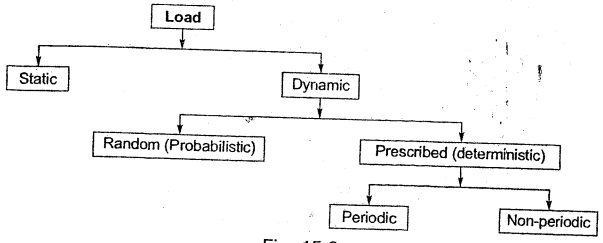


Fig. 15.8

D. Conceptual Design

- The designer should have thorough knowledge of the form, shape, material to be used, functional planning and cost requirements, for safety against critical failure during an earthquake.
- Conceptual planning, functional planning and the framing systems improves overall performance of the structure during an earthquake.

15.10 Continuous Load Transfer Path

- Earthquake induced vertical inertia forces are generally small. Hence lateral inertia forces are of prime concern.
- There should be one or more (preferable) continuous load path with sufficient strength, stiffness and ductility from the origin of initial load manifestation to the final lateral load resisting elements.
- Structures having more than one load path exhibits redundancy.
- Continuous load path improves the performance of structure under earthquake.

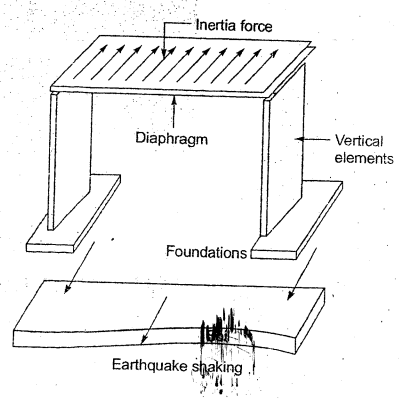


Fig. 15.9 Flow of seismic inertia forces through the structural components

15.10.1 Sequence of Lateral Load Transfer Path

- Majority of structural mass is concentrated at the floor level. Thus maximum inertia forces generates at floors. These are resisted by the diaphragm (horizontal systems such as RC floors and horizontal bracing systems). Then these inertia forces are transmitted to the vertical structural system such as braced frame, moment resisting frame and shear walls; that is in simple words, to vertical elements such as walls and columns. Finally then forces are transferred to foundation system and then to the ground and connection between them should be designed to safely transfer these inertia forces through them.

NOTE

- Seismic horizontal forces are directly proportional to masses of structural elements.
- Seismic forces generated at different floor levels of a structure, are transferred to ground by the shortest possible path.
- Earthquake inertia forces tends to be greater with increase in height.
- Walls and column are most critical elements in transferring the inertia forces.

15.11 Symmetric and Uniform Structure

- For good performance during an earthquake, structure should posses four main attributes, namely.
  (i) Simple and regular configuration
  (ii) Adequate lateral strength
  (iii) Stiffness
  (iv) Ductility
- Building possessing simple regular geometry and uniformly distributed mass and stiffness in plan as well as in elevation, suffers lesser damage than structure with irregular configuration.
- Structure should have simple rectangular, square, circular or octagonal plan.
- Structure should be symmetric so that centres of mass and rigidity coincide with each other. Otherwise structure will undergo torsion.
- Structure having parts of different rigidities can be separated through crumple sections.

Do You Know?

1. **Separation section:** A gap of specified width between adjacent buildings or parts of the same building, either left uncovered or covered suitably to permit movement in order to avoid hammering due to earthquake.
2. **Crumple Section:** A separation section filled with appropriate material which can crumple or fracture in an earthquake.

- The length of such building between separation sections should not exceed three times the width.

NOTE

Alternative to separation section to reduce torsional moments, the centre of rigidity of the building may be brought close or coincident to the centre of mass by adjusting the locations and sizes of columns and walls.

- Building having plans with shapes like L, T, E and Y should be separated into rectangular parts by providing separation sections at appropriate places.

NOTE

- Building having plan shapes like L, T, E and Y need not to be separated, if the projection does not exceed 15 to 20 percent of the total dimension of the building in the direction of the projection.
- For minor asymmetrical building in plan and elevation, separation section may be omitted.

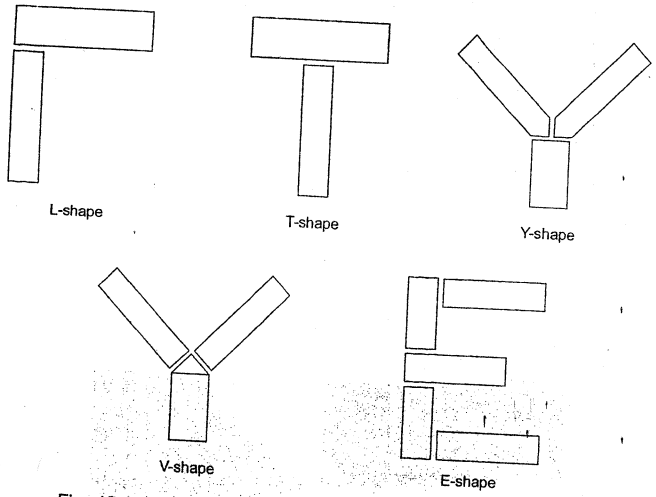


Fig. 15.10 Typical shapes of building with separation sections

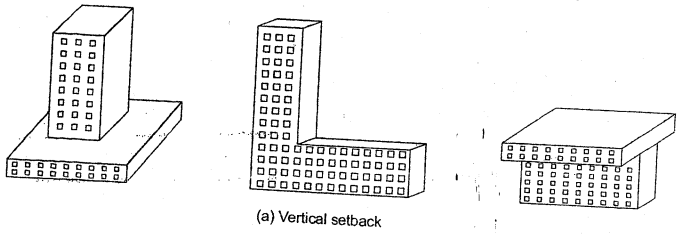
- Too much elongated or tall structures do not perform well during earthquake.

15.12 Stiffness and Strength

- Building having uniform distribution of strength and stiffness in plan and elevation performs well during earthquake.
- Strength and stiffness is a measure of resistance against forces and displacement respectively.

NOTE

- If two elements possessing different stiffness, are subjected to a force, then stiffer element will carry more of the total force because it takes more force to deflect it by the same amount as the element with lesser stiffness.
- Sudden changes in stiffness, strength and mass in either vertical and horizontal planes of a structure creates difficulty in determining earthquake stresses through computerized analysis and problem in detailing of the structure.



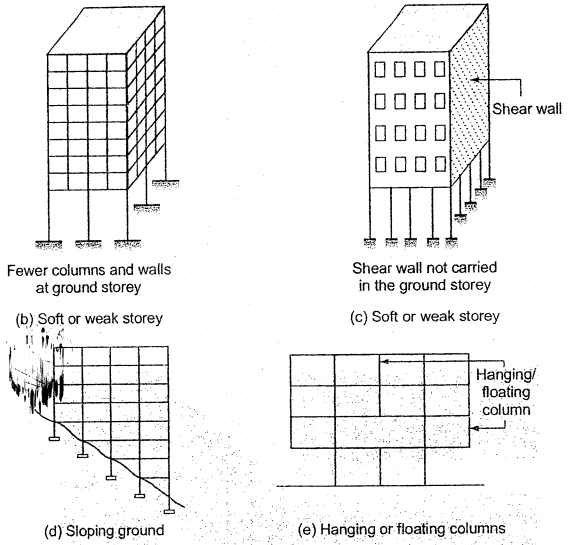


Fig. 15.11 Discontinuation in vertical configuration of buildings (not recommended)

Note: Shear wall is designed to resist lateral forces acting in its own plane.

Do You Know?

Soft storey is that storey whose lateral stiffness is less than 70 percent of that in the storey above or less than 80 percent of the average lateral stiffness of the three storeys above.
Weak storey is that storey whose lateral strength is less than 80 percent of that in the storey above.

15.13 Horizontal and Vertical Members

- There are two arrangements possible in a structure that is weak-beam-strong-column and strong-beam-weak column arrangement.

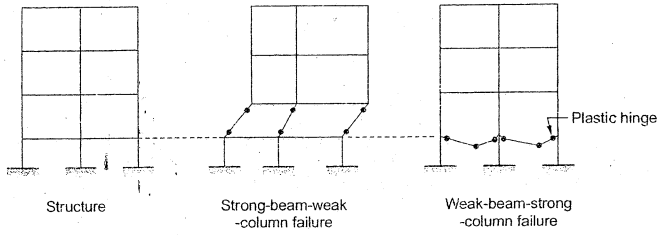


Fig. 15.12 Type of failure of structures

- Designers always opt for weak-beam-strong-column arrangement as it does not cause complete collapse as the case with strong-beam-weak-column arrangement.
- Weak-beam-strong-column arrangement insures safety of life and property outside the building, as during earthquake people will run outside the building and nearby earthquake resistant structures such as hospitals etc. will be safe from the partial inside collapse of the building.

15.14 Twisting of Buildings

- Torsion is due either to eccentricity in the building layout, when centre of mass of the structure does not coincide with the centre of rigidity or structure loaded unequally in plan.
- During torsion, structures rotates about centre of rigidity.
- The twist induces greater damage in the columns and walls on the side that moves greater.

Note:
- Centre of mass is the point through which the resultant of masses of system acts.
- The point through which resultant of restoring forces of system acts is called centre of stiffness.

15.15 Ductility

- The higher the value of ductility, the greater is the chances of sustaining seismic forces by the structure without collapse.
- Figure shown below illustrate the relationship between lateral design forces for an elastic structure and for a yielding ductile structure. Greater design forces are required for an elastic structure without ductility.

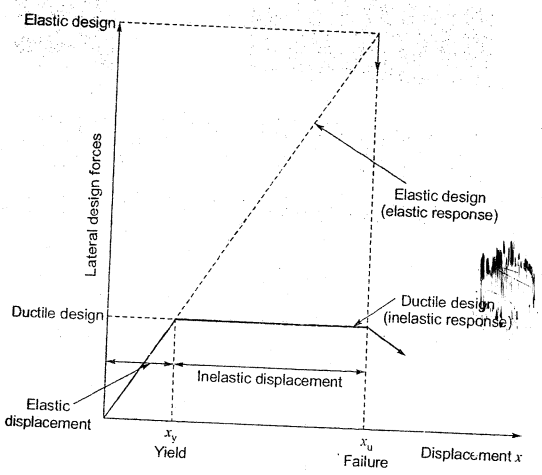


Fig. 15.13 Lateral forces and ductility

15.16 Lateral Load Resisting Elements

- It is part of the structural system assigned to resist lateral forces.

15.16.1 Moment Resisting Frame

- Frame whose members and joints are capable of resisting forces primarily by flexure.

15.16.1.1 Ordinary Moment Resisting Frame

- Moment resisting frame not meeting special detailing requirements for ductile behaviour.

15.16.1.2 Special Moment Resisting Frame

- Moment resisting frame specially detailed to provide ductile behaviour and comply with recommendations of IS 4326 or IS 13920 or SP6(6).

E. Design Earthquake Loads

15.17 Design Horizontal Earthquake Load

- When the lateral load resisting elements are oriented along orthogonal horizontal direction, the structures should be designed for the effects due to full design earthquake load in one horizontal direction at time.
- When the lateral load resisting elements are not oriented along the orthogonal horizontal directions, the structure should be designed for the effects due to full design earthquake load in one horizontal direction plus 30 percent of the design earthquake load in the other direction. For example, the structure should be designed for
  (i) $EL = \pm EL_x \pm 0.3\, EL_y$ and
  (ii) $EL = \pm 0.3\, EL_x \pm 0.3\, EL_y$
  where x and y are two orthogonal horizontal directions, EL is earthquake load.

15.18 Design Vertical Earthquake Load

- The design vertical earthquake load is calculated by considering the vertical acceleration as two-thirds of the horizontal acceleration, that is design vertical acceleration spectrum is taken as two-thirds of the design horizontal acceleration spectrum.

15.19 Combination of Two or Three Component Motion

- When responses from three earthquake components are to be considered, then responses are combined assuming that when maximum response from one component occurs, the response from the other two components is 30% of their maximum and all possible components including variations in sign should be considered.
- Thus, response due to earthquake force (EL) is maximum of the following:
  (i) $\pm EL_x \pm 0.3\, EL_y \pm 0.3\, EL_z$
  (ii) $\pm 0.3\, EL_x \pm EL_y \pm 0.3\, EL_z$
  (iii) $\pm 0.3\, EL_x \pm 0.3\, EL_y \pm EL_z$
  where, x and y are two horizontal orthogonal directions and z is vertical direction.

- Alternatively, combined response (EL) can be obtained on the basis of 'square root of the sum of the square (SRSS)'

$$EL = \sqrt{(EL_x)^2 + (EL_y)^2 + (EL_z)^2}$$

F. Basic Load Combinations

- Structure is analysed and designed for the combination that yields the most critical value.
- Terms DL, IL, and EL are the response quantities due to dead load, imposed load and earthquake load respectively.

15.20 Load Factors for Plastic Design of Steel Structures

Following load combinations should be accounted for:

(i) $1.7\,(DL + IL)$ (ii) $1.7\,(DL \pm EL)$

(iii) $1.3\,(DL + IL \pm EL)$

15.21 Partial Safety Factor for Limit State Design of Reinforced Concrete and Prestressed Concrete Structures

Following load combinations should be accounted for:

(i) $1.5\,(DL + IL)$ (ii) $1.2\,(DL + IL \pm EL)$

(iii) $1.5\,(DL \pm EL)$ (iv) $0.9\,DL \pm 1.5\,EL$

where, 1.5, 1.2 and 0.9 are partial safety factors.

G. Permissible Stresses

- When seismic forces are considered along with other normal design forces, the permissible stresses in material, in the elastic method of design, may be increased by one-third.
- However, for steels having a definite yield point, the stress should be limited to the yield stress. Whereas for steels without a definite yield point, the stress will be limited to 80 percent of the ultimate strength or 0.2 percent proof stress, whichever is smaller.
- For prestressed concrete members, tensile stress in the extreme fibres of the concrete may be permitted so as not to exceed two-thirds of the modulus of rupture of concrete.
- While considering earthquake forces the allowable bearing pressure should be increased, depending upon type of soil and type of foundation of the structure.
- In soil deposit consisting of submerged loose sands and soils falling under classification SP with standard penetration N-values less than 15 in seismic zones III, IV, V and less than 10 in seismic zone II, the vibration caused by earthquake may cause liquefaction or excessive total and differential settlements.

H. Analysis Methods and Design Approaches (Based on Codes)

- After structural modelling, we perform analysis to determine the seismically induced forces in the structure.

- Basis of analysis can be external force, the behaviour of structure and type of structural model.
- Based on the type of external action and behaviour of structure, the analysis can be further classified as linear static analysis, linear dynamic analysis, non-linear static analysis or non-linear dynamic analysis.

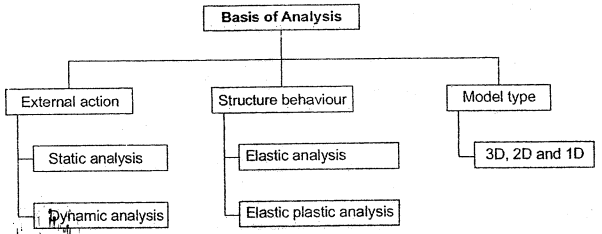


Fig. 15.14 Analysis process

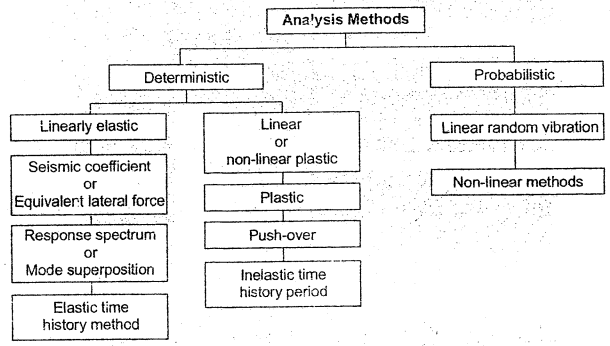


Fig. 15.15 Methods of analysis

- Linear static analysis or equivalent lateral force analysis is used for regular structures with limited height.
- Linear dynamic analysis can be performed in two ways, either by the response spectrum method or by the elastic time-history method.
- The major difference between linear static and linear dynamic analysis in the level of the forces and their distribution along the height of the structure.
- Non-linear static analysis is an improvement over the linear static or linear-dynamic analysis as it allows inelastic behaviour of the structure.

Note: A non-linear dynamic analysis or inelastic time-history analysis is the best suitable method to describe the actual behaviour of a structure during an earthquake.

Do You Know?

As per the IS 1893 (Part-I) : 2002, dynamic analysis is done for:

(i) **Regular buildings:** For height greater than 40 m in zones IV and V and height greater than 90 m in zones II and III.

(ii) **Irregular buildings:** For height greater than 12 m in zones IV and V and height greater than 40 m in zones II and III. However if height is lesser than 40 m in zones II and III, dynamic analysis is not mandatory, but is recommended.

- We will limit discussion to elastic analysis method i.e., the seismic coefficient method, response spectrum method and time history method.

Assumptions made for earthquake resistant design of structures:

(a) Earthquake causes complex and irregular ground motions, impulsive in nature, changing in period, and amplitude each lasting for a small duration. Therefore resonance would not occur as it requires time to buildup such amplitudes.
However, resonance like conditions have seen to occur in case of long distance waves and tall structures founded on deep soft soils.

(b) Earthquake does not occur simultaneously with wind or maximum flood or maximum sea waves.

(c) Elastic modulus of materials is used for static analysis unless a more definite value is available.

15.22Elastic Analysis Methods

- Elastic analysis is either carried out by the static method (the equivalent lateral force method) or dynamic method (the response spectrum method or elastic time history method).
- For preliminary purposes, the equivalent lateral force method, that is, seismic coefficient method is done and then it is further used are more refined methods namely response spectrum analysis and elastic time history method.

15.22.1 Seismic Coefficient Method (Equivalent Lateral Force Method)

- The base shear (is equal to total horizontal force on the structure) is calculated on the basis of the structural mass, the fundamental period of vibration and the corresponding mode shape.
- As per the code, the base shear is distributed along the height of structure, in terms of lateral loads.
- Orthogonal directions are analysed separately. The two results of analysis in orthogonal horizontal direction and torsional motion effects of the structure are combined.
- It is applicable to low to medium height structure with regular configuration.

15.22.2 Response Spectrum Analysis (Mode Superposition Method)

- It is applicable to assymmetrical and irregular structures and for multi-storey buildings for moderate earthquakes (causing large responses of structures).
- Response spectrum terminologies:
  - (i) **Normal Mode:** A system is in normal mode of it is vibrating such that all its masses attain maximum values of displacement and rotations simultaneously and pass through equilibrium positions simultaneously.
  - (ii) **Natural period:** Time period of undamped free vibration of a structure.
  - (iii) **Fundamental natural period:** The first (longest) modal time period of vibration.
  - (iv) **Modal natural period (T_k):** The modal natural period of mode K is the time period of vibration in mode K.

  - (v) **Modal Mass (M_k):** Modal mass of a structure is part of total seismic mass of the structure subjected to ground motion, which is effective in mode K vibration.
    Modal mass for a given mode has a unique value irrespective of scaling of mode shape.
  - (vi) **Modal participation factor (P_k):** Modal participation factor of mode K of vibration is the amount by which mode K contributes to overall vibration of the structures during time the earthquake. The value of this factor depends on the scaling used for mode shapes.

15.22.3 Elastic Time History Method

- This method overcomes all the disadvantages of modal response spectrum analysis, provided non-linear behaviour is not involved.
- In this method, relative signs of response quantities are preserved in the response history, which helps in considering interaction effects of stress resultants.

NOTE

- The main difference between equivalent lateral force and response spectrum analysis is the base shear magnitude and lateral forces distribution.
- In response spectrum analysis method, calculation of forces in done on the basis of compound periods and mode shapes of various modes of vibration.
  whereas in equivalent lateral force method, forces are calculated based on fundamental period and distribution of forces of a regular structure (in terms of mass and stiffness) along the height is done by simple formulae.

15.23Seismic Analysis Factors

- Factors accounting for design lateral forces and the design response spectrum are explained below:

15.23.1 Zone Factor (Z) (IS 1893 : 2002 Part-I)

- It is used to obtain design spectrum or design seismic forces depending upon the potential seismic hazard.
- It is an estimate of effective peak ground acceleration considering maximum considering earthquake (MCE) and service life of structure in each seismic zone.

Table 15.2: Zone factor, Z

Seismic zone	II	III	IV	V
Seismic intensity	Low	Moderate	Severe	Very severe
Z	0.10	0.16	0.24	0.36

15.23.2 Importance Factor (I) (IS 1893 : 2002 Part-I)

- It is used to obtain design seismic force depending upon the functional use of the structure.

Table 15.3: Importance factor, I

Sl. No.	Structure	Importance factors
(i)	Important service and community buildings, such as hospitals, schools, monumental structures; emergency buildings like telephone exchange, television stations, ratio stations, railway stations, fire station buildings, large community halls like cinemas, assembly halls and subway stations, power stations.	1.5
(ii)	All other buildings	1.0

Note: Design engineer may opt higher value of importance factor (I) than those mentioned above in table.

15.23.3 Response Reduction Factor (R) (IS 1893 : 2002 Part-I)

- In case of strong ground motions; since structure is allowed to get damaged but without collapse, thus design seismic forces should be lesser in value compared to the forces calculated considering structure remaining linearly elastic, during design basis earthquake (DBE).
- Response reduction factor (R) is a factor by which actual base shear is reduced to obtain the design lateral forces.

Table 15.4: Response reduction factor, R for building systems

Sl. No.	Lateral Load Resisting System	R
	Building Frame Systems	
(i)	Ordinary RC moment-resisting frame (OMRF)	3.0
(ii)	Special RC moment-resisting frame (SMRF)	5.0
(iii)	Steel frame with	
	(a) Concentric braces	4.0
	(b) Eccentric braces	5.0
(iv)	Steel moment resisting frame designed as per SP 6	5.0
	Building with Shear Walls	
(v)	Load bearing masonry wall buildings	
	(a) Unreinforced	1.5
	(b) Reinforced with horizontal RC bands	2.5
	(c) Reinforced with horizontal RC bands and vertical bars at corners of rooms and jambs of openings	3.0
(vi)	Ordinary reinforced concrete shear walls	3.0
(vii)	Ductile shear walls	4.0
	Buildings with Dual Systems	
(viii)	Ordinary shear wall with OMRF	3.0
(ix)	Ordinary shear wall with SMRF	4.0
(x)	Ductile shear wall with OMRF	4.5
(xi)	Ductile shear wall with SMRF	5.0

Note: Over strength, redundancy and ductility are those factors which lowers the value of design seismic forces, and response reduction factor accounts for these factors.

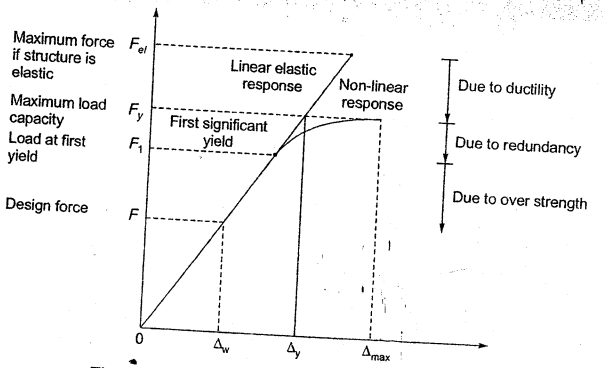


Fig. 15.16 Concept of response reduction factor

15.23.4 Fundamental Natural Period (T_a) (IS 1893 : 2002, Part-I)

- The fundamental natural period (T_a) is the first (longest) modal time period of vibration.
- For moment resisting frame building without bricks infill panels, T_a may be estimated as
  $T_a = 0.075\, h^{0.75}$ for RC frame building
  $T_a = 0.085\, h^{0.75}$ for steel frame building
- And for all other building including moment resisting frame buildings with brick infill panels, may be estimated as

$$T_a = \frac{0.09h}{\sqrt{d}}$$

where, h is height of building in meters, excluding the basement storeys, where basement walls are connected with the ground floor deck or fitted between the building columns. But it includes the basement storeys, when they are not so connected. d is base dimension of the building at the plinth level, in meters, along the considered direction of lateral forces.

15.23.5 Design Acceleration Spectrum (IS 1893 : 2002, Part-I)

- The design response spectrum (smooth response spectrum) specifies the seismic resistance required for design.
- Design acceleration spectrum is smoothened average plot of maximum acceleration as a function of frequency or time period of vibration for specified damping ratio at the base.
- The structural response factor (S_a/g) is ordinate of acceleration spectrum divided by acceleration due to gravity.
- S_a/g is average response acceleration coefficient for rock and soil sites, as shown in figure below.

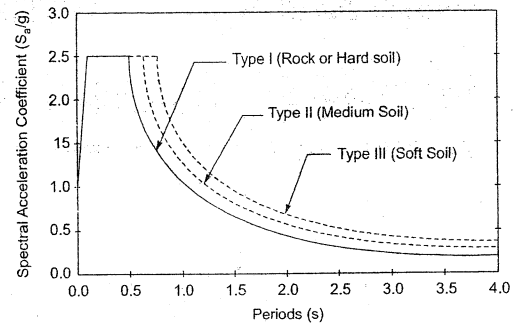


Fig. 15.17 Response spectrum for rock and soil sites for 5% damping

- Table below gives the multiplyign factors for obtaining spectral values for various other dampings.

Table 15.5: Multiplying factor for obtaining values for other damping

Damping %	0	2	5	7	10	15	20	25	30
Factors	3.20	1.40	1.00	0.90	0.80	0.70	0.60	0.55	0.50

Note: In case design spectrum is specifically prepared for a structure at a particular project site, same may be used for design at the discretion of authorities.

15.24 Design Seismic base Shear (V_B)

- The total design lateral force or design seismic base shear (V_B) along any principal direction is calculated as:

$$V_B = A_h W$$

where, A_h is design horizontal acceleration spectrum using the fundamental natural period (T_a) in the considered direction of vibration. W is seismic weight of the building.

- The design horizontal seismic coefficient (A_h) for a structure is calculated as:

$$A_h = \frac{ZIS_a}{2Rg}$$

Provided if T (undamped natural period of free vibration) ≤ 0.1s, then A_h value should not be less than $Z/2$, for any value of I/R.

NOTE

- Zone factor (Z) given in table is for the maximum considered earthquake (MCE) and service life of structure in a zone. The factor 2 in the denominator of Z is used to reduce MCE zone factor to the factor for design basis earthquake (DBE).
- The ratio I/R should not be greater than 1.0.
- The maximum value of response acceleration coefficient (Sa/g) as per IS 1893 : 2002 is 2.5 g, obtained for short natural period ranging between 0.10 to 0.40 s.
- For steel and RC building, the value of damping is taken as 2 and 5 percent of critical respectively.

Do You Know?

- For underground structures and foundations at depths of 30 m or below, the design horizontal acceleration spectrum value should be taken as half the value calculated from the expression of A_h.
- And for those placed between the ground level and 30 m depth, the design horizontal acceleration spectrum value should be linearly interpolated between A_h and 0.5 A_h.
- The design vertical spectrum, may be taken as two-third of A_h.

15.25 Seismic Weight (IS 1893 : 2002, Part-I)

- The seismic weight of the whole structure is the sum of the seismic weights of all the floors.
- The seismic weight of each floor is its full dead load plus appropriate percentage of imposed load (show in table below)
- For seismic weight calculation of each floor, the weight of columns and wall in any storey should be equally distributed to the floors above and below the storey.
- For the design seismic force calculation, the imposed load on the roof need not be considered.

Table 15.6: Percentage of imposed load to be considered in seismic weight calculation

Imposed Uniformity Distributed Floor Loads (kN/m^2)	Percentage of Imposed Load
Upto and including 3.0	25
Above 3.0	50

15.26 Distribution of Design Force (IS 1893 : 2002, Part-I)

- Structure and their elements are designed to resist the effects of design lateral force (V_B) as a minimum.
- Initially the design lateral force is calculated for whole building and then afterwards it is distributed to various floor levels.
- Again, the design seismic force at each floor level is distributed to individual lateral load-resisting systems (depending upon diaphragm action).

15.26.1 Seismic Coefficient Method (Equivalent lateral force method or equivalent static method)

- This method does not requires dynamic analysis, however it accounts structural dynamics in an approximate way.
- It is simplest method, requires less computational efforts. It is based on formulae given in the code.

Vertical distribution of base shear to different floor levels:

The design base shear (V_B) calculated, is distributed along the height of the building as:

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{i=1}^{n} W_i h_i^2}$$

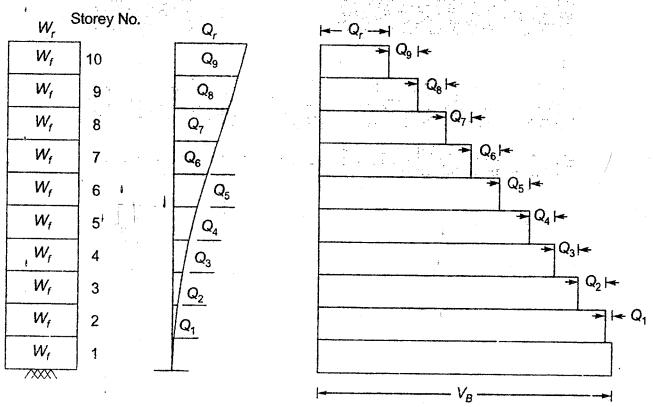
where,

Q_i is design lateral force at floor, i

W_i is seismic weight of floor i

h_i is height of floor i measured from the base

n is number of storeys in the building that is number of levels at which the masses are located.



(a) Frame (b) Distribution of forces (c) Distribution of shears

Fig. 15.18

Distribution of horizontal design lateral force to different lateral force resisting elements

- In case if buildings whose floors are capable of providing rigid horizontal diaphragm action, the total shear in any horizontal plane should be distributed to the various vertical elements of lateral force resisting system (assuming the floors to be infinitely rigid in the horizontal plane).
- Otherwise, if building whose floor diaphragms cannot be treated as infinitely rigid in their own plane, the lateral shear at each floor should be distributed to the vertical elements resisting the lateral forces, considering the in-plane flexibility of the diaphragms.

NOTE

- A floor diaphragm is considered to be flexible, if it deforms such that maximum lateral displacement measured from the chord of the deformed shape at any point of the diaphragm is more than 1.5 times the average displacement of the entire diaphragm.
- Reinforced concrete monolithic slab-beam floors or those consisting of prefabricated/precast elements with topping reinforced screed can be taken as rigid diaphragms.

15.26.2 Response Spectrum Method

- It is a dynamic analysis method which is done using design spectrum.
- In this method, peak response of a structure during seismic action is obtained directly from design response spectrum, which in-turn is used for structural design purposes.
- Here, the multiple modes of response of a structure to an seismic action are taken into account.
- For each mode, a response is read from the design spectrum, depending upon the modal frequency and the modal mass.
- Then responses of different modes are combined to obtain total response of the structure using modal combination methods.
- In dynamic analysis, the design base shear (V_B) is compared with a base shear ($\overline{V}_B$) (calculated using a fundamental period T_a). If V_B is less than $\overline{V}_B$, then all the response quantities (member forces, displacement, storey forces, storey shears and base reactions) should be multiplied by $\frac{\overline{V}_B}{V_B}$.

NOTE

- The number of modes to be used in analysis should be such that the total sum of modal masses of all modes considered is atleast 90% of the total seismic mass.
- If modes with natural frequency beyond 33 Hz is considered, then modal combination should be carried out upto 33 Hz. The higher modal effects should be included by considering missing mass correction following established procedures.
- In dynamic analysis, when number of modes are considered, then the value of A_h for each mode is determined using the natural period of vibration of that mode.

Modal Analysis

- Structures with regular or nominally irregular plan configurations may be modelled as a system of masses lumped at the floor levels with each mass having one degree of freedom, that is lateral displacement under considered direction.

The following expressions are used for analysis:

(a) **Modal Mass (M_k):** The modal mass (M_k) of mode k is given as

$$M_k = \frac{\left[\sum_{i=1}^{n} W_i \phi_{ik}\right]^2}{g\sum_{i=1}^{n} W_i (\phi_{ik})^2}$$

where,

g is acceleration due to gravity

ϕ_{ik} is mode shape coefficient at floor i in mode k

W_i is seismic weight of floor i.

(b) **Modal participation factor (P_k):** The modal participation factor of mode k is given as

$$P_k = \frac{\sum_{i=1}^{n} W_i \phi_{ik}}{\sum_{i=1}^{n} W_i (\phi_{ik})^2}$$

(c) **Design lateral force at each floor in each mode:** The peak lateral force (Q_{ik}) at floor i in mode k is given as

$$Q_{ik} = A_k \phi_{ik} P_k W_i$$

where

A_k is design horizontal acceleration spectrum using natural period of vibration (T_k) of mode k.

(d) **Shear forces in each mode:** The peak shear force (V_{ik}) acting in storey i in mode k is given as:

$$V_{ik} = \sum_{j=i+1}^{n} Q_{ik}$$

(e) **Storey shear forces due to all modes considered:** The peak storey shear force (V_i) in storey i due to all modes considered is obtained by combining those due to each mode in accordance with modal combinations.

NOTE

For storey shear forces due to all modes considered, approximate methods such as **square root of sum of squares (SRSS) or maximum absolute response (ABS)** are used. The ABS method is conservative and seldom used, whereas the SRSS method is preferred for total maximum response.

(f) **Lateral forces at each storey due to all modes considered:** The design lateral forces, F_{roof} and F_i, at roof and at floor i respectively is given as

$$F_{roof} = V_{roof}$$
$$F_i = V_i - V_{i+1}$$

Modal Combinations

The peak response quantities (e.g., member forces, displacement, storey forces, storey shears and base reactions) is combined as per complete quadratic combination (CQC) method.

$$\lambda = \sqrt{\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i \rho_{ij} \lambda_j}$$

where

r is number of modes being considered.

ρ_{ij} is cross-modal coefficient

λ_i is response quantity is mode i (including sign)

λ_j is response quantity in mode j (including sign)

Also

$$\rho_{ij} = \frac{8\zeta^2(1+\beta)\beta^{1.5}}{(1+\beta^2)^2 + 4\zeta^2(1+\beta)^2}$$

where, ζ = Modal damping ratio (in fraction)

β = Frequency ratio = $\frac{\omega_j}{\omega_i}$

ω_i = Circular frequency in i^{th} mode

ω_j = Circular frequency in j^{th} mode

Alternatively

The peak response quantities may be combined by the SRSS method or ABS method.

SRSS Method: If the building does not have closely spaced modes, the peak response quantity λ due to all modes considered is obtained as

$$\lambda = \sqrt{\sum_{k=1}^{r}(\lambda_k)^2}$$

where,

λ_k is absolute value of the quantity in mode k,

r is number of modes considered.

ABS Method: If the building has a few closely spaced modes, the peak response quantity λ^* due to these modes is obtain as

$$\lambda^* = \sum_{c}^{r}\lambda_c$$

where, summation is for closely spaced modes only. This peak response quantity due to the closely spaced modes λ^* is then combined with those of the remaining well separated modes by the SRSS method.

15.26.3 Time History Method

- The time history method represent the most sophisticated method of dynamic analysis.
- In this method, mathematical model of structure is subjected to acceleration from expected earthquake recorded datas at the base of the structure.

- It consists of a step by step direct integration over a time interval.
- The equation of motion are solved with the displacement, velocities and accelerations of the previous step serving as initial functions.
- The equation of motion is written as

$$kx(t) + c\dot{x}(t) + m\ddot{x}(t) = p(t)$$

where

k is stiffness matrix

c is damping coefficient matrix

m is diagonal mass matrix

x is displacement

$\dot{x}$ is velocity

$\ddot{x}$ is acceleration

$p(t)$ is applied load (here in case of earthquake, $p(t)$ includes ground acceleration)

I. Torsion (IS 1893 : 2002, Part-I)

- There is increase in shear forces on the lateral force resisting elements resulting from horizontal torsional moment arising due to eccentricity between the centre of mass and centre of rigidity.
- The calculated design forces are applied at the centre of mass, which is displaced appropriately so as to cause design eccentricity between the displaced centre of mass and centre of rigidity. However negative torsional shear should be neglected.
- The design eccentricity, e_{di}, used for floor i is

$$e_{di} = \begin{cases} 1.5e_{si} + 0.05b_i \\ \text{or} \\ e_{si} - 0.05b_i \end{cases}$$

whichever of these gives the more severe effect in the shear of any frame,

where

e_{si} = static eccentricity at floor i that is distance between centres of mass and rigidity.

b_i = floor plan dimensions of floor i, perpendicular to force direction.

Note: The factor 1.5 represents dynamic amplification factor, when 3D analysis is not carried out. However for 3D analysis, amplification factor is 1. And 0.05 represent the extent of accidental eccentricity

J. Soft Storeys in Construction

For building with a flexible storey,such as ground story consisting of open spaces for parking that is stilt building, special arrangement needs to be made to increase the lateral strength or stiffness of soft storey.

The following design criteria is adopted after carrying out the earthquake analysis, neglecting the effect of infill wall in other storeys:

(i) The columns and beams of the soft storey are to be designed for 2.5 times the storey shears and moments calculated under seismic loads.

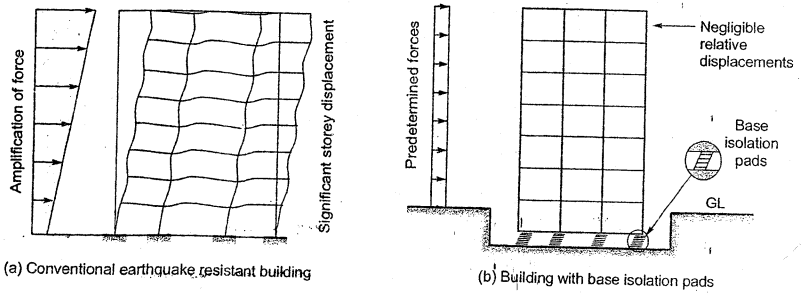
(ii) The shear walls are designed for 1.5 times the lateral storey shear force and are placed symmetrically in both directions of the building, as far away from the centre of the building as possible.

K. Storey Drift

- The storey drift is evaluated as a difference of deflections of the floors at the top and the bottom of the storey under consideration.
- Due to minimum specified designed lateral force, the storey drift with partial load factor of 1.0, should not exceed 0.004 times the storey height.

L. Seismic Response Control

- The dynamic behaviour of a structure can be modified in order to minimize structural response.
- Modifications can be done to structural dynamics or energy absorption capacity of the structure.
- **Base isolater devices** are used to prevent seismic force from acting on a structure, is based on **input reduction approach.** e.g. Rubber pads and devices using dry friction.
- **Seismic dampers devices** are used to absorb part of seismic energy, is based on **damping augmentation approach.** e.g. Hydraulic dampers, Metallic dampers etc.
- Seismic base isolation principle is to increase the structure natural time period, which decreases its natural frequency of vibration. Hence fundamental period of vibration shifts away from predominant period of seismic ground motion, thus lesser forces are transmitted to the structures.
- Seismic dampers helps in dissipating seismic energy as the structure sways back and forth due to seismic loading and helps in lowering the overall displacements of the structure.



(a) Conventional earthquake resistant building (b) Building with base isolation pads

Fig. 15.19 Response of buildings with and without isolation

M. Ductile Detailing of Reinforced Concrete Structures Subjected to Seismic Forces

Provision given below should be adopted for structures located in zone III, IV and V.

15.27 General Recommendations

- The design and construction of RC structures is governed by IS 456 : 2000, except as modified by the provision of IS 13920 : 1993.
- Minimum grade of concrete for structures having more than three storeys should be M20 i.e., $f_{ck} = 20$ MPa.
- Steel grade used is limited to Fe415 or less for better ductility.
- Thermo mechanically treated (TMT) bars (high strength deformed bars) of grades Fe500 and Fe550 may also be used for reinforcement if they have elongation more than 14.5 percent, and conform to other requirements of IS 1786 : 1985.
- Cold worked steel is not recommended.

15.28 Flexural Members

15.28.1 General

- The member should have width to depth ratio of more than 0.3.
- The width of the member should not be less than 200 mm.
- The depth of the members should not be more than one fourth of the clear span.
- The factored axial stress on the member under seismic loading should not exceed $0.1\, f_{ck}$.

15.28.2 Longitudinal Reinforcement

- The member should have atleast two bars, both at top and bottom, throughout the length.
- The tension steel ratio on any face, at any section, should not be less than

$$\rho_{min} = \frac{0.24\sqrt{f_{ck}}}{f_y}$$ where f_{ck} and f_y are in MPa

- The maximum steel ratio on any face at any section, should not exceed $\rho_{max} = 0.025$.
- The positive steel at a joint face should be atleast equal to half the negative steel at that face.
- The steel provided at each top and bottom face of the member at a section along its length should be equal to at least one fourth of the maximum negative moment steel provided at the face of either joint.
- In an external joint, both the top and the bottom bars of the beam should be provided with anchorage length, beyond the inner face of the column, equal

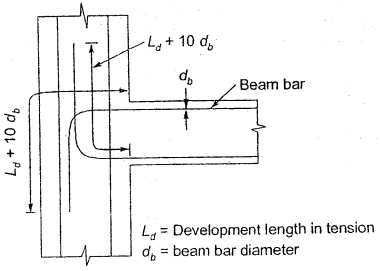
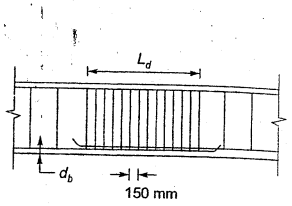


Fig. 15.20 Anchorage of beam bars in an external joint

to development length in tension plus 10 times the bar diameter minus the allowance for 90 degree bends (shown in figure below). In an internal joint both face bars of the beam should be taken continuously through the column.

15.28.3 Lap Splices

- The longitudinal bars should be spliced, only if hoops are provided over the entire splice length, at a spacing not exceeding 150 mm (as shown in figure below).
- The lap length should not be less than the bar development length in tension.
- Lap splices should not be provided:
  (a) Within a joint.
  (b) Within a distance of 2d from joint face.
  (c) Within a quarter length of the member.
  where flexural yielding may generally occur under the effect of earthquake forces.
- Not more than 50% of the bars should be spliced at one section.



L_d = development length in tension
d_b = Bar diameter

Fig. 15.21 Lap, Splice in Beam

15.28.4 Web Reinforcement

- Web reinforcement should consist of vertical hoops. A vertical hoop is a closed stirrup having a 135° hoop with a 10 diameter extension (but not < 75 mm) at each end that is embedded in confined core (shown in figure, a, below).
- Sometimes, it may also be made up of two pieces of reinforcement; U-stirrup with a 135° hook and a 10 diameter extension (but not < 75 mm) at each end, embedded in the confined core.

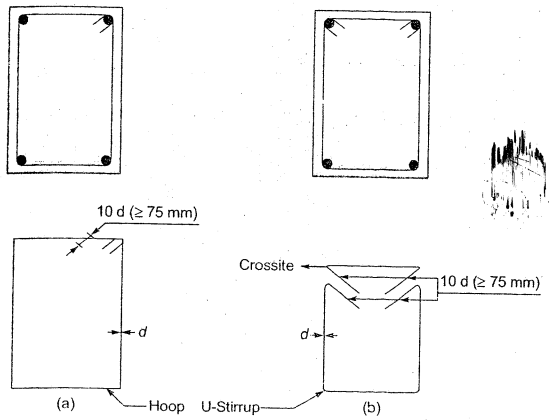


Fig. 15.22 Beam web reinforcement

- The minimum diameter of the bar forming a hoop should be 6 mm. However for beams with clear span > 5 m, the minimum bar diameter should be 8 mm.
- The shear force to be resisted by the vertical hoop should be maximum of
  (i) Calculated factored shear force as per the analysis.
  (ii) Shear force due to the formation of plastic hinges at both ends of the beam plus the factored gravity load on the span (as shown in figure below).

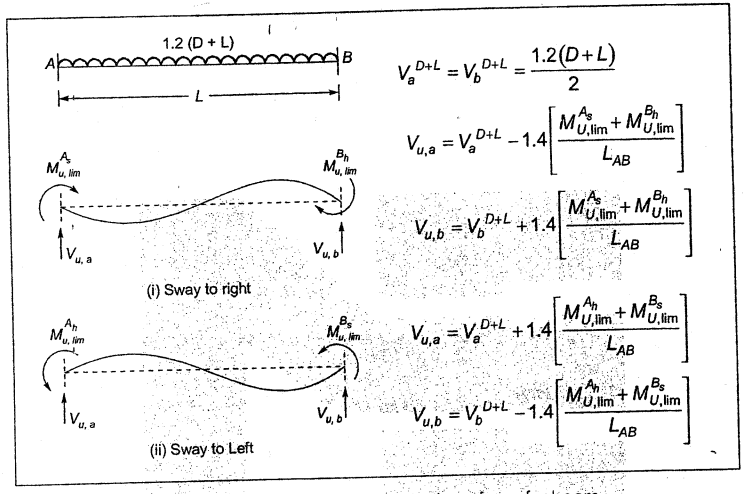


Fig. 15.23 Calculation of design shear force for beam

where; $M^{A_s}_{u,lim}$, $M^{A_h}_{u,lim}$ and $M^{B_s}_{u,lim}$, $M^{B_h}_{u,lim}$ are the sagging and hogging moment of resistance of beam section at ends A and B, respectively.

These are calculated as per IS 456 : 2000.

L_{AB} is clear span of beam.

V_a^{D+L} and V_b^{D+L} are the shears at ends A and B respectively due to vertical load with partial safety factor of 1.2 on loads.

The design shear at end A and B should be the larger of the two values of $V_{U,a}$ and $V_{U,b}$ computed above (in Fig. 23) respectively.

NOTE

The factor 1.4 is obtained by assuming stress in flexural reinforcement as 1.25 f_y instead of 0.87f_y for plastic moment capacity calculation.

$$F = \frac{1.25}{0.87} = 1.4$$

- The contribution of bent up bars and inclined hoops to shear resistance of the section should not be considered.

- The spacing of hoops over a length of $2d$ at either end of a beam should not exceed
  (a) $d/4$
  (b) 8 times the diameter of the smallest longitudinal bar.
  However it should not be less than 100 mm, as shown in figure below.
- The first hoop should be at a distance not exceeding 50 mm from the joint face. Vertical hoops at the same spacing as above should also be provided over length equal to $2d$ on either side of a section where flexural yielding may occur under the effect of seismic forces. Elsewhere, the beam should have vertical hoops at a spacing not exceeding d/2.

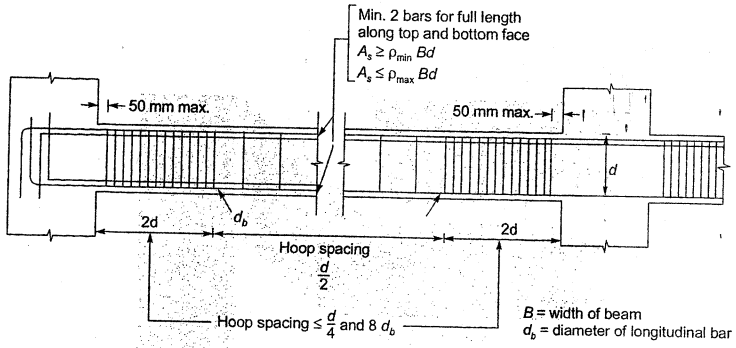


Fig. 15.24 Beam reinforcement

15.29 Slab

- Thickness should not be less than $1/50^{th}$ of the clear distance between the footings, but not less than 100 mm in any case.
- Minimum reinforcement area ratio should be 0.15% for mild steel bars or 0.12% for high strength deformed bars in each direction placed symmetrically at top and bottom.

15.30 Columns and Frame Members subjected to Bending and Axial Load

15.30.1 General

- The requirements apply to those frame members which have factored axial stress in excess of $0.1 f_{ck}$ under the effect of seismic forces.
- The minimum dimension of member should not be less than 200 mm. However, for frames which have beams with centre to centre span exceeding 5 m or columns of unsupported length exceeding 4 m, the shortest dimension of the column should not be less than 300 mm.
- The ratio of the shortest cross-sectional dimension to the perpendicular dimension should not be less than 0.4.

15.30.2 Longitudinal Reinforcement

- Lap splices should be provided only in the central half of the member length. It should be proportioned as a tension splice. Hoops should be provided over the entire splice length at spacing not exceeding 150 mm centre to centre. Not more than 50% of the bars should be spliced at a section.
- Any area of a column that extends more than 100 mm beyond the confined core due to architectural requirements should be detailed as:
  If contribution of this area of strength has been considered, then minimum longitudinal and transverse reinforcement should be provided as per IS 13920 : 1993. However if this area is treated as non-structural, the minimum longitudinal and transverse reinforcement should be as per IS 456 : 2000.

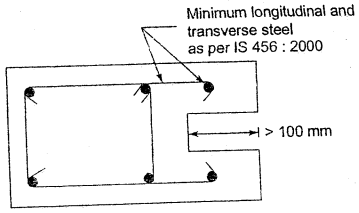
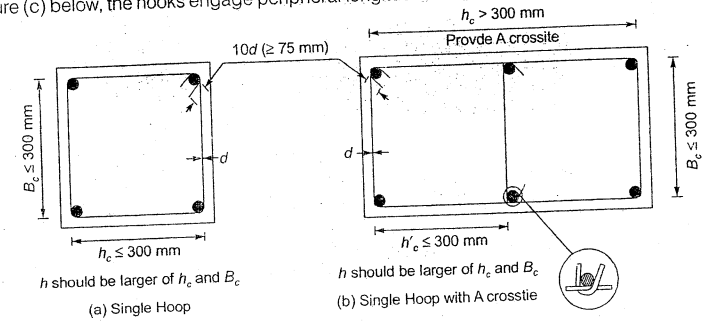


Fig. 15.25 Reinforcement requirement for column with more than 100 mm projection beyond core

15.30.3 Transverse Reinforcement

- Transverse reinforcement for circular columns should consist of circular or spiral hoops. For rectangular columns, rectangular hoops can be used.
- Parallel legs of rectangular hoop should be spaced not more than 300 mm centre to centre. If the length of any side of the hoop exceeds 300 mm, a crosstie should be provided, as shown in figure (b) below. Alternatively, a pair of overlapping hoops may be provided within the column, as shown in figure (c) below, the hooks engage peripheral longitudinal bars.



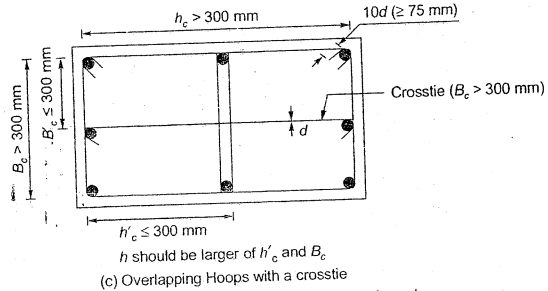
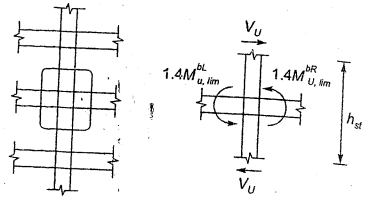


Fig. 15.26 Transverse reinforcement in column

- The spacing of hoops should not exceed half the least lateral dimension of the column except where special confining reinforcement is provided.
- The design shear force for columns should be maximum of
  (i) Calculated factored shear force as per analysis
  (ii) A factored shear force, given as

$$V_u = 1.4\left[\frac{M_{u,lim}^{bL} + M_{u,lim}^{bR}}{h_{st}}\right]$$

where, $M_{u,lim}^{bL}$ and $M_{u,lim}^{bR}$ are moment of resistance (of opposite sign) of beams framing into columns from opposite faces (as shown in figure below) and h_{st} is storey height. The beam moment capacity is evaluated as per IS 456 : 2000.



$$V_U = 1.4\left(\frac{M_{U,lim}^{bL} + M_{U,lim}^{bR}}{h_{st}}\right)$$

Fig. 15.27 Calculation of design shear force for column

15.30.4 Special Confining Reinforcement

- This requirement should be done unless a larger amount of transverse reinforcement is required from shear strength considerations.
- Special confining reinforcement should be provided over a length l_0 from each joint face, towards midspan, and on either side of any section, where flexural yielding may occur under the effect of seismic forces (as shown in figure below) The length l_0 should not be less than
  (i) larger lateral dimension of the member at the section where yielding occurs.
  (ii) 1/6 of the clear span of the member
  (iii) 450 mm

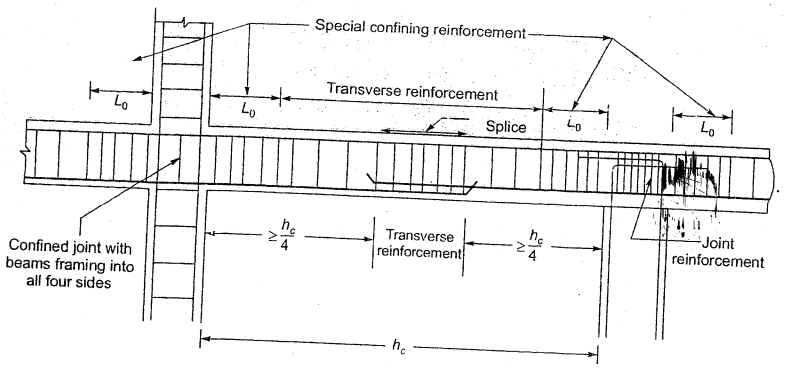


Fig. 15.28 Column and joint detailing as per IS 13920

- When column terminates into a footing, special confining reinforcement should extend atleast 300 mm into the footing (see figure below)

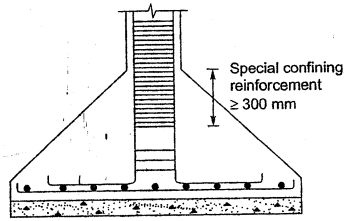


Fig. 15.29 Provision of special confining reinforcement in footings

- Special confining reinforcement should be provided over the full height of the column, if the contraflexure point (due to gravity and seismic forces) is not within the middle half of the member clear height.
- Columns supporting reactions from discontinued stiff members such as walls should be provided with special confining reinforcement over their full height (see figure below).
- This reinforcement should also be placed above the discontinuity for atleast the development length of the largest longitudinal bar in the column. Where column is supported on wall, this reinforcement should be provided over the full height of the column; it should also be provided below the discontinuity for the same development length.

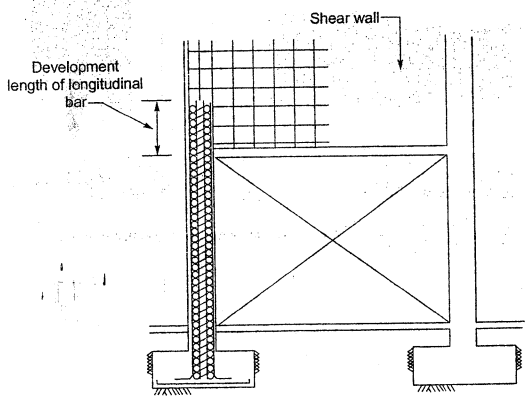
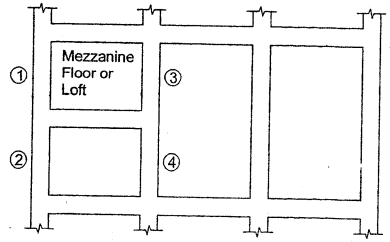


Fig. 15.30 Special confining reinforcements for columns under discontinued walls

- Column which has significant variation in stiffness along its height, should be provided with special confining reinforcement. This variation in stiffness can be due to presence of bracing, a mezzanine floor or a RCC wall on either side of the column that extends only over a part of column height as shown in figure below.

- The spacing of hoops used as special confining reinforcement should not exceed 1/4 of the minimum member dimension but should not be less than 75 mm nor more than 100 mm.



(1), (2), (3) and (4) are relatively stiff columns: They attract large seismic shear force

Fig. 15.31 Columns with Varying Stiffness

- For bar used as special confining reinforcement in form of circular hoops or spirals, area should not be less than

$$A_{sh} = 0.09SD_k\frac{f_{ck}}{f_y}\left[\frac{A_g}{A_k}-1.0\right]$$

where,

A_{sh} is area of bar cross-section

S is pitch of spiral or spacing of hoops

D_k is diameter of core measured to the outside of spiral or hoop

A_g is gross area of column

A_k is area of concrete core $= \frac{\pi}{4}D_k^2$

- Area of cross-section of the bar forming rectangular hoop, to be used as special confining reinforcement should not be less than

$$A_{sh} = 0.18Sh\frac{f_{ck}}{f_y}\left[\frac{A_g}{A_k}-1.0\right]$$

where

h is longer dimension of the rectangular confining hoop measured to its outer face.

It should not exceed 300 m

A_k is area of confined concrete core in the rectangular hoop measured to its outside dimension.

Note: The dimension h of the hoop can be reduce by introducing crossties (shown in figure)

15.31 Joints of Frames

- A special confining reinforcement as required at the end of column should be provided through the joint as well, unless the joint is confined by; beams farming into all vertical faces of it and where each beam width is atleast 3/4 of the column width, may be provided with half the special confining reinforcement required at the end of the column. The spacing of hoops should not exceed 150 mm.

15.32 Shear Walls

15.32.1 General Requirements

- Following requirements apply to shear walls, which are part of the lateral forces resisting systems.
- The thickness of any part of the wall should not be less than 150 mm.
- The effective flange width used in the design of the flanged wall sections, should be assumed to extend beyond the face of the web for a distance which should be smaller of
  - (i) half of distance to an adjacent shear wall web
  - (ii) 1/10th of the total wall height
- Reinforcement is provided in both longitudinal and transverse directions in the plane of wall. Reinforcement should be uniformly distributed across the cross-section of the wall. The minimum reinforcement ratio should be 0.0025 of the gross area in each direction.
- If factored shear stress in the wall exceeds $0.25\sqrt{f_{ck}}$ or if the wall thickness is more than 200 mm, reinforcement should be provided in two curtains, each having bars running in the longitudinal and transverse direction in the plane of the wall.
- The diameter of the bars should not exceed 1/10th of the thickness of that part of the wall.
- The maximum spacing of reinforcement in either direction should not exceed the smaller of
  - (i) $l_w/5$
  - (ii) $3\,t_w$
  - (iii) 450 mm

  where l_w is horizontal length of wall and t_w is the thickness of the wall web.

15.32.2 Shear Strength

- The nominal shear stress (τ_v) is given as

$$\tau_v = \frac{V_u}{t_w d_w}$$

where,

V_u is factored shear force

t_w is thickness of the web

d_w is effective depth of wall section (may be taken as 0.8 l_w for rectangular sections)

- The design shear strength of concrete (τ_c) is calculated as per IS 456 : 2000. And nominal shear stress in the wall (τ_v) should not exceed $\tau_{c\,max}$ (given as per IS 456 : 2000)
- If τ_v is less than τ_c then minimum shear reinforcement should be provided.
- If $\tau_v > \tau_c$, then area of horizontal shear reinforcement (A_h) is provided with a vertical spacing, S_v. given as

$$V_{us} = \frac{0.87f_yA_hd_w}{S_v}$$

where, $V_{us} = (V_u - \tau_c t_w d_w)$ is the shear force to be resisted by the horizontal reinforcement. However horizontal reinforcement should be greater than minimum reinforcement.

- The vertical reinforcement (uniformly distributed) in the wall should not be less than the horizontal reinforcement (as calculated above).

Summary

- India is classified into four seismic zones that is zone II, zone III, zone IV and zone V.
- Earthquake magnitude is measure of energy released in an earthquake. Whereas earthquake intensity is measure of strength of shaking and extent of damage caused during the earthquake.
- For noticeable earthquakes (strong ground motion) we opt accelerograph for measuring acceleration. The ground velocity and displacement can be obtained by integrating accelerogram.
- Seismic lateral forces are directly proportional to masses of structural elements.
- Twisting of a structure occurs when centre of mass does not coincide with centre of rigidity.
- A non-linear dynamic analysis or inelastic time history analysis is the best suitable method to describe the actual behaviour of a structure during on earthquake.
- The main differences between two elastic methods that is static method and response spectrum method are the base shear magnitude and lateral force distribution.
- In static method (seismic coefficient/equivalent lateral load method), base shear is calculated on the basis of structural mass, fundamental period of vibration and corresponding modal shape.
- In response spectrum method, calculation of forces is done on the basis of compound periods and mode shapes of various modes of vibration.
- Response reduction factor is used in the design to account for ductility, over strength and redundancy.
- The number to modes to be used in dynamic analysis should be such that the total sum of modal masses of all modes considered is atleast 90% of the total seismic mass.
- For flexural members, then tension steel ratio on any face, at any section, should not be less than $\rho_{min} = 0.24\frac{\sqrt{f_{ck}}}{f_y}$, where f_{ck} and f_y are in MPa.

Objective Brain Teasers

Q.1 The quantitative measure of earthquake is
(a) seismic response
(b) seismic intensity
(c) seismic magnitude
(d) seismic damage to structures

Q.2 The qualitative measure of an earthquake is
(a) seismic response
(b) seismic intensity
(c) seismic magnitude
(d) seismic damage to structures

Q.3 Accelerograph has frequency ratio (ω/ω_n) i.e., ratio of frequency of vibration of earthquake to natural frequency of accelerograph
(a) can have any value
(b) larger than 1
(c) smaller than 1
(d) equal to 1

Q.4 IS 1893 : 2002 (Part-I) address which effect of the earthquake
(a) Seismic loads on structures
(b) Flood caused due to seismicity
(c) Liquefaction of soil strata
(d) Landslides due to earthquake

Q.5 Acceleration response spectrum is a plot of
(a) Ground acceleration versus natural period
(b) Ground acceleration versus time
(c) Response acceleration versus time
(d) Response acceleration versus natural period

Q.6 In the design response spectra according to IS 1893 : 2002, the maximum value of response acceleration is
(a) 4.5 g (b) 1.5 g
(c) 3.0 g (d) 2.5 g

Q.7 In a response spectrum, the zone of small period ($T < 0.5$ s) is known as
(a) Displacement sensitive region
(b) Velocity sensitive region
(c) Acceleration sensitive region
(d) Depends upon response acceleration value

Q.8 In a response spectrum, the zone of large period ($T > 3$s) is known as
(a) Displacement sensitive region
(b) Velocity sensitive region
(c) Acceleration sensitive region
(d) Depends upon response acceleration value

Q.9 The zone factor is an reasonable estimate of
(a) Peak magnitude of earthquake
(b) Peak ground velocity
(c) Peak intensity of earthquake
(d) Peak ground acceleration

Q.10 Most rigid element in a structure receives
(a) Most of the seismic lateral load
(b) Least of the seismic lateral load
(c) Same seismic lateral load as the other elements
(d) Seismic lateral load whose value depends on other factors

Q.11 Moment resistant rigid joints in the structure will
(a) Increase the ductility
(b) Decrease the ductility
(c) No affect on the ductility
(d) May increase or decrease

Q.12 In the moment resistant rigid framing system, for good performance under seismic action, it is desirable to have
(a) Moment capacity of beam equal to that of the column
(b) Moment capacity of beam greater than that of the column
(c) Moment capacity of column greater than that of the beam
(d) Can have any arrangement of moment capacities of beams and columns

Q.13 Which structures has least lateral force resistance
(a) Ordinary moment-resisting RC frame
(b) Special moment-resisting RC frame
(c) Shear wall structure
(d) Coupled shear wall structure

Q.14 The base isolaters in structure
(a) Increase the strength of the structure
(b) Decrease the strength of the structure
(c) Decrease the natural period of the structure
(d) Increase the natural period of the structure

Q.15 The seismic weight for a floor with dead load of 5 kN/m^2 and live load of 4 kN/m^2 is
(a) 9 kN/m^2 (b) 7 kN/m^2
(c) 6 kN/m^2 (d) 5.5 kN/m^2

Q.16 The seismic weight for a floor with dead load of 4.8 kN/m^2 and live load of 1 kN/m^2 is
(a) 4.8 kN/m^2 (b) 5 kN/m^2
(c) 5.05 kN/m^2 (d) 5.8 kN/m^2

Q.17 Soft storey is one which has
(a) $k_i < 0.7\ k_{i+1}$ (b) $k_i > 0.7\ k_{i+1}$
(c) $k_i < 0.8\ k_{i+1}$ (d) $k_i > 0.8\ k_{i+1}$
Where, k_i is lateral stiffness of soft storey and k_{i+1} is lateral stiffness of storey above to it.

Q.18 In response spectrum method, the evaluation of storey shear is based on
(a) Normal mode
(b) First mode

(c) Number of modes having atmost 80% modal mass
(d) Number of modes having at-least 90% modal mass

Q.19 The acceleration response spectrum for RC building according to IS 1893 : 2002 value of critical damping is of
(a) 1% (b) 2%
(c) 3% (d) 5%

Q.20 With increase in redundancy, the response reduction factor
(a) Increases
(b) Decreases
(c) Unaltered
(d) May increase or decrease

Q.21 For given damping and natural period ($T_n > 0.4$s) the structures with foundations of soft soils have response acceleration
(a) greater than that for rocky foundations
(b) lesser than that for rocky foundations
(c) equal to that for rocky foundations
(d) can be greater or lesser than that for rocky foundations

Q.22 With increase in damping (>5%) of structure, the spectral acceleration coefficient (S_a/g) values will
(a) increase
(b) decrease
(c) unaltered
(d) may increase or decrease

Q.23 For short natural period range ($T_n < 0.4$ s), the spectral acceleration coefficient values are
(a) smaller for the soft soil site in comparison with the hard soil site
(b) smaller for the medium soil site in comparison with the hard soil site
(c) greater for the medium soil site in comparison with the hard soil site
(d) same for the soft, medium and hard rocky soil sites

Q.24 Through dynamic analysis on a 3-storey building; the lateral forces on three floors (from the top) are 10 kN, 40 kN, 25 kN. If base shear obtained by static method is 100 kN, then the revised lateral forces at storey levels (from the top) are
(a) 13.33, 53.33 and 33.33 kN
(b) 100, 100 and 100 kN
(c) 53.33, 13.33 and 33.33 kN
(d) 33.33, 33.33 and 33.33 kN

Q.25 The response reduction factors accounts for
(a) Ductility
(b) Redundancy
(c) Over strength of structure
(d) All of the above

Q.26 The definition of the weak storey mainly considers the
(a) mass (b) natural period
(c) strength (d) damping

Q.27 Ductility of a building is defined as
(a) Ratio of plastic strength to plastic deformation
(b) Ratio of plastic strength to elastic strength
(c) Ratio of plastic deformation to elastic defomation
(d) Ratio of elastic strength to plastic deformation

Q.28 Ductility of a structure
(a) increases damping
(b) increases deformation
(c) decreases seismic force
(d) All of the above

Q.29 Special confining reinforcement is
(a) Lateral reinforcement
(b) Longitudinal reinforcement
(c) Shrinkage reinforcement
(d) Temperature reinforcement

Q.30 In columns, the laps for reinforcement are desirable at
(a) the mid height (b) the ends
(c) at the joint (d) any section

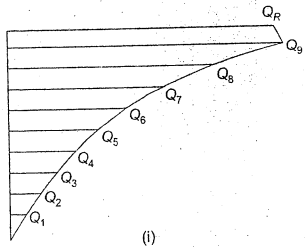
Q.31 For the continuous beam, the lap splices of top reinforcement should not be provided
(a) at the ends of span
(b) at mid span

(c) away from joints
(d) None of the above

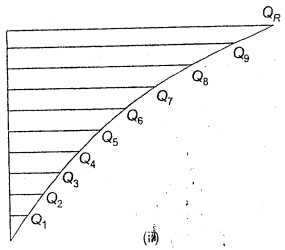
Q.32 For a column, the lap splices of main reinforcement should not be provided
(a) at the end (b) at mid height
(c) away from joints (d) None of the above

Q.33 The Indian Standard provides different expressions for the evaluation of fundamental natural periods of structures considering or neglecting the stiffness of brick infill. Considering the brick infill stiffness
(a) increases seismic forces
(b) decreases seismic forces
(c) may increase or decrease
(d) unaltered

Q.34 Which of following figure or figures illustrating distribution of forces along the height of 10-storey building may be correct?



(i)



(ii)

(a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) None of the above

Q.35 The RC hospital building having ordinary moment resisting frame has the following design parameters. For seismic zone III, zone factor is 0.16. Response acceleration coefficient is 2.5. The design horizontal seismic coefficient is
(a) 0.001 (b) 0.01
(c) 0.1 (d) 1

Q.36 The design horizontal seismic coefficient is 0.083 and seismic weight is 1020 kN. The design base shear is
(a) 63.88 kN (b) 75.61 kN
(c) 80.32 kN (d) 84.66 kN

Q.37 The regular three-storey building has following plan dimensions; 20 m x 20 m. Dead load on each floor is 5 kN/m^2 and dead load on roof is 4 kN/m^2. The live load on each floor is 3.5 kN/m^2 and live load on roof is 1 kN/m^2. The total seismic weight is
(a) 7000 kN (b) 7500 kN
(c) 9000 kN (d) 9800 kN

Q.38 For mode k:
Design horizontal acceleration spectrum is 0.04, mode shape coefficient at a floor is 0.5, modal participation factor is 0.2 and seismic weight on that floor is 1500 kN. Then peak lateral force at concerned floor in mode k is
(a) 5 kN (b) 6 kN
(c) 7 kN (d) 11 kN

Q.39 The peak storey shears acting on a storey in modes 1, 2 and 3 are 6 kN, –5 kN and 10 kN. The storey shear force due to all modes considered, using square root of sum of squares, is
(a) 9.5 kN (b) 10.2 kN
(c) 12.7 kN (d) 15.1 kN

Answers

1. (c)	2. (b)	3. (c)	4. (a)	5. (d)
6. (d)	7. (c)	8. (a)	9. (d)	10. (a)
11. (b)	12. (c)	13. (a)	14. (d)	15. (b)
16. (c)	17. (a)	18. (d)	19. (d)	20. (a)
21. (a)	22. (b)	23. (d)	24. (a)	25. (d)
26. (c)	27. (c)	28. (d)	29. (a)	30. (a)
31. (a)	32. (a)	33. (a)	34. (c)	35. (c)
36. (d)	37. (a)	38. (b)	39. (c)	

Hints and Explanations:

15. (b)
Seismic weight=Dead load + 0.5 × Live load
(Because percentage of imposed load is taken as 50% for uniformly distributed imposed floor loads greater than 3 kN/m²)
⇒Seismic weight = 5 + 0.5 × 4 = 7 kN/m²

16. (c)
Seismic weight=Deal load + 0.25 × Live load
(Because percentage of imposed load is taken as 25% for uniformly distributed imposed floor loads lesser and equal to 3 kN/m²)
Seismic weight = 4.8 + 1 × 0.25
= 5.05 kN/m²

20. (a)
Response reduction factor is used in design to account for redundancy, ductility and over strength.

24. (a)
Base shear (by dynamic analysis)
= 10 + 40 + 25 = 75 kN
V_B' (100 kN) > V_B (75 kN)
⇒ Storey forces (from top)

$$10\times\frac{100}{75} = 13.33\text{ kN}$$

$$40\times\frac{100}{75} = 53.33\text{ kN}$$

and $$25\times\frac{100}{75} = 33.33\text{ kN}$$

33. (a)
If brick infill stiffness is considered, then natural period reduces and consequently response acceleration (S_a/g) increases and hence the seismic forces increases.

34. (c)
For figure (i) comparatively less roof shear can be obtained if there is relatively less seismic load on it.

35. (c)

$$A_h = \frac{ZIS_a}{2Rg} = \frac{0.16\times1.5\times2.5}{2\times3} = 0.1$$

(For ordinary moment resisting frame, $R = 3$ and for hospital, $I = 1.5$)

36. (d)
The design base shear (V_B)
= $A_h W$ = 0.083 × 1020 = 84.66 kN

37. (a)
Floor area = 20 × 20 = 400 m²
Seismic weight of each floor
= (DL + 0.5 LL) × 400
= (5 + 0.5 × 3.5) × 400
= 6.75 kN/m² × 400
= 2700 kN
Seismic weight of roof is
= DL × 400
= 4 × 400
= 1600 kN
Total seismic weight
= 2 × 2700 + 1600
= 7000 kN

38. (b)

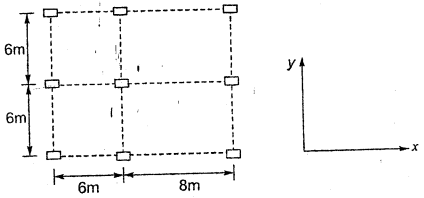
$$Q_{ik} = A_k \phi_{ik} P_k W_i$$
$$= 0.04 \times 0.5 \times 0.2 \times 1500$$
$$= 6\text{ kN}$$

39. (c)

$$V = \sqrt{6^2+(-5)^2+10^2} = 12.7\text{ kN}$$

Conventional Practice Questions

Q.1 The plan of a one storey building is shown below. All the columns and the beams have same cross-section. Then calculate centre of stiffness, taking bottom left corner as origin.



Solution:
In y-direction there are three identical frames located at equal spacing of 6 m. Thus the y-coordinate of centre of stiffness in located at 6 m from left bottom corner.
In x-direction, there are three identical frames having equal stiffness and unequal spacing. Thus x-coordinate of center of stiffness (assuming stiffness of each frame to be k) is

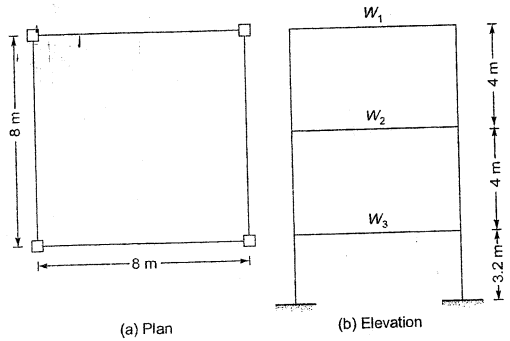
$$x = \frac{k\times0+k\times6+k\times14}{k+k+k} = \frac{20k}{3k} = 6.67\text{ m}$$

⇒ Center of stiffness coordinates are (6.67, 6)

Q.2 The plan and elevation of a RCC school building is shown in figure. The building is located in zone IV and founded on hard soil. It is proposed to design the building with special moment resisting frame. The seismic weights on the floor are W_1 = 650 kN, W_2 = 700 kN and W_3 = 700 kN, Determine the design seismic load by static analysis
For hard soil, average response acceleration coefficient is

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00\le T\le 0.10 \\ 2.50 & 0.10\le T\le 0.40 \\ 1.00/T & 0.40\le T\le 4.00 \end{cases}$$

where, T is,natural time period



(a) Plan (b) Elevation

Solution:

Static analysis: Equivalent lateral force method

Design parameters:

For seismic zone IV, zone factor (Z) = 0.24

Importance factors $I = 1.5$

Response reduction factor, $R = 5$ (For school)
(because of special moment resisting frame)

Fundamental period of vibration, $T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09\times11.2}{\sqrt{8}} = 0.356s$

$\Rightarrow$ Response acceleration coefficient $\left(\frac{S_a}{g}\right)$ for hard soil = 2.50 (for T_a = 0.356 s)

Seismic weight:

Total seismic weight of the building,

$$W = \sum W_i = 650+700+700 = 2050 \text{ kN}$$

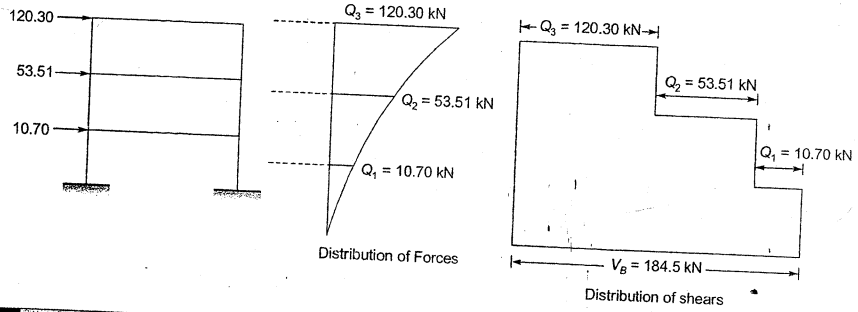
Thus, design horizontal seismic coefficient,

$$A_h = \left(\frac{Z}{2}\right)\left(\frac{I}{R}\right)\left(\frac{S_a}{g}\right) = \frac{0.24}{2}\times\frac{1.5}{5}\times2.5 = 0.09$$

$\Rightarrow$ Design base shear, $V_B = A_h W = 0.09 \times 2050 = 184.5$ kN

The lateral force distribution along the height of building is given in table and figure below:

Storey level	W_i (kN)	h_i (in m from the base)	$W_i h_i^2$	$\frac{W_i h_i^2}{\sum_{i=1}^{3} W_i h_i^2}$	$Q_i = V_B \frac{W_i h_i^2}{\sum_{i=1}^{3} W_i h_i^2}$
3	650	11.2	81536	0.652	120.30
2	700	7.2	36288	0.29	53.51
1	700	3.2	7168	0.058	10.70
			$\sum_{i=1}^{3} W_i h_i^2 = 124992$		$\sum_{i=1}^{3} Q_i = 184.5$kN

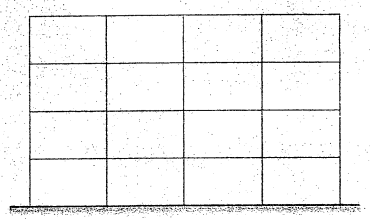


Q.3 The RCC having ordinary moment resisting frame building is shown in figure below. Importance factor =1, loads on the floors: dead load is 7 kN/m^2 on each floor and 6 kN/m^2 on roof; live load is 3 kN/m^2 each floor and 1.5 kN/m^2 on roof. The storey heights are: ground storey is 4.2 m, first, second and third storey are 3.2 m. The building frame is of four bays of each 4 m in both directions and located in zone V, founded on hard soil. The plan dimensions of the building are 16 m × 16 m and the total height of the building is 13.8 m. The damping in the structure is 10%. Determine the seismic forces by static method. Neglect stiffness of brick masonry in fill.

For hard soil, average response accleration coefficient is

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.40 \\ 1.00/T & 0.40 \le T \le 4.00 \end{cases}$$

where T is natural time period in seconds.



Solution:

Design Parameters:

Importance factor, $I = 1$

Z (zone V) = 0.36

Response reduction factor, $R = 3$ (because of ordinary moment resisting frame)

Natural period, $T_n = 0.075\, h^{0.75}$

(because stiffness of brick masonry in fill is neglected)

$= 0.075\,(13.8)^{0.75}$

$= 0.537$ seconds

$$\frac{S_a}{g} \text{ (for hard soil)} = \frac{1}{T_n} = 1.862 \quad \text{(For } T_n > 0.4 \text{ s)}$$

Multiplication factor for 10% damping = 0.8

$\Rightarrow$ $$A_h = \frac{ZIS_a}{2Rg} = \frac{0.36\times1\times(1.862\times0.8)}{2\times3} = 0.089$$

Total Seismic Weight:

Floor area = 16 × 16 = 256 m^2

For floor, 25% of live load is to be considered and for roof, no live load need to be considered for the evaluation of seismic weight.

Seismic weight of each floor = $A \times W_i$

$= 256 \times (7 + 0.25 \times 3)$

$= 1984$ kN

Seismic weight of roof $= 256 \times 6 = 1536$ kN

Total seismic weight, $W = 3 \times 1984 + 1536 = 7488$ kN

The base shear is

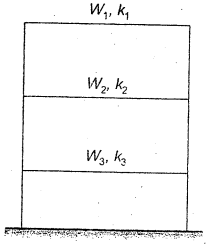
$$V_B = A_h W = 0.089 \times 7488 = 666.432 \text{ kN}$$

Computation of lateral forces at each storey:

Storey	Weight, W_i (kN)	h_i (m) from the base	$W_i h_i^2$	$\frac{W_i h_i^2}{\sum W_i h_i^2}$	$Q_i = V_B \frac{W_i h_i^2}{\sum W_i h_i^2}$
4	1536	13.8	292516	0.44	293.23
3	1984	10.6	222922	0.34	226.59
2	1984	7.4	108644	0.165	109.96
1	1984	4.2	34998	0.055	36.65
$\sum$	7488		659080		666.43

Therefore the base shear (V_B') = 666.43 kN and the lateral forces at the storey levels are $Q_4 = 293.23$ kN, $Q_3 = 226.59$ kN, $Q_2 = 109.96$ kN and $Q_1 = 36.65$ kN.

Q.4 A residential RCC (special moment resisting frame) building frame is shown in figure. The seismic weights are $W_1 = 290$ kN, $W_2 = 1860$ kN and $W_3 = 1050$ kN. The storey stiffness are $k_1 = 98000$ kN/m, $k_2 = 98000$ kN/m and $k_3 = 44000$ kN/m. The storey heights are ground storey = 4 m and first and second storey = 3.2 m. The building is founded on hard soil strata and situated in zone IV. The free vibration results are: frequencies (ω) = (10.02 40.20 64.18) rad/s and mode shapes: (ϕ_1) = (1.00 0.95 0.75), (ϕ_2) = (1.00 0.50 −1.30) and (ϕ_3) = (1.00 −0.25 0.08). Determine the seismic force by dynamic analysis method. For hard soil, average response accleration coefficient is



$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.40 \\ 1.00/T & 0.40 \le T \le 4.00 \end{cases}$$

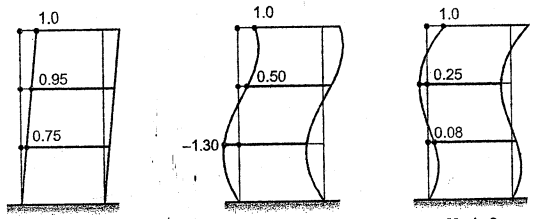
where T is natural time period, in seconds

Solution:

Dynamic analysis: Response spectrum method

Free vibration results for the structure

Natural period (s) $\left(\frac{2\pi}{\omega}\right)$	Mode 1	Mode 2	Mode 3
	0.627	0.156	0.098
	Modal shapes		
Roof	1.0	1.0	1.0
Second floor	0.95	0.5	−0.25
First floor	0.75	−1.30	0.08



Mode Shapes

Computation of model mass and model participation factors

Storey	Weight W_i (kN)	Mode, k = 1			Mode, k = 2			Mode, k = 3		
		ϕ_1	$W_i\phi_1$	$W_i\phi_1^2$	ϕ_2	$W_i\phi_2$	$W_i\phi_2^2$	ϕ_3	$W_i\phi_3$	$W_i\phi_3^2$
Roof	290	1.0	290	290	1.0	290	290	1.00	290	290
Second Storey	1860	0.95	1767	1679	0.50	930	465	−0.25	−465	116
First Storey	1050	0.75	788	591	−1.30	−1365	1775	0.08	84	7
Summation (Σ)	3200		2845	2560		−145	2530		−91	413
Modal mass (M_k)		$\frac{2845^2}{2560 \times g} \times 1000 = 322297$ kg			847 kg			2044 kg		
% of Total weight		$\frac{322297 \times 9.81}{3200 \times 1000} \times 100 = 98.8\%$			0.26%			0.63%		
Modal participation factor (P_k)		$\frac{2845}{2560} = 1.11$			−0.057			−0.22		

Note:

The formula used is, Modal mass $(M_k) = \frac{\left[\sum (W_i \phi_{ik})\right]^2}{g \sum W_i \phi_{ik}^2}$

Mass participation factor, $(P_k) = \frac{\sum (W_i \phi_{ik})}{\sum W_i \phi_{ik}^2}$

Computation of horizontal acceleration coefficient:

Mode	k=1	k=2	k=3	
Natural period, T_a	0.627	0.156	0.098	
S_a/g	1.595	2.5	2.47	
Z	0.24	0.24	0.24	Zone IV
I	1	1	1	Residential Building
R	5	5	5	Special moment resistant building
$A_k = \left(\frac{Z}{2}\right)\left(\frac{I}{R}\right)\left(\frac{S_a}{g}\right)_k$	0.038	0.06	0.059	For each mode

Computation of lateral loads by SRSS modal combinations

Storey	Weight W_i (kN)	Mode, k = 1			Mode, k = 2			Mode, k = 3			SRSS Method
		ϕ	Q_{i1}	V_{i1}	ϕ	Q_{i2}	V_{i2}	ϕ	Q_{i3}	V_{i3}	$V_i = \left(\sum V_{ik}^2\right)^{1/2}$
Roof	290	1.0	12.232	12.232	1.0	–0.992	–0.992	1.00	–3.764	–3.764	12.84
Second Storey	1860	0.95	74.532	86.764	0.50	–3.181	–4.173	–0.25	6.036	2.272	86.89
First Storey	1050	0.75	33.217	119.981	–1.30	4.668	0.495	0.08	–1.09	1.182	120.00

For calculation we used,

$$Q_{ik} = A_{hk}\phi_{ik}P_k W_i$$

$$V_{ik} = \Sigma Q_{ik}$$

where, Q_{ij} is the peak lateral force of floor i in k^{th} mode and V_{ik} is the peak shear force acting in storey i in k^{th} mode.

Calculation:

Roof:

Mode 1: $Q = 0.038 \times 1 \times 1.11 \times 290 = 12.232$ kN

Mode 2: $Q = 0.06 \times 1 \times (-0.057) \times 290 = -0.992$ kN

Mode 3: $Q = 0.059 \times 1 \times (-0.22) \times 290 = -3.764$ kN

Second Storey

Mode 1: $Q = 0.038 \times 0.95 \times 1.11 \times 1860 = 74.532$ kN

Mode 2: $Q = 0.06 \times 0.5 \times (-0.057) \times 1860 = -3.181$ kN

Mode 3: $Q = 0.059 \times (-0.25) \times (-0.22) \times 1860 = 6.036$ kN

First Storey:

Mode 1: $Q = 0.038 \times 0.75 \times 1.11 \times 1050 = 33.217$ kN

Mode 2: $Q = 0.06 \times (-1.30) \times (-0.057) \times 1050 = 4.668$ kN

Mode 3: $Q = 0.059 \times 0.08 \times (-0.22) \times 1050 = -1.09$ kN

However only the first mode (modal mass = 98.8%) is sufficient, as it is greater than 90%; we have considered all the three modes and combined them by SRSS rule for illustration.

The combined individual contribution of each mode is (shown in figure below)

$$V_1 = \sqrt{12.232^2 + (0.992)^2 + 3.764^2} = 12.84 \text{ kN}$$

$$V_2 = \sqrt{86.764^2 + 4.173^2 + 2.272^2} = 86.89 \text{ kNm}$$

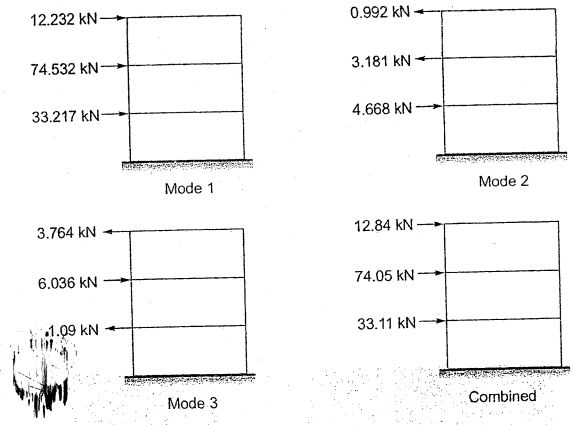
$$V_3 = \sqrt{119.981^2 + 0.495^2 + 1.182^2} = 120 \text{ kN}$$

The base shear, V_B is 120 kN and the lateral forces at the storey levels are

$$Q_1 = V_1 = 12.84 \text{ kN}$$

$$Q_2 = (V_2 - V_1) = (86.89 - 12.84) = 74.05 \text{ kN}$$

$$Q_3 = (V_3 - V_2) = (120 - 86.89) = 33.11 \text{ kN}$$



NOTE

As per IS 1893 : 2002, the base shear obtained by dynamic analysis (V_B = 119.98 kN) need to be compared with the base shear obtained by static method (V_B'). If $V_B' > V_B$ then response quantities will be scaled up by a factor (V_B'/V_B).

Suppose $\frac{V_B'}{V_B} = \alpha$ (Numerical value)

Then, $Q_1 = 12.84\,\alpha$ kN

$Q_2 = 74.03\,\alpha$ kN

$Q_3 = 33.11\,\alpha$ kN

From static analysis

Let say plan area of building is 8 m × 8 m

Natural period, $T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 10.4}{\sqrt{8}} = 0.331s$

Response acceleration coefficient, $\frac{S_s}{g} = 2.5$

Horizontal seismic coefficient, $A_h = \frac{Z}{g}\frac{I}{R}\frac{S_a}{g} = \frac{0.24 \times 1 \times 2.5}{2 \times 5} = 0.06$

Base shear, $V_B' = A_h W = 0.06 \times 3200 = 192$ kN

So, $a = \frac{V_B'}{V_B} = \frac{192}{120} = 1.6$

APPENDIX A

Masonry Design

Introduction

Masonry in general is defined as any structural assemblage of masonry units like stones, bricks, blocks etc. with a binding material which is known as mortar. The walls of the masonry building and the building itself are designed to be stable, strong and durable enough to withstand the most severe combination of loads called as design load. Masonry construction is flexible in terms of building layout and is quite economical, provides thermal and acoustic insulation and as well as it is fire and weather resistant. Masonry units that have been previously used shall not be reused in brickwork or block work construction unless they are thoroughly cleaned and conform to relevant code. The minimum compressive strength of masonry units to be used in reinforced masonry shall be 7 N/mm^2. But the strength of bricks in India varies from region to region depending on the nature of local soil, technique adopted in moulding and burning.

NOTE

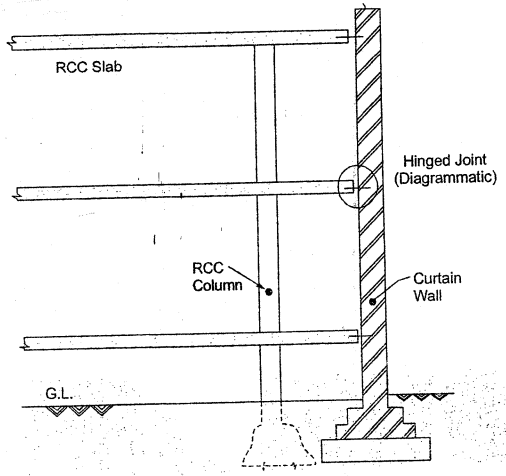
At present, in India, the IS code for masonry construction is **IS 1905:1987 (reaffirmed 1998) "Structural Use of Unreinforced Masonry"**. The corresponding detailed hand book on masonry design and construction is **SP 20**. The IS code for **Structural Use of Reinforced Masonry** is under preparation. **IS 4326** provides some guidelines for construction of reinforced masonry. **IS 13828:1998** provides guideline for earthquake resistant low strength masonry buildings.

Terminologies

Bed Block: A block bedded on a wall, a column or a pier in order to disperse a concentrated load on a masonry element.

Cavity wall: It is the wall consisting of two leaves with each leaf separated by a cavity and ties together with metal ties or the bonding units in order to ensure that the two leaves act as one unit. The space between the two leaves is either left free as a continuous cavity or filled with non-load bearing insulating or water proofing material.

Curtain wall: It is a non load bearing wall subjected to lateral loads only. It may be laterally supported by vertical or horizontal structural members.



Curtain Wall

ECB: Engineered Concrete Block

Effective height: It is the height of wall or column which is required for computing the slenderness ratio.

EMU: Engineered Masonry Unit – These are special units for architectural purpose (colour, shape, texture etc.), physical purpose (density etc.), structural requirement (strength, elasticity, durability etc.).

Free Standing Wall: It is the compound wall or parapet wall. It is acted upon by wind force which tends to over turn it. This tendency to overturning is resisted by gravity force due to self weight of wall, and also by flexural moment of resistance on account of tensile strength of masonry.

Grout: It the mixture of cement (or any other binding material), sand and water with a pourable consistency for filling the voids.

Grouted cavity reinforced masonry: Two parallel single leaf walls spaced at least 50 mm apart effectively tied together with wall ties. The cavity contains steel reinforcement and is filled with concrete so as to have a common action with masonry under load.

Grouted hollow masonry unit: It is that form of grouted masonry construction wherein certain designated cells of hollow units are continuously filled with grout.

Grouted multi wythe unit: It is that form of grouted masonry construction wherein certain the space between the wythes is solidly or periodically filled with grout.

HCB: Hollow Concrete Block – A masonry unit in which net cross sectional area in any plane parallel to the bearing surface is less than 75% of its gross cross sectional area measured in the same said plane.

Leaf: Inner or outer section of a cavity wall is called as leaf.

Load bearing wall: It is the wall which is designed to carry an imposed vertical load in addition to its own self weight along with lateral loads, if any. The following table gives maximum slenderness ratio for reinforced load bearing wall.

Table: *Maximum slenderness ratio for reinforced load bearing wall*

End Condition	Ratio of span to effective depth
Simply supported	35
Continuous	45
Spanning in two directions	45
Cantilever	18

Masonry unit: Individual units bonded together with mortar to form a masonry element like column, wall etc.

Panel wall: It is the exterior non-load bearing wall in framed constructions wholly supported at each storey and subjected to lateral loads in an out plane direction like wind loads, earthquake etc.

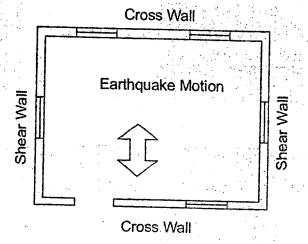
Partition wall: It is an interior non-load bearing wall which is one storey or part storey in height.

Prism: It is an assemblage of masonry units bonded by mortar with or without grout used as a test specimen for determining the properties of masonry (with height to thickness ratio of 2 to 5).

RM: Reinforced masonry

SCB: Solid Concrete Block

Shear wall and cross wall: It is the wall designed to carry the horizontal forces acting in its plane with or without vertical imposed loads. The walls normal to shear walls are referred to as cross walls.



SMB: Stabilized Mud Block

Specified compressive strength of masonry: It is the minimum compressive strength expressed as force per unit net cross sectional area required in the masonry to be used in construction.

TMB: Table Moulded Brick

URM: Unreinforced masonry

Wall tie: It is a metal fastener which connects wythes of masonry to each other or to other materials.

WCB: Wire Cut Brick

Wythe: It is the continuous vertical tie of one masonry unit in thickness.

Masonry Reinforcement

For general load bearing masonry construction, Fe 415 steel is quite acceptable. But for earthquake resistant masonry design, a variety of reinforcement is used.

Table: *Reinforcement specification in load bearing construction*

Tensile Strength	
MS Bars confirming to IS 432 (Part I)	140 MPa for diameter ≤ 20 mm 130 MPa for diameter > 20 mm
HYSD Bars (IS 1786)	230 MPa
Compressive Strength	
Size and spacing of reinforcement	130 MPa
The maximum size of reinforcement used in masonry shall be 25 mm diameter bars and minimum size shall not be less than 5 mm.	

Remember

The reinforcement diameter shall not exceed one-half the least clear dimension of the cell, bond beam or collar joint in which it is placed. Clear distance between parallel bars shall not be less than the diameter of bar or 25 mm whichever is more. In columns, clear distance between the vertical bars shall not be less than 1.5 times the bar diameter nor shall be less than 35 mm.

Design Loads

Loads to be considered in the design of masonry structures are:

1. Dead load of walls, columns, floors and roofs as per **IS 875 (Part-I)**.
2. Live load on floors and roof as per **IS 875 (Part-II)**.
3. Wind loads on walls and sloping roofs as per **IS 875 (Part-III)**.
4. Seismic loads as per **IS 1893 (Part-I)** and snow loads as per **IS 875 (Part-IV)**.

Load Combinations

In structural design of masonry structures, the most commonly adopted design philosophy is the allowable stress design method. As per this design method, the structure shall be designed for the following load combinations:

1. DL + IL
2. DL + IL + WL (or EQL)
3. DL + WL
4. 0.9DL + EQL

DL: Dead Load, **IL:** Imposed Load, **WL:** Wind Load, **EQL:** Earthquake Load

Permissible Loads and Stresses

In the design load combination wherein wind load (or earthquake load) is being considered, there permissible stresses may be increased by 33.33%.

Alternatively, instead of increasing the permissible stresses, we can use 25% reduced load for load combinations which involves wind load (or earthquake load) and take into account the full permissible stresses. Thus the modified design load combinations for loads involving wind load (or earthquake load) are:

1. 0.75(DL + IL + WL (or EQL))
2. 0.75 (DL + WL)
3. 0.75 (0.9DL + EQL)

Effective Height of Walls

It is the height of wall or column which is required for computing the slenderness ratio.

Table: *Effective height of wall*

S.No.	Condition of Supports	Effective Height
1.	Lateral as well as rotational restraint (that is, full restraint) at top and bottom. For example, when the floor/roof spans on the walls so that reaction to load of floor/roof is provided by the walls, or when an RCC floor/roof has bearing on the wall (minimum 9 cm), irrespective of the direction of the span (foundation footings of a wall give lateral as well as rotational restraint).	0.75 H
2.	Lateral as well as rotational restraint (that is, full restraint) at one end and only lateral restraint (that is, partial restraint) at the other. For example, RCC floor/roof at one end spanning or adequately bearing on the wall and timber floor/roof not spanning on wall, but adequately anchored to it, on the other end.	0.85 H
3.	Lateral restraint, without rotational restraint (that is, partial restraint) on both ends. For example, timber floor/roof, not spanning on the wall but adequately anchored to it on both ends the wall, that is, top and bottom.	1.00 H
4.	Laterai restraint as well as rotational restraint (that is, full restraint) at bottom but have no restraint at the top. For example, parapet walls with RCC roof having adequate bearing on the lower wall, or a compound wall with proper foundation on the soil.	1.50 H

Where,

H = Height of wall between centres of support in case of RCC slabs and timber floors.

In case of footings or foundation block, height (H) is measured from top of footing or foundation block.

In case of roof truss, height (H) is measured up to bottom of the tie beam.

In case of beam and slab construction, height (H) should be measured from centre of bottom slab to centre of top beam.

Effective Height of Masonry between the Openings

When openings occur in a wall such that masonry between the openings is by definition a column shall be reckoned as follows:

(a) When wall has full restraint at the top:

1. Effective height for the direction perpendicular to the plane of the wall equals 0.75 H plus 0.25 H_1, where H is the distance between supports and H_1 is the height of the tallest openings; and
2. Effective height for the direction parallel to the wall equals H, that is, the distance between the supports.

(b) When wall has partial restraint at the top:

1. Effective height for the direction perpendicular to plane of wall equals H when height of neither opening exceeds 0.5 H and it is equal to 2 H when height of any opening exceeds 0.5 H; and
2. Effective height for the direction parallel to the plane of the wall equals 2 H.

Effective Length of Walls

The effective length of masonry wall is computed as per the table below:

Table: *Effective length of wall*

S.No.	Conditions of Support	Effective Length
1.	Where a wall is continuous and is supported by cross wall, and there is no opening within a distance of H/8 from the face of cross wall. or Where a wall is continuous and is supported by piers/buttresses.	0.8 L
2.	Where a wall is supported by a cross wall at one end and continuous with cross wall at other end. or Where a wall is supported by a pier/buttress at one end and continuous with pier/buttress at other end.	0.9 L
3.	Where a wall is supported at each end by cross wall or Where a wall is supported at each end by a pier/buttress.	1.0 L
4.	Where a wall is free at one end and continuous with a cross wall at the other end. or Where a wall is free at one end and continuous with a pier/buttress at the other end.	1.5 L
5.	Where a wall is free at one end and supported at the other end by a cross wall. or Where a wall is free at one end and supported at the other end by a pier/buttress.	2.0 L

Where,

$H \rightarrow$ Actual height of wall between centres of adequate lateral suppport

$L \rightarrow$ Length of wall from or between centres of cross wall, piers or buttresses

NOTE: In case there is an opening taller than 0.5 H in a wall, ends of the wall at the opening shall be considered as free.

APPENDIX B

Objective Brain Teasers

Q.1 In case of brick masonry
(a) mortar strength should be more than the brick strength
(b) mortar strength should be less than the brick strength
(c) mortar strength should be equal to the brick strength
(d) mortar strength and brick strength are not at all related to each other

Q.2 Consider the following statements:
1. Masonry containing rich cement mortar although is having good strength but is more prone to surface cracks.
2. Lime mortar has poor workability and also has high shrinkage.
3. Masonry with lime mortar has better resistance against rain penetration and is less prone to crack as compared to masonry with cement mortar.

Which of the above statements are true?
(a) 1 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

Q.3 It is required to protect the brick masonry walls and columns of a building from earthquake. For this, earthquake proofing is done by providing.
(a) less openings
(b) steel band at corners above windows below ceiling
(c) cross walls
(d) shear walls

Q.4 Mortar containing both lime and cement is called as
(a) cement mortar
(b) lime mortar
(c) light weight mortar
(d) gauged mortar

Q.5 The major drawback of lime mortar is that it
(a) does not set quickly
(b) swells
(c) is plastic
(d) is not durable

Q.6 Consider the following statements with respect to cement mortar is brick
1. In most of the cases, cracking in brick masonry is due to differential movement of structure.
2. Rich mix cement mortar is brick masonry makes the structure too rigid.
3. A small amount of hydrated lime in cement mortar reduces shrinkage cracks.

Which of the above statements are correct?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.7 Consider the following statements:
1. Strength of brick masonry depends on type of mortar used.
2. Brick masonry with lime mortar gains full strength earlier than cement mortar masonry.
3. Mortar strength decides the strength of masonry.

Which of the above statement(s) is/are true?
(a) 1 and 3 (b) 1 only
(c) 1, 2 and 3 (d) 2 only

Q.8 As per IS code, the minimum wall thickness of any load bearing wall in case of public buildings should be less than
(a) $\frac{1}{2}$ brick (b) 2 brick
(c) $1\frac{1}{2}$ brick (d) 1 brick

Q.9 According to masonry code, the stiffening coefficient for walls stiffened by piers, buttresses or intersecting walls can be
(a) 0 to 1 (b) > 2
(c) 1 to 2 (d) equal to 1

Q.10 If effective length, effective height and effective thickness of a masonry wall are l, h and t respectively, then slenderness ratio of wall should be
(a) Larger of $\frac{l}{t}$ and $\frac{h}{t}$
(b) Smaller of $\frac{l}{t}$ and $\frac{h}{t}$
(c) $\frac{l}{h}$
(d) $\frac{h}{t}$

Answers

1. (c)	2. (a)	3 (b)	4. (d)	5. (a)
6. (b)	7. (a)	8. (d)	9. (c)	10. (b)

